# ES, IS, DIS, SIDIS and Hadron Structure

Casual Nuclear Physics Lecture

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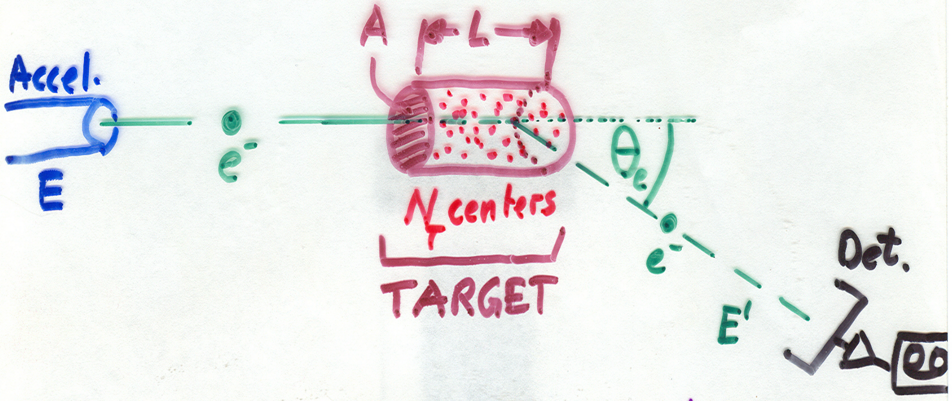
ODU, Spring 2015

# Electron Scattering – what can we measure?




What is the likelihood to find the electron scattered into the detector?

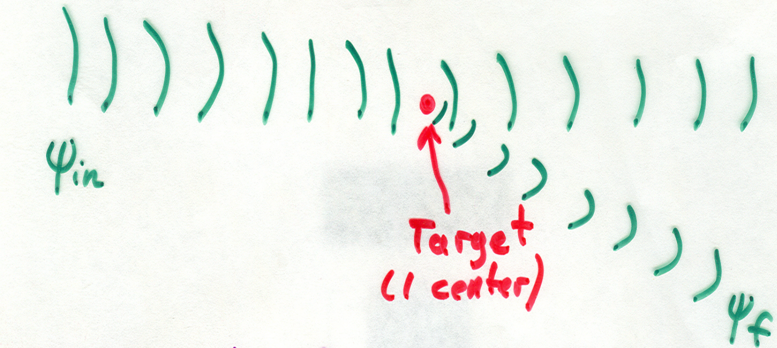
$$P \sim n_T \cdot L = \frac{N_T}{A\,L} \cdot L = \frac{N_T}{A} \; {}^*$$

$\Rightarrow$ call $\Delta\sigma = P / \left(\frac{N_T}{A}\right)$ (cross section)

$\Delta\sigma$ DEPENDS on the kinematics $(E, E', \theta_e)$ and is $\approx$ proportional to SIZE of kinematic bin spanned by the detector

*Note: $\frac{N_T}{A} = \rho \left[\frac{g}{cm^3}\right] \cdot L\,[cm] \cdot \frac{\text{Avogadro}}{\text{Atomic Weight [u]}}$

# Electron Scattering – Theorist's View




What is the transition rate $W_{i\to f}$?

$$\dot N_{e,f} = \dot N_{e,in} \cdot P(i\to f) = I_{e,in} \cdot \frac{N_T}{A} \cdot \Delta\sigma$$

$$= \frac{I_{e,in}}{A} \cdot N_T \cdot \Delta\sigma = \left(\vec{j}_{e,in}\right)_z \cdot N_T \cdot \Delta\sigma$$

$$\Rightarrow W_{i\to f} = j_{in} \cdot \Delta\sigma$$

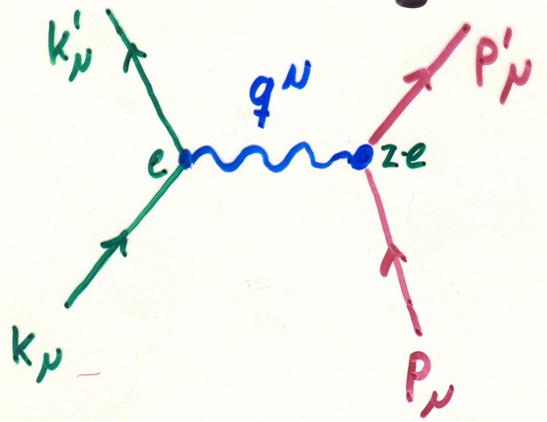
Fermi's GOLDEN Rule:

$$W_{i\to f} = \frac{2\pi}{\hbar} |\mathcal{M}_{fi}|^2 \Delta\phi$$

← Phase space spanned by detector/kinematic bin

$$\mathcal{M}_{fi} = \langle \psi_f | H_{int} | \psi_{in} \rangle$$

## Elastic Scattering – Feynman diagram




$\mathcal{M}_{fi} = e\, j_\nu \left(-\frac{1}{q^\mu q_\mu}\right) z j^\nu$ $\quad Q^2 := -q^\mu q_\mu$

Based on $\Box^2 A_\mu = j_\nu \Rightarrow A_\mu = \left(\frac{1}{iq}\right)^2 j_\nu$

and $H_{int} = A_\mu j^\mu = V\rho - \vec{A}\cdot\vec{j}$

$$\Rightarrow \Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4}\left(1-\beta^2\sin^2\frac{\theta}{2}\right)^{*)}\left(1+2\frac{\nu^2}{Q^2}\tan^2\frac{\theta}{2}\right)^{*1)} \cdot$$

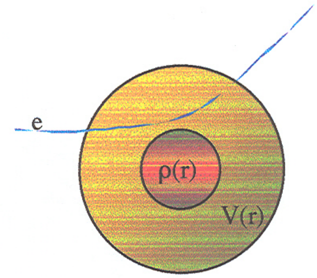
$$\cdot \int d^3\vec{k}'\,\delta(E'-E'_{el}) \left(= E'^2 \cdot \Delta\Omega\right)$$

*) magnetic interaction due to electron spin

*1) magnetic interaction due to target spin

# Form Factors

**Low-medium energy: Distribution of charge and magnetism inside the hadron**




$$\vec{\nabla}^2 V(\mathbf{r}) = -4\pi\rho(\mathbf{r}) \quad \Rightarrow$$

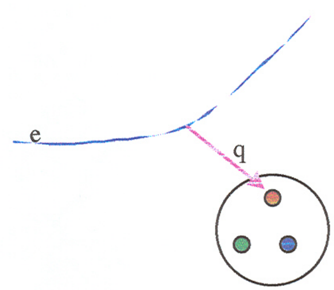
$$q^2 V \propto \int e^{i\mathbf{qr}}\rho(\mathbf{r})d^3\mathbf{r} = F(q)$$

$$\mathcal{H} \approx -eV \propto \frac{F(q)}{q^2}$$

$$\frac{d\sigma}{d\Omega} \propto \left|\langle f|\mathcal{H}|i\rangle\right|^2 \propto \frac{F^2(q)}{q^4}$$

Ex: $\rho(r) = a\cdot e^{-\alpha r} \Rightarrow F(q^2) = (1+q^2/\alpha^2)^{-2}$ (Dipole Form)

**High energy: "Stability" of internal structure against hard "blows"**




**Can the hadron absorb a high momentum virtual photon without breaking apart?**

## Some kinematics – in the lab system

$$k^\mu = (E, 0, 0, E) = (E, \vec{k})$$

$$k'^\mu = (E', E'\sin\theta, 0, E'\cos\theta) = (E', \vec{k}')$$

$$q^\mu = (E-E', -E'\sin\theta, 0, E - E'\cos\theta) = (\nu, \vec{q})$$

$$Q^2 = -q^\mu q_\mu = -\nu^2 + \vec{q}^{\,2} = (E - E'\cos\theta)^2 + E'^2\sin^2\theta - (E-E')^2$$

$$= E^2 - 2EE'\cos\theta + E'^2 - E^2 - E'^2 + 2EE'$$

$$= 2EE'(1-\cos\theta) = 4EE'\sin^2\frac{\theta}{2}$$

$$p^\mu = (M, 0, 0, 0) = (M, \vec{0})$$

$$p'^\mu = p^\mu + q^\mu = (M+\nu, \vec{q})$$

$$p'^\mu p'_\mu =: W^2 = M^2 + 2M\nu + \nu^2 - \vec{q}^{\,2}$$

$$= M^2 + 2M\nu - Q^2$$

Elastic Scattering: $W^2 \stackrel{!}{=} M^2$

$$\Rightarrow \nu_{el} \stackrel{!}{=} \frac{Q^2}{2M} \quad \text{or} \quad x_{el} = \frac{Q^2}{2M\nu_{el}} \stackrel{!}{=} 1$$

## Elastic Cross section – final form

$$\Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4}\cos^2\frac{\theta}{2}\left[\underbrace{\frac{G_E^2 + \tau G_M^2}{1+\tau}}_{\text{LONGITUDINAL (charge)}} + \underbrace{2\tau\tan^2\frac{\theta}{2}\,G_M^2}_{\text{TRANSVERSE (magnetic)}}\right] \cdot E'^2\,\Delta\Omega\,\underbrace{\frac{E'}{E}}_{\text{Recoil}}$$

$\tau = \frac{\nu^2}{Q^2}$ , $G_E(Q^2)$, $G_M(Q^2)$ : Form Factors

Dirac Particle: $G_E = G_M = 1$ (const.)

Anomalous magnetic moment: $G_M \approx (1+\kappa)G_E$

Extended Charge distribution:

$G_E(Q^2) \simeq$ Fourier transform of $\rho(r)$

Ex: $\rho(r) \simeq \frac{a^3}{8\pi}e^{-ar} \Rightarrow G_E(Q^2) \simeq \left(\frac{1}{1+\frac{Q^2}{a^2}}\right)^2$

(Dipole Form). $p: a^2 \simeq 0.71\,\text{GeV}^2$

Elastic scattering

$$\frac{\Delta\sigma}{\Delta\Omega} = \frac{4\alpha^2(\hbar c)^2 E'^2 cos^2\frac{\theta}{2}}{Q^4}\frac{E'}{E}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau}+2\tau\, tan^2\frac{\theta}{2}\, G_M^2(Q^2)\right)$$

(

where $\tau = \nu^2/Q^2$.

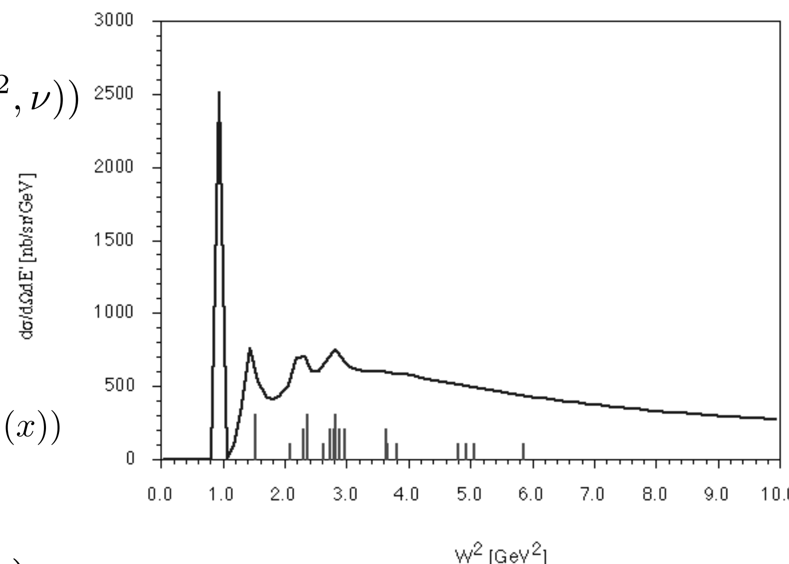
Inelastic Scattering

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(W_2(Q^2,\nu)+2\,tan^2(\theta/2)\,W_1(Q^2,\nu))$$

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}\frac{W_1(Q^2,\nu)}{\epsilon(1+\tau)}(1+\epsilon R(Q^2,\nu))$$

with $\epsilon = (1+2(1+\tau)tan^2(\theta/2))^{-1}$

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu}F_2(x)+2\,tan^2(\theta/2)\frac{1}{M}F_1(x))$$




$F_1(x) = MW_1(Q^2,\nu) \qquad F_2(x) = \nu W_2(Q^2,\nu)$

# Deep Inelastic Scattering (DIS)

Reminder: Elastic scattering

$$\frac{\Delta\sigma}{\Delta\Omega} = \frac{4\alpha^2(\hbar c)^2 E'^2 cos^2\frac{\theta}{2}}{Q^4}\frac{E'}{E}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau\, tan^2\frac{\theta}{2}\, G_M^2(Q^2)\right)$$

Elastic scattering from quarks:

$$\Delta\sigma = \frac{4\pi z_q^2\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(q(x)\Delta x + 2\nu^2/Q^2 tan^2(\theta/2)q(x)\Delta x)\Delta Q^2. \tag{12}$$

We can use the relation $\Delta x = -Q^2/(2M\nu^2)\Delta\nu = -x\Delta\nu/\nu$ to rewrite this as

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{x}{\nu}z_q^2 q(x) + \frac{1}{M}tan^2(\theta/2)z_q^2 q(x)). \tag{13}$$

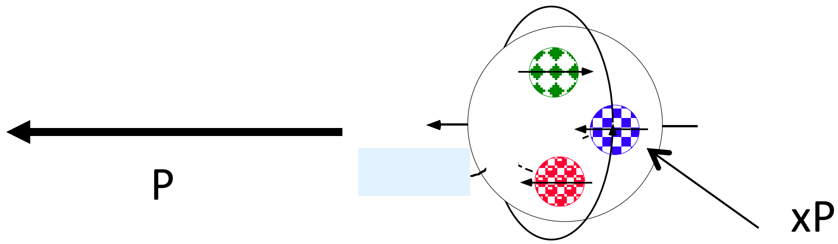
Reminder: IN-Elastic scattering

$$\frac{\Delta\sigma}{\Delta Q^2\Delta\nu} = \frac{4\pi\alpha^2(\hbar c)^2 E' cos^2(\theta/2)}{Q^4 E}(\frac{1}{\nu}F_2(x) + 2\, tan^2(\theta/2)\frac{1}{M}F_1(x))$$

$$\Rightarrow \quad F_1(x) = \frac{1}{2}\left(\frac{4}{9}\left[u(x)+\bar{u}(x)\right] + \frac{1}{9}\left[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)\right] + ...\right)$$

No $Q^2$!

# Quark-Parton Structure of the Proton




q(x) $\sim \left\langle P,s \middle| \bar{q}\gamma^{\mu} q \middle| P,s \right\rangle$

Δq(x) $= q\Uparrow\uparrow(x) - q\Uparrow\downarrow(x) + \bar{q}\Uparrow\uparrow(x) - \bar{q}\Uparrow\downarrow(x) \sim \left\langle P,s \middle| \bar{q}\gamma^{\mu}\gamma^{5} q \middle| P,s \right\rangle$

“axial charge”, similarly G(x) and ΔG(x) for gluons

Spin Sum Rule:

$$S_p = \frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \Delta G + L_q + L_G$$

$\Delta\Sigma$

# Simple (Constituent) Quark Model

| Flavor | Isospin $I$ | $I_3$ | Strangeness $S$ | Charge $Q$ | Baryon Number $B$ |
|---|---|---|---|---|---|
| $U$ | $1/2$ | $+1/2$ | $0$ | $+2/3$ | $1/3$ |
| $D$ | $1/2$ | $-1/2$ | $0$ | $-1/3$ | $1/3$ |
| $S$ | $0$ | $0$ | $-1$ | $-1/3$ | $1/3$ |

$$|\Delta^{++}\uparrow\rangle = |U\uparrow U\uparrow U\uparrow\rangle$$

$$|\Delta^{+}\uparrow\rangle = 1/\sqrt{3}\,(|U\uparrow U\uparrow D\uparrow\rangle + |U\uparrow D\uparrow U\uparrow\rangle + |D\uparrow U\uparrow U\uparrow\rangle)$$

The case of the proton is a bit more complicated, since the wave function cannot be symmetric in spin and flavor separately. The most intuitive way to derive the proton wave function is by observing that 2 of the 3 quarks are equal ($U$), and therefore their relative spin wave function should be symmetric also. This leads to the conclusion that the two $U$–quarks couple their spins to a total spin of one. Let's denote the case where this spin has a z-projection of $+1$ as $(UU\Uparrow) := |U\uparrow U\uparrow\rangle$, while the projection with $S_z = 0$ will be indicated by $(UU\Rightarrow) := 1/\sqrt{2}\,(|U\uparrow U\downarrow\rangle + |U\downarrow U\uparrow\rangle)$. We can now combine the spin $1/2$ of the remaining $D$ quark with the spin 1 of the $UU$ pair in two ways to get total spin and projection $1/2$; the proper way follows simply from insertion of the correct Clebsch-Gordon coefficients:

$$|P\uparrow\rangle = 1/\sqrt{3}\left(\sqrt{2}|(UU\Uparrow)D\downarrow\rangle - |(UU\Rightarrow)D\uparrow\rangle\right). \qquad (2)$$

# Quark Model:

- SU(6)-symmetric wave function of the proton in the quark model:

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\left(3u\uparrow\left[ud\right]_{S=0} + u\uparrow\left[ud\right]_{S=1} - \sqrt{2}u\downarrow\left[ud\right]_{S=1} - \sqrt{2}d\uparrow\left[uu\right]_{S=1} - 2d\downarrow\left[uu\right]_{S=1}\right)$$

- In this model: d/u = 1/2, Δu/u = 2/3, Δd/d = -1/3 for all *x*
  => $A_{1p}$ = 5/9, $A_{1n}$ = 0, $A_{1D}$ = 1/3 *)

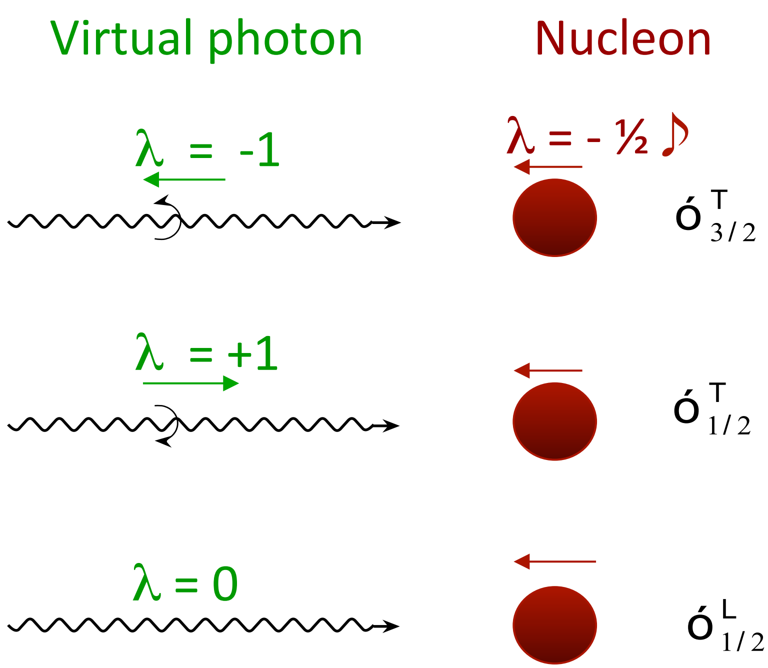
- Hyperfine structure effect: S=1 suppressed => d/u = 0, Δu/u = 1, Δd/d = -1/3
  for *x* -> 1 => $A_{1p}$ = 1, $A_{1n}$ = 1, $A_{1D}$ = 1

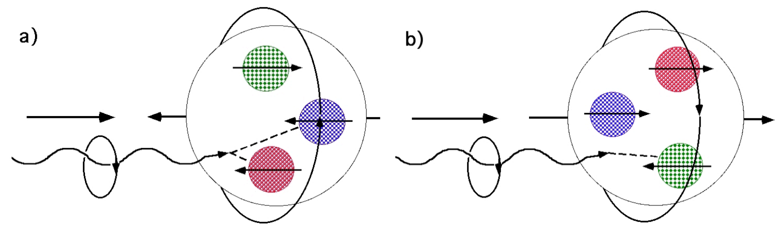
- pQCD: helicity conservation (q↑↑p) => d/u =2/(9+1) = 1/5, Δu/u = 1, Δd/d = 1
  for *x* -> 1

- Wave function of the neutron via isospin rotation:
  replace u -> d and d -> u => using experiments with protons and neutrons one can extract information on u, d, Δu and Δd in the valence quark region.

*) $$A_{1p} = \frac{4/9\cdot u\cdot \Delta u/u + 1/9\cdot d\cdot \Delta d/d}{4/9\cdot u + 1/9\cdot d} = \frac{4\cdot \Delta u/u + \left(d/u\right)\cdot \Delta d/d}{4+\left(d/u\right)}$$

# Virtual Photon Asymmetries




$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_T} \qquad A_2 = \frac{\sigma_{LT'}}{\sigma_T}$$

related to quark polarizations
Δq/q

# Spin Structure Functions

$$\frac{d\sigma}{dE'd\Omega}\downarrow\Uparrow - \frac{d\sigma}{dE'd\Omega}\uparrow\Uparrow = \frac{4\alpha^2 E'}{M\nu Q^2 E}\left[\left(E + E'\cos\theta\right)\mathbf{g_1} - 2xM\mathbf{g_2}\right]$$

Unpolarized: $F_1(x,Q^2)$ and $F_2(x, Q^2)$

Polarized: $g_1(x,Q^2)$ and $g_2(x, Q^2)$

Parton model:

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x) \text{ and } F_2(x) = 2xF_1(x)$$

$$g_1(x) = \frac{1}{2}\sum_i e_i^2 \Delta q_i(x) \text{ and } g_2(x) = 0$$

$i =$ quark flavor

$e_i =$ quark charge

the structure functions $\mathbf{g_1}$ and $\mathbf{g_2}$ are linear combinations of $\mathbf{A_1}$ and $\mathbf{A_2}$

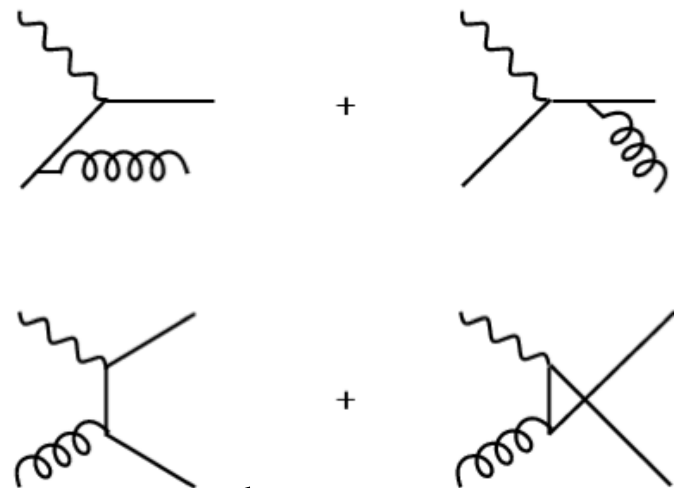
$$g_1(x,Q^2) = \frac{\tau}{1+\tau}\left(A_1 + \frac{1}{\sqrt{\tau}}A_2\right)F_1$$

$$g_2(x,Q^2) = \frac{\tau}{1+\tau}\left(\sqrt{\tau}A_2 - A_1\right)F_1$$

$$\tau = \frac{\nu^2}{Q^2}$$

# Parton Distribution Functions and NLO pQCD

Two effects modify simple parton picture:




1) (Gluon) radiative corrections change elementary cross section

$$g_1(x,Q^2)_{pQCD} = \frac{1}{2}\sum_q^{N_f} e_q^2 [(\Delta q + \Delta \bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q) + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \frac{\delta C_G}{N_f}]$$

$\delta C_q, \delta C_G - Wilson$ coefficient functions

2) pQCD evolution makes PDFs $Q^2$-dependent