THE APPLICATION OF PARTICLE SWARM OPTIMIZATION TO ADAPTIVE IIR PHASE EQUALIZATION

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ABSTRACT

This paper investigates the application of particle swarm optimization techniques to infinite impulse response (IIR) adaptive phase equalizers. Particle swarm optimization (PSO) is similar to the genetic algorithm (GA) in that it utilizes a population based search suitable for optimizing multimodal error surfaces where gradient-based algorithms tend to fail, such as those generated by IIR adaptive filters. This paper will investigate PSO for the phase equalization of minimum phase surface acoustic wave (SAW) filters used in CDMA receivers.

1. INTRODUCTION

Adaptive IIR phase equalizers are useful in many communications and acoustical channel equalization problems because they can provide a better approximation of a long impulse response using a fewer number of coefficients compared to FIR filters. IIR filters have a nonlinear phase response, but can be configured to have an all-pass characteristic where the magnitude is unaffected. This property makes IIR all-pass filters ideal for nonlinear phase equalization applications.

The drawback to IIR adaptive filter structures is that they tend to generate error surfaces that are multimodal. When the error surface is multimodal, local optimization techniques that work well for FIR adaptive filters, such as versions of gradient descent algorithms, are not suitable because they are likely to get trapped in a local minimum solution.

An alternative to a gradient-based optimization technique is a structured stochastic search of the error space. These types of global searches are structure independent because a gradient is not calculated and the adaptive filter structure does not directly influence the parameter updates – aside from the error computation. Due to this property, these types of algorithms are potentially capable of globally optimizing any class [4] of adaptive filter structures or objective functions. Several structured stochastic search approaches have appeared in the IIR adaptive filtering literature, most notably simulated annealing [6] and evolutionary algorithms such as the GA [6,7,10]. PSO is another structured stochastic search algorithm that has recently gained popularity for optimization problems.

This paper will discuss particle swarm optimization techniques for adaptive all-pass IIR filters with application to the motivating example of the phase equalization of a minimum phase surface acoustic wave (SAW) filter.

1.1 Phase Equalization for SAW Filters

Surface acoustic wave (SAW) filters are commonly used in the RF and IF stages of mobile communications devices because of their linear frequency response over the pass-band and high stop-band attenuation. The drawback to SAW filters is that their frequency response relies on their physical characteristics, which cannot be further miniaturized to keep up with the ever-shrinking mobile communications devices. However, the physical size of SAW filter packages can be minimized without compromising their frequency response by performing some compensation through adaptive signal processing.

This decrease in physical size can be achieved by using minimum phase SAW filters, which are a more compact alternative to the larger linear phase SAW filters commonly used in communications devices. Though these minimum phase SAW filters exhibit the shortest possible propagation delays, their phase response is nonlinear and fails to meet the IS-95-A CDMA specifications. This nonlinear phase characteristic can be compensated for with an adaptive all-pass IIR phase equalizer [11].

1.2 Adaptive IIR Phase Equalizers

The general structure of a multiple pole/zero adaptive all-pass phase equalizer is given as follows:

$$H(z) = \prod_{n=1}^{N} \frac{a_n^* + z^{-1}}{1 - a_n z^{-1}} \tag{1}$$

where * denotes the complex conjugate.

By cascading an all-pass adaptive after a minimum phase SAW filter, the non-linear phase response of the SAW filter can be adaptively compensated without altering the magnitude response. It has been shown [11] that a single first order stage (N=1) of the adaptive IIR equalizer given by equation (1) is capable of providing enough non-linear phase compensation to meet the IS-95-A standards, but higher order IIR filters can further improve the performance of the system. Reference [11] considered only gradient based approaches, specifically the decision directed (DDA) and the constant modulus algorithms (CMA). Though these algorithms have demonstrated success with a single first-order stage, they often encounter local minima that will become increasingly limiting when the filter order is increased.

Some work has been done to eliminate the local minimum problem in similar higher order SAW filter phase equalizers using the simulated annealing and the genetic algorithm [2,8]. Particle swarm optimization is a novel algorithm that demonstrates several advantages over SA and the GA for adaptive IIR filtering [4], which especially can be exploited for higher order all-pass phase equalization.

2. PARTICLE SWARM OPTIMIZATION

Particle swarm optimization was first developed in 1995 by Eberhart and Kennedy [3], rooted on the notion of swarm intelligence of insects, birds, etc. The swarm of particles represents multiple parameter estimates, analogous to the population of individuals in the GA. The standard PSO algorithm begins by initializing a random swarm of M particles, each having N unknown parameters to be optimized. At each iteration, the fitness of each particle is evaluated according to the selected fitness function. The algorithm stores and progressively replaces the most fit parameters of each particle ($pbest_i$, $i=1,2,...,M$) as well as a single most fit particle (*gbest*) as better fit parameters are encountered. The parameters of each particle (p_i) in the swarm are updated at each iteration (n) according to the following equations:

$$\overline{vel}_i(n) = w * \overline{vel}_i(n-1) + acc_1 * diag\left[e_1, e_2, ..., e_N\right]_{i1} * (gbest - p_i(n-1)) + acc_2 * diag\left[e_1, e_2, ..., e_N\right]_{i2} * (pbest_i - p_i(n-1)) \tag{2}$$

$$p_i(n) = p_i(n-1) + \overline{vel}_i(n) \tag{3}$$

where

$\overline{vel}_i(n)$ =velocity vector of particle i
e_r=random values $\in$ (0,1)
acc_1=acceleration coefficient toward *gbest*
acc_2=acceleration coefficient toward $pbest_i$
w=inertia weight

The trajectory of each particle is influenced in a direction determined by the previous velocity and the location of *gbest* and $pbest_i$. The two acceleration coefficients combined form what is analogous to the step size of an adaptive algorithm. Small acceleration coefficients tend to give a better search with slower convergence, while larger coefficients give a lesser search and faster convergence. The random e_i vectors provide the randomness of the step between *gbest* and $pbest_i$. The inertia weight controls the influence of the previous velocity. A single particle update is graphically illustrated in two dimensions in Figure 1. The new particle coordinates can lie anywhere within the bounded region, depending upon the weights and random components associated with each vector.

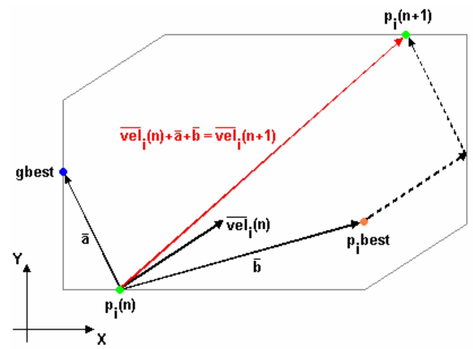

Fig. 1 Possible search region for a single particle

As new *gbests* are encountered during the update process, all other particles begin to swarm toward the new *gbest*, continuing to search along the way. The search regions continue to constrict as new $pbest_i$*s* are encountered. The algorithm is terminated when all of the particles in the swarm have converged to *gbest* or a suitable minimum error condition is met.

For adaptive IIR phase equalization, the fitness function is chosen as the CMA error-generating function. This error metric is chosen because a training signal is not available for the error formulation in adaptive phase equalization but the constant modulus is known for digital signal transmissions regardless of phase or polarity. This property makes it an effective metric for tracking time-varying phase. The general CMA error-generating function for IIR adaptive phase equalization is as follows:

$$J_i(n)_{CMA} = \frac{1}{K}\sum_{k=0}^{K}\left[\left(\delta^r - \left|y_i(n-k)\right|^r\right)^q\right] \tag{4}$$

$$Y_i(z) = \left[\prod_{m=1}^{N} \frac{p_{i,m}^* + z^{-1}}{1 - p_{i,m} z^{-1}}\right] X(z) \tag{5}$$

where N *is* the order of the all-pass filter, K is the length of the window over which the error is averaged, the p_i's are the tap weights of each particle, and q and r are integer parameters. When $J(n)$ is minimized, the coefficients provide the optimum phase equalization for the given filter. A more detailed description of PSO for general IIR and nonlinear adaptive filtering, as well as suggested modifications and a comparison to the GA is outlined in a previous paper [4].

3. SIMULATIONS

The experimental system consists of a basic CDMA communication system model. The transmitted signal is an IS-95-A signal, which consists of a bipolar data stream spread by two pseudorandom (PR) bipolar sequences; one forms the in-phase (real) channel, while the other forms the quadrature (imaginary) channel. The transmitter consists of a length 48 pulse shaping filter, matched to the SAW filter, obtained from [5]. These coefficients were interpolated by a factor of 4 to obtain 196 coefficients, in order to correct for the up-sampled pulses in the bit stream.

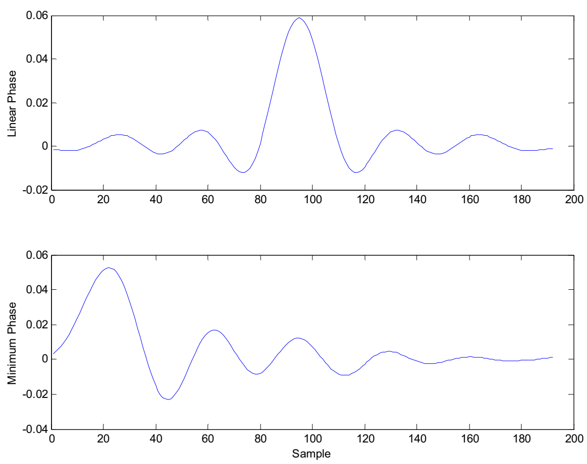

Fig. 2. Impulse responses of the linear and minimum phase pulse shaping filters, matched to the SAW filters

The transmitted in-phase and quadrature signals each pass through the same channel model before reaching the receiver. The simple channel model adds independent complex additive white Gaussian noise with zero mean. The receiver consists of the minimum-phase SAW filter, given in Figure 2. The long analog baseband filter in the receiver is modeled by a linear-phase FIR filter, of order 201, also obtained from [5].

A modified PSO (MPSO) algorithm [4] was used to update the IIR phase equalization filter that directly follows the cascade of the minimum phase SAW filter and the baseband filter. MPSO uses the standard PSO algorithm as a base, with several added modifications to improve efficiency that involve re-randomization and mutation of the particles, detailed in [4].

Since the signal is bipolar, the modulus of equation (4) is chosen to be $\delta=1$, and $r=q=2$. Both PSO and MPSO are initialized with the same population of real-valued parameters $\in$ (-0.5,1.5) and allowed to evolve.

3.1 First-order IIR Phase Equalizer

In this example, a first-order (N=1) IIR equalizer trained with the MPSO algorithm. The acceleration constants were selected to be 0.5 in all cases. The CMA error is plotted with respect to the single filter coefficient in Figure 3. The convergence plots of the filter coefficients, averaged over 50 trails, is given in Figure 4 for different swarm sizes (M=10, 20, 30). The straight line at the top of Figure 4 indicates the optimal solution, given by the global minimum in Figure 3.

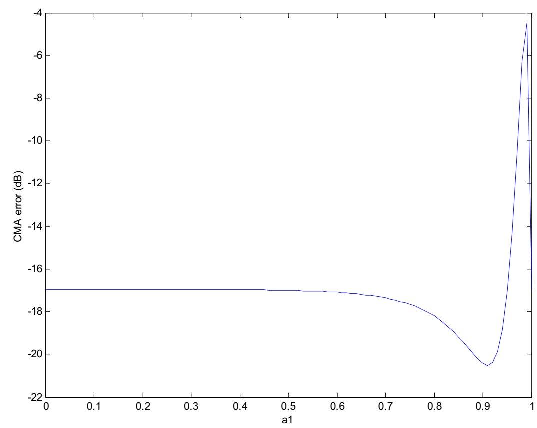

Fig. 3. Error surface of the first-order phase equalizer

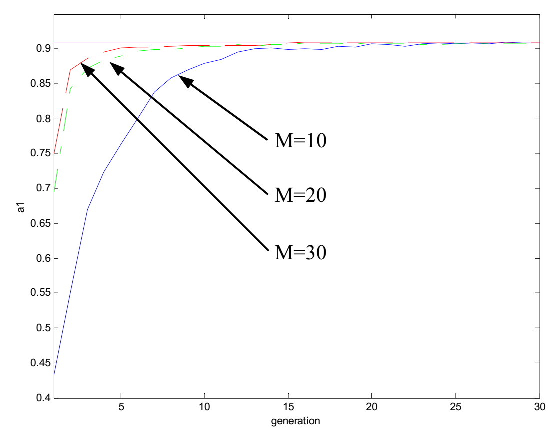

Fig. 4. Coefficient learning curves of different swarm sizes for the first-order phase equalizer

3.2 Second-order IIR Phase Equalizer

In this example, a second-order (N=2) IIR equalizer is trained with the MPSO algorithm. The acceleration constants were selected to be 0.5 in all cases. The CMA error contour is plotted with respect to the two filter coefficients in Figure 5. Because there is no significance to the order of the stages in the cascade, this error surface in is symmetric about a 45° manifold in the parameter space. Because the global minimum occurs on the diagonal, both coefficients will converge to the same value. Therefore, only one of the coefficients is plotted for each case. These coefficient convergence plots, averaged over 50 trails, are given in Figure 6 for different swarm sizes (M=20, 30, 40). The straight line at the top of Figure 6 indicates the optimal coefficients, given at the center of the global minimum region in Figure 5.

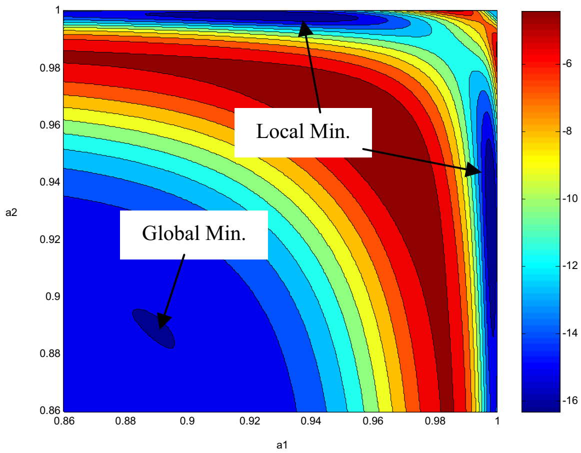

Fig. 5. Portion of the multimodal error surface contour for second-order phase equalizer

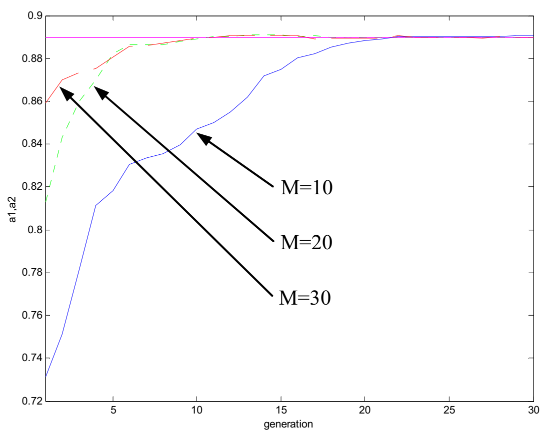

Fig. 6. Coefficient learning curves of different swarm sizes for the second-order phase equalizer

4. DISCUSSION

One curious observation from Figures 4 and 6 is that the coefficient curves always begin at a lower value and converge upward toward the optimum, despite the fact that the coefficient vectors are randomly initialized. This can be attributed to the fact that the PSO algorithm will always select the *gbests* with the lowest error value. It can be observed from Figures 3 and 5 that the largest regions with relatively low error values are located where the coefficients are lower in value than the optimum. Therefore, in these examples, it is more likely for the initial coefficients to start lower and increase toward the optimum.

Adding all-pass stages offers a performance improvement with diminishing returns as the number of stages increases. Therefore, 2-3 stages should be sufficient to exceed the IS-95-A standards with improved performance over the single stage. Since the numerator and denominator coefficients are the same for each stage in the all-pass equalizer, the number of particle parameters is equal to the number of stages, which will generally be very low. Because the swarm size is directly related to the dimensionality of the search space, PSO is well suited for these minimal parameter cases because it can give fast convergence with a relatively small swarm size, as indicated by Figures 4 & 6. This property makes PSO ideal for fast on-line phase adaptation in communications devices.

When compared to alternative stochastic search algorithms such as the SA and the GA [2,8] for the same problem, PSO is able to give a more robust search with better convergence properties for a given population size. This can be directly attributed to the inherent update mechanism and adjustable step size of PSO, which is especially effective for low-dimensional real-valued on-line adaptive problems such as all-pass phase equalization.

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