Standard Model

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1 Weak Hypercharge and Weak Isospin

If one was to consider the world without gravity and the strong nuclear force, there would be 2 symmetries: The weak hypercharge Y_w and the weak iso-spin I_w.

There are three chiral (left-handed) iso-spin doublets for the quarks:

$$\begin{pmatrix} \text{u} \\ \text{d'} \end{pmatrix} \begin{pmatrix} \text{c} \\ \text{s'} \end{pmatrix} \begin{pmatrix} \text{t} \\ \text{b'} \end{pmatrix}$$

And three doublets for the leptons:

$$\begin{pmatrix} \nu_e \\ \text{e}^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Where $S = \frac{1}{2}$ and the third component of the weak iso-spin $I_3 = +\frac{1}{2}$ for the top rows and $I_3 = -\frac{1}{2}$ for the bottom rows.

The weak Isospin has 12 chiral (right-handed) singlets with $I = I_3 = 0$: $u_R, d_R, s_R, c_R, b_R, t_R, \nu_R, e^-_R, \nu_{\mu R}, \mu^-_R, \nu_\tau, \tau_R$

The weak hypercharge can be defined by the equation: $Q = I_3 + \frac{1}{2}Y_w$

With this information the fermions can be arranged in the table below:

Fermion Family	Left-chiral Fermions	Electric Charge Q	Weak Isospin T_3	Weak Hyper-Charge Y_W	Right-chiral Fermions	Electric Charge Q	Weak isospin T_3	Weak Hyper-Charge Y_W
Leptons	ν_e, ν_μ, ν_τ	0	$+1/2$	-1	No interaction (if they exist)			
	e^-, μ^-, τ^-	-1	$-1/2$	-1	e^-_R, μ^-_R, τ^-_R	-1	0	-2
Quarks	u, c, t	$+2/3$	$+1/2$	$+1/3$	u_R, c_R, t_R	$+2/3$	0	$+4/3$
	d, s, b	$-1/3$	$-1/2$	$+1/3$	d_R, s_R, b_R	$-1/3$	0	$-2/3$

2 Gauge Bosons

The gauge bosons correspond to operators in the weak hypercharge/isospin space. The W^+ boson corresponds to the I_3^w lowering operator likewise: $W^- \rightarrow I_3^w$ raising, and $W^0 \rightarrow I_3^w$

Each boson couples to the isospin with the same coupling strength g and forms an isospin triplet: $\begin{pmatrix} \mathrm{W}^+ \\ \mathrm{W}^0 \\ \mathrm{W}^- \end{pmatrix}$

3 Mass Eigenstates

In experiment we don't measure isospin and hypercharge, but rather the mass eigenstates. The Higgs interactions causes the quark weak eigenstates to mix and form the mass eigenstates in the following way:

$$\begin{pmatrix} \text{d'} \\ \text{s'} \\ \text{b'} \end{pmatrix} = \begin{pmatrix} \text{Cabbibo Mixing Matrix} \end{pmatrix} \begin{pmatrix} \text{d} \\ \text{s} \\ \text{b} \end{pmatrix}$$

Where the mixzing matreix can be formed with three mixing angles and a phase shift.

The neutrinos, however,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \text{Mixing Matrix} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

The bosons also have mass eigenstates formed by a mixing of the weak eigenstates

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} sin(\theta) & cos(\theta) \\ cos(\theta) & -sin(\theta) \end{pmatrix} \begin{pmatrix} \mathrm{W}^0 \\ \mathrm{B}^0 \end{pmatrix}$$