PHYSICS 722/822 - Fall Semester 2018 - ODU

**Nuclear Physics - Problem Set 9 – Due TUESDAY 11/27**

Note: You have LOTS of time (until after Thanksgiving) for this problem but don't wait too long to start – you can ask me questions on Monday 11/19
P.S.: Please forgive the funny symbols for vectors – instead of an arrow, they ended up looking like an "r" above the verctors.

**Problem 1)**

I claim that $\psi = \dfrac{e^{-mr}}{r}$ is a solution for the static Klein-Gordon equation with source term: $\left(-\vec{\nabla}^2 + m^2\right)\psi = 4\pi\delta^3(\vec{r})$. Because of the singularity at r=0, this cannot be proven directly using the expression for the Laplacian in cylindrical coordinates. Instead, use the fact that $\vec{\nabla}^2 t = \vec{\nabla}\cdot\vec{\nabla} t = div(grad\,t)$ and Gauß' theorem $\iiint\limits_{Volume} div\vec{v}\,d^3\vec{r} = \iint\limits_{Surface} \vec{v}\cdot d\vec{a}$ to integrate both $-\vec{\nabla}^2 \dfrac{e^{-mr}}{r}$ (l.h.s.) and $4\pi\delta^3(\vec{r}) - m^2 \dfrac{e^{-mr}}{r}$ (r.h.s.) over a sphere of radius R, and show that both integrals agree, independent of the size of R. This proves my claim.
*Hint:* The form of the gradient in spherical coordinates is given by $\vec{\nabla} t = \dfrac{\partial t}{\partial r}\hat{r} + \dfrac{1}{r}\dfrac{\partial t}{\partial \theta}\hat{\theta} + \dfrac{1}{r\sin\theta}\dfrac{\partial t}{\partial \varphi}\hat{\varphi}$ (of course, the last two terms yield zero for our spherically symmetric solution).

**Problem 2)**

I also claim that the Tensor operator, $S_{12} = 3\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2$, connects L=0 states like $^3S_1$ with L=2 states like $^3D_1$. Prove this explicitly by applying this operator to the state $\left|{}^3S_1, m_j = +1\right\rangle = Y_{00}\left|\uparrow\uparrow\right\rangle$ (where the two arrows indicate a state where both nucleon spins are "up"). Show that the resulting state is a multiple of

$$\left|{}^3D_1, m_j = +1\right\rangle = \sqrt{\frac{3}{5}}Y_{22}\left|\downarrow\downarrow\right\rangle - \sqrt{\frac{3}{10}}Y_{21}\left|\frac{\uparrow\downarrow + \downarrow\uparrow}{\sqrt{2}}\right\rangle + \sqrt{\frac{1}{10}}Y_{20}\left|\uparrow\uparrow\right\rangle.$$

*Hint:* You will need to use Clebsch-Gordan coefficients (like the ones above) and the specific form of the spherical harmonics (see Particle Data Group collection). Also, it is advantageous to use the following relationships (check them for yourselves):

$$\vec{\sigma}\cdot\hat{r} = \sigma^{+}\left(\frac{x-iy}{r}\right) + \sigma^{-}\left(\frac{x+iy}{r}\right) + \sigma^{z}\frac{z}{r}; \quad \vec{\sigma}_1\cdot\vec{\sigma}_2 = 2\sigma_1^{+}\sigma_2^{-} + 2\sigma_1^{-}\sigma_2^{+} + \sigma_1^{z}\sigma_2^{z}$$

Note:

Pauli matrices:

$$\sigma^{x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma^{y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma^{z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma^{+} = \frac{1}{2}\left(\sigma^{x} + i\sigma^{y}\right) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$ (raises 3rd component of

$$\sigma^{-} = \frac{1}{2}\left(\sigma^{x} - i\sigma^{y}\right) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ (lowers 3rd component o

Spherical Harmonics:

$$Y_{00} = \sqrt{\frac{1}{4\pi}}; Y_{20} = \sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right); Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\varphi}; Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{2i}$$