PHYSICS 722/822 - Fall Semester 2018 - ODU

Nuclear Physics - Problem Set 6 – Solution

Problem 1)

All you have to do is to take the magnetic moment operator $\boldsymbol{\mu}_p = \boldsymbol{\mu}_u + \boldsymbol{\mu}_u + \boldsymbol{\mu}_d$ and "sandwich" it between the proton wave functions given (one "bra" to the left and one "ket" to the right). Since the three components of the wave function are orthogonal, you can simply square them and add up. The first component gives $2/3(2\,\boldsymbol{\mu}_u - \boldsymbol{\mu}_d)$ and the second and third give no contribution from the u-quarks and $1/3\,\boldsymbol{\mu}_d$ from the d-quark. Adding it all gives $4/3\,\boldsymbol{\mu}_u - 1/3\,\boldsymbol{\mu}_d$ (see Eq. 15.17). My formula gives $\boldsymbol{\mu}_u = 1.9\,\mu_N$ and $\boldsymbol{\mu}_d = -0.95\,\mu_N$. Plugging it all in yields $\boldsymbol{\mu}_p = 2.85\,\mu_N$. The experimental value is 2.79 - pretty good!
To do the same thing for neutrons, you simply use the fact that d's and u's change places in the wave function, so I get $\boldsymbol{\mu}_n = 4/3\,\boldsymbol{\mu}_d - 1/3\,\boldsymbol{\mu}_u = -1.90\,\mu_N$. (Experiment: - 1.91 - again very good).

Problem 2)

This one is even easier. The first expression in the wave function gives a net spin fraction of +4/3 carried by the u quarks (the second 2 expressions give nothing). Vice versa, the net spin carried by d-quarks comes out as 2/3(-1) + 1/3(+1) = -1/3. Adding them straight yields 1 of course, since in the simple model given they carry the total spin of the proton. Weighing with the squared quark charges and dividing by 2 gets us 0.278. The experimental result is 0.12 -- 0.13 (approximately), which is quite different. Most of the difference is explained by relativity (which reduces the quark spin), orbital angular momentum (which is ignored in the wave function), and the rest by some combination of gluon and sea quark contributions.
For the neutron I get $1/2(4/3\cdot 1/9 - 1/3\cdot 4/9) = 0$. That is actually not so bad given that the experimental value is -0.04 -- -0.06 or so.

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Table 1
Results for the first moments of the spin structure functions g_1 from different experiments. Each experiment evolved its data to a fixed value of Q^2 which is indicated. The results for the deuteron are *not* corrected for its D-state, but are "per nucleon".

Q^2 (GeV^2)	Γ_1^p	Γ_1^n	Γ_1^d	$\Gamma_1^p - \Gamma_1^n$	Ref.
10	0.120 ± 0.016	–	0.019 ± 0.015	0.198 ± 0.023	SMC [28]
3	–	-0.033 ± 0.011	–	–	E142 [31]
3	0.133 ± 0.010	-0.032 ± 0.018	0.047 ± 0.007	0.164 ± 0.023	E143 [34]
5	–	-0.056 ± 0.009	–	0.168 ± 0.010	E154[a] [38]
5	0.118 ± 0.008	-0.058 ± 0.009	–	0.176 ± 0.008	E155[a] [36]
2.5	0.120 ± 0.009	-0.028 ± 0.009	0.043 ± 0.004	0.148 ± 0.017	HERMES[b] [44]
3	–	–	0.046 ± 0.006	–	COMPASS [49]
5	Bjorken sum rule			0.182 ± 0.002	

[a] From an NLO analysis.
[b] Over measured region $x > 0.021$ only.