Physics
Old Dominion UNIVERSITY

Nucleon Resonances

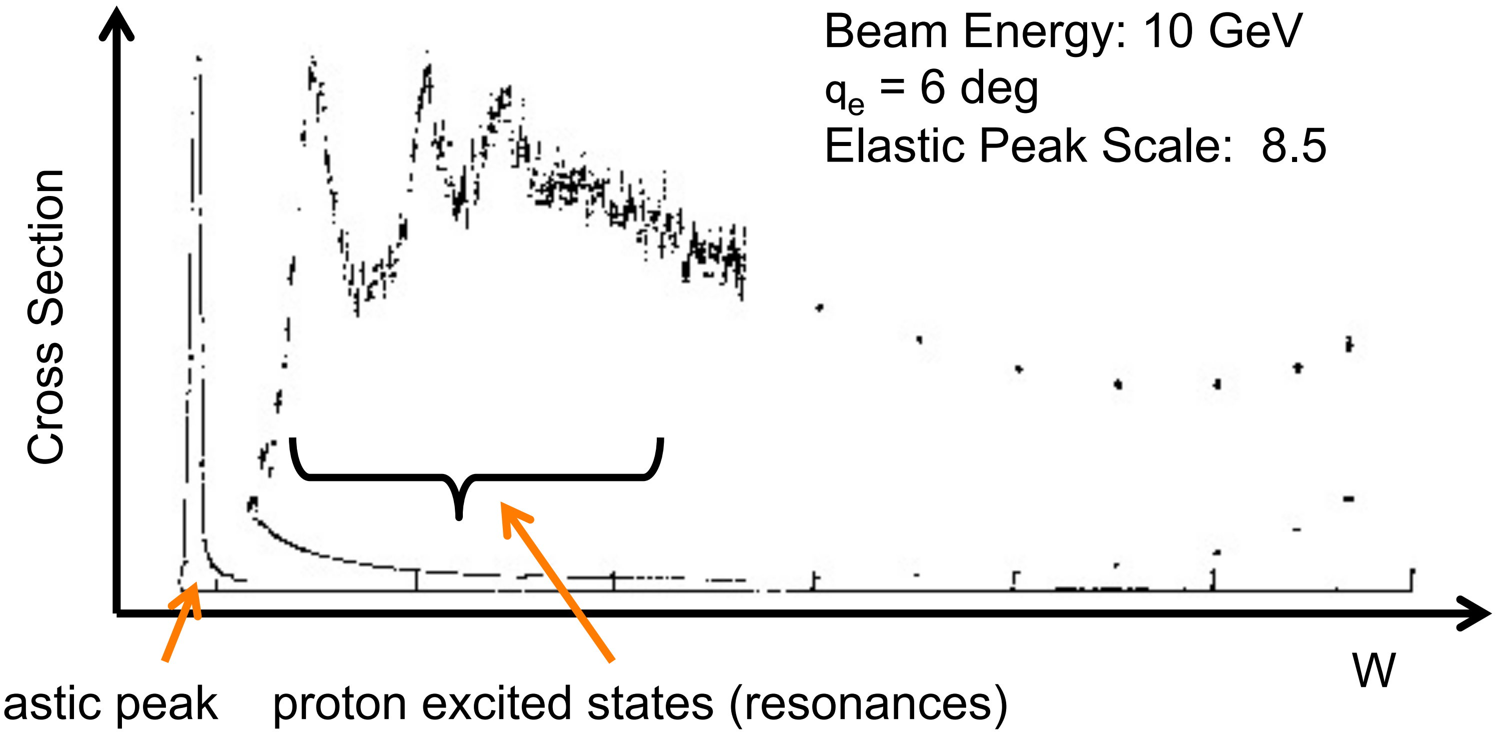
Gail Dodge

- Notation, PDG
- S-matrix, Breit-Wigner
- Partial Wave Analysis, Argand Plots
- Quark Model
- Missing Resonances & new data
- The "Complete" Experiment
- Roper
- PWIA fits to world data

Nuclear Seminar April 2, 2015

Inclusive Electron Scattering on the Proton

Physics
Old Dominion UNIVERSITY



First excited state is a Delta resonance (Δ)

Friedman and Kendell, Ann. Rev. Nucl. Sci. 22, 203-254 (1972)

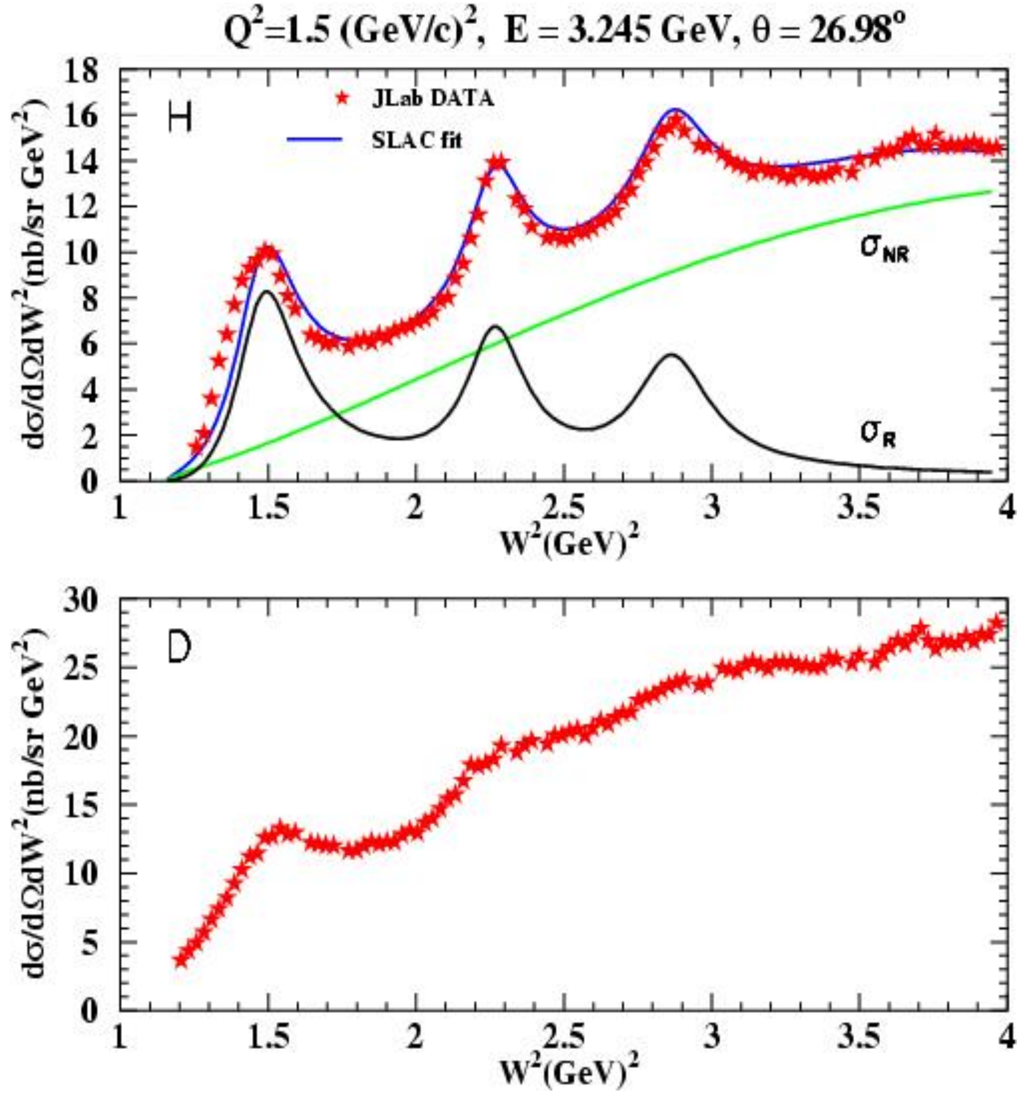
Neutron excitations are harder to study

Physics
Old Dominion UNIVERSITY

Elastic peak not shown

Note W^2 on the x-axis

Fermi motion of the proton and neutron in the deuteron smears out the structure



Notation

Physics
Old Dominion UNIVERSITY

Originates with πN scattering
L is the orbital angular momentum between the π and nucleon
Label resonance by via production mechanism:

$$L_{2I,2J}$$

Pion has isospin 1, spin 0

In 2012 PDG changed to label resonance by the spin and parity of the state:

Originally : Δ(1232) P_{33}

Now PDG lists: Δ(1232) $3/2^+$ Parity:

More notation: N or N* is a resonance with isospin = ½ (strangeness = 0)
Δ has isospin = 3/2 (strangeness = 0)

Roberts and Crede, 2013

Resonances in the PDG (2014)

"Resonances are defined by poles of the S-matrix, whether in scattering, production or decay matrix elements. These are poles in the complex plane in s, as discussed in the new review on *Resonances*. As traditional we quote here the pole positions in the complex energy $w = \sqrt{s}$ plane. Crucially, the position of the pole of the S-matrix is independent of the process, and the production and decay properties factorize. This is the rationale for listing the pole position first for each resonance."

$\Delta(1232)\ 3/2^+$

$$I(J^P) = \tfrac{3}{2}(\tfrac{3}{2}^+)$$

Breit-Wigner mass (mixed charges) = 1230 to 1234 ($\approx$ 1232) MeV
Breit-Wigner full width (mixed charges) = 114 to 120 ($\approx$ 117) MeV
$p_{\text{beam}} = 0.30$ GeV$/c$ $\qquad 4\pi\lambda\!\!\!^{-2} = 94.8$ mb
Re(pole position) = 1209 to 1211 ($\approx$ 1210) MeV
-2Im(pole position) = 98 to 102 ($\approx$ 100) MeV

PDG 2014

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2$
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	***	Σ^0	$1/2^+$	****	Ξ^-	$1/2$
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Xi(1530)$	$3/2$
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1620)$	
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$		*	$\Xi(1690)$	
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	**	$\Sigma(1560)$		**	$\Xi(1820)$	$3/2$
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$	
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	*	$\Xi(2030)$	$\geq \frac{5}{2}$
$N(1685)$		*	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2120)$	
$N(1700)$	$3/2^-$	***	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1670)$	$3/2^-$	****	$\Xi(2250)$	
$N(1710)$	$1/2^+$	***	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1690)$		**	$\Xi(2370)$	
$N(1720)$	$3/2^+$	****	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1730)$	$3/2^+$	*	$\Xi(2500)$	
$N(1860)$	$5/2^+$	**	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1750)$	$1/2^-$	***		
$N(1875)$	$3/2^-$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1770)$	$1/2^+$	*	Ω^-	$3/2$
$N(1880)$	$1/2^+$	**	$\Delta(2200)$	$7/2^-$	*	$\Sigma(1775)$	$5/2^-$	****	$\Omega(2250)^-$	
$N(1895)$	$1/2^-$	**	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1840)$	$3/2^+$	*	$\Omega(2380)^-$	
$N(1900)$	$3/2^+$	***	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1880)$	$1/2^+$	**	$\Omega(2470)^-$	
$N(1990)$	$7/2^+$	**	$\Delta(2390)$	$7/2^+$	*	$\Sigma(1900)$	$1/2^-$	*		
$N(2000)$	$5/2^+$	**	$\Delta(2400)$	$9/2^-$	**	$\Sigma(1915)$	$5/2^+$	****		
$N(2040)$	$3/2^+$	*	$\Delta(2420)$	$11/2^+$	****	$\Sigma(1940)$	$3/2^+$	*		
$N(2060)$	$5/2^-$	**	$\Delta(2750)$	$13/2^-$	**	$\Sigma(1940)$	$3/2^-$	***		
$N(2100)$	$1/2^+$	*	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2000)$	$1/2^-$	*		
$N(2120)$	$3/2^-$	**				$\Sigma(2030)$	$7/2^+$	****		
$N(2190)$	$7/2^-$	****	Λ	$1/2^+$	****	$\Sigma(2070)$	$5/2^+$	*		
$N(2220)$	$9/2^+$	****	$\Lambda(1405)$	$1/2^-$	****	$\Sigma(2080)$	$3/2^+$	**		

3 and 4 star resonances are listed in the PDG Tables

Physics Old Dominion UNIVERSITY

N BARYONS
$(S = 0, I = 1/2)$

$p, N^+ = uud; \quad n, N^0 = udd$

p $\qquad I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 1.00727646681 \pm 0.00000000009$ u
Mass $m = 938.272046 \pm 0.000021$ MeV [a]

Λ BARYONS
$(S = -1, I = 0)$

$\Lambda^0 = uds$

Λ $\qquad I(J^P) = 0(\frac{1}{2}^+)$

Mass $m = 1115.683 \pm 0.006$ MeV

Δ BARYONS
$(S = 0, I = 3/2)$

$\Delta^{++} = uuu, \quad \Delta^+ = uud, \quad \Delta^0 = udd, \quad \Delta^- = ddd$

$\Delta(1232)\ 3/2^+$ $\qquad I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$

Σ BARYONS
$(S = -1, I = 1)$

$\Sigma^+ = uus, \quad \Sigma^0 = uds, \quad \Sigma^- = dds$

Σ^+ $\qquad I(J^P) = 1(\frac{1}{2}^+)$

Mass $m = 1189.37 \pm 0.07$ MeV $\quad (S = 2.2)$
Mean life $\tau = (0.8018 \pm 0.0026) \times 10^{-10}$ s

Ω BARYONS
$(S = -3, I = 0)$

$\Omega^- = sss$

CHARMED BARYONS
$(C = +1)$

$\Lambda_c^+ = udc, \quad \Sigma_c^{++} = uuc, \quad \Sigma_c^+ = udc, \quad \Sigma_c^0 = ddc,$
$\Xi_c^+ = usc, \quad \Xi_c^0 = dsc, \quad \Omega_c^0 = ssc$

S-Matrix, Breit-Wigner

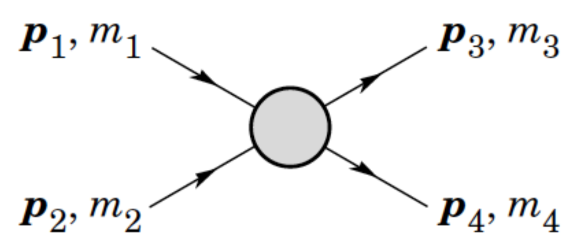
$$S_{ab} = I_{ab} - 2i\sqrt{\rho_a}\mathcal{M}_{ab}\sqrt{\rho_b} \qquad \rho_a(s) = \frac{1}{16\pi}\frac{2|\vec{q}_a|}{\sqrt{s}}$$

$$\mathcal{M} = \mathcal{M}^{\mathrm{b.g.}} + \mathcal{M}^{\mathrm{pole}}$$

$\sqrt{s_R} = M_R - i\Gamma_R/2$ Mass and width of resonance

The lifetime of the resonance/state is $\tau = \frac{\hbar}{\Gamma}$

Location of pole for resonance R in complex s- plane



$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

In the limit of one isolated resonance $M_R = M_{BW}$

$$\mathcal{M}^{\mathrm{pole}}_{ba}\Big|_{N=1} = -\frac{g_b\, g_a}{s - M_{\mathrm{BW}}^2 + i\sqrt{s}\Gamma_{\mathrm{BW}}}$$

g_a = coupling of resonance to channel a

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s}\frac{1}{|\boldsymbol{p}_{1\mathrm{cm}}|^2}|\mathcal{M}|^2$$

A Breit-Wigner fit is problematic if there are overlapping resonances and/or thresholds for new channels opening up and backgrounds that vary over the width of the resonance. Some theorists use the K-matrix or T-matrix.

For comparison, see Workman, PRC 79, 038201 (2009)
Workman, PRC 59, 3441 (1999)

PDG 2014

Physics
Old Dominion UNIVERSITY

Partial Wave Analysis

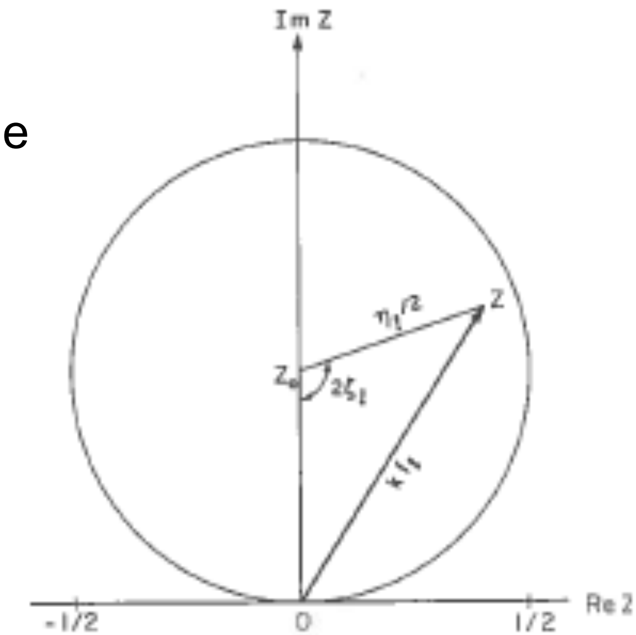
$$f(\theta)=\sum_{\ell}\sqrt{4\pi(2\ell+1)}f_lY_{l0}(\theta)$$

f_ℓ is the complex scattering amplitude for the *l*th partial wave

$$f_\ell=\frac{1}{2ik}(\eta_\ell e^{2i\delta}-1)$$

k is the c.m. momentum
η_l is the inelasticity parameter (=1 for elastic scattering)
δ_l is the phase shift

Plot $z=kf_\ell=\frac{1}{2i}(\eta_l e^{2i\delta_l}-1)$ in the complex plane

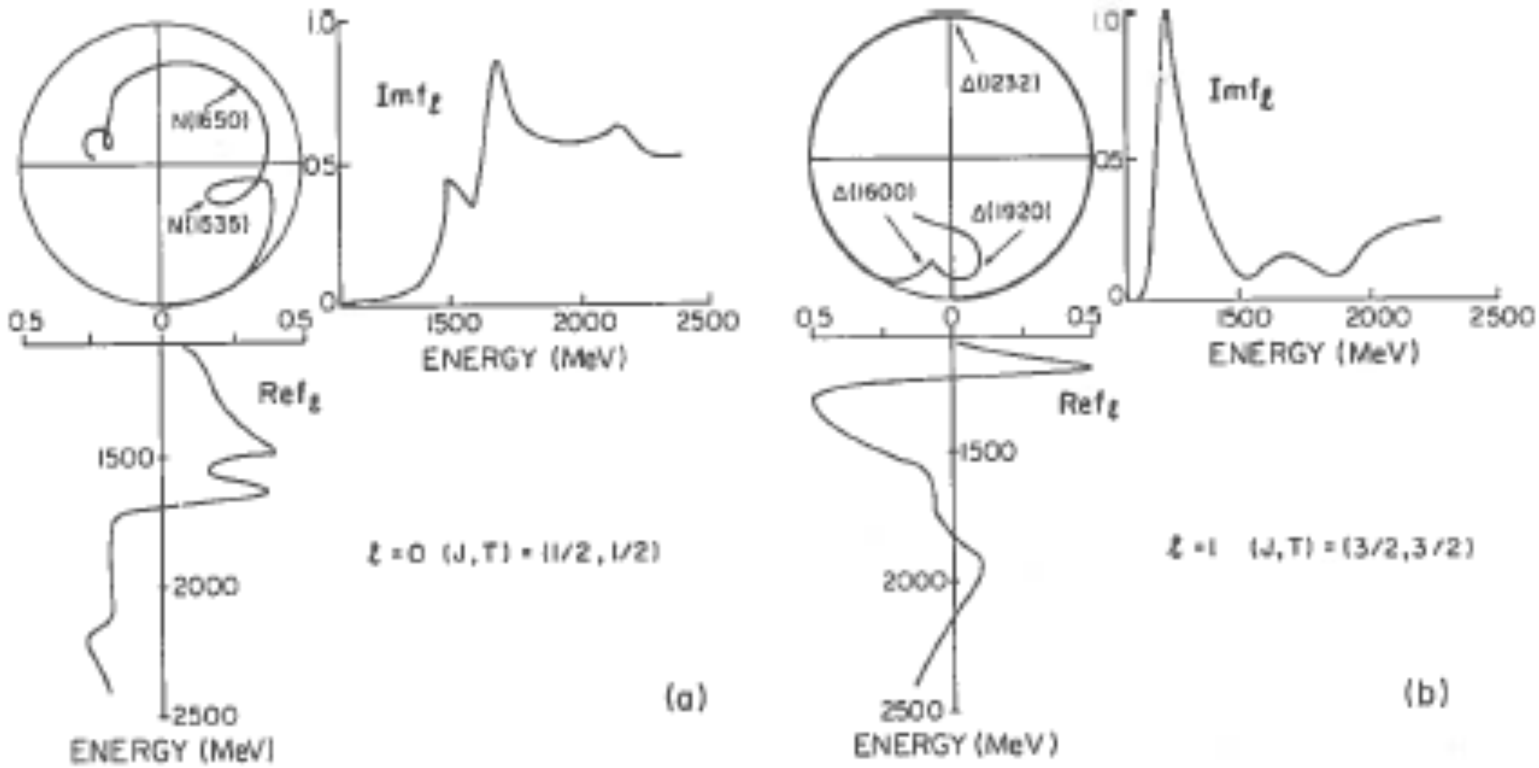


Unit circle centered at (0,½*i*)
If η_l = 1, z traces the circumference of the circle

If the phase shift is small, then *z* is almost entirely real. If phase shift is π/2, *z* is almost entirely imaginary and there may be a resonance.

Wong

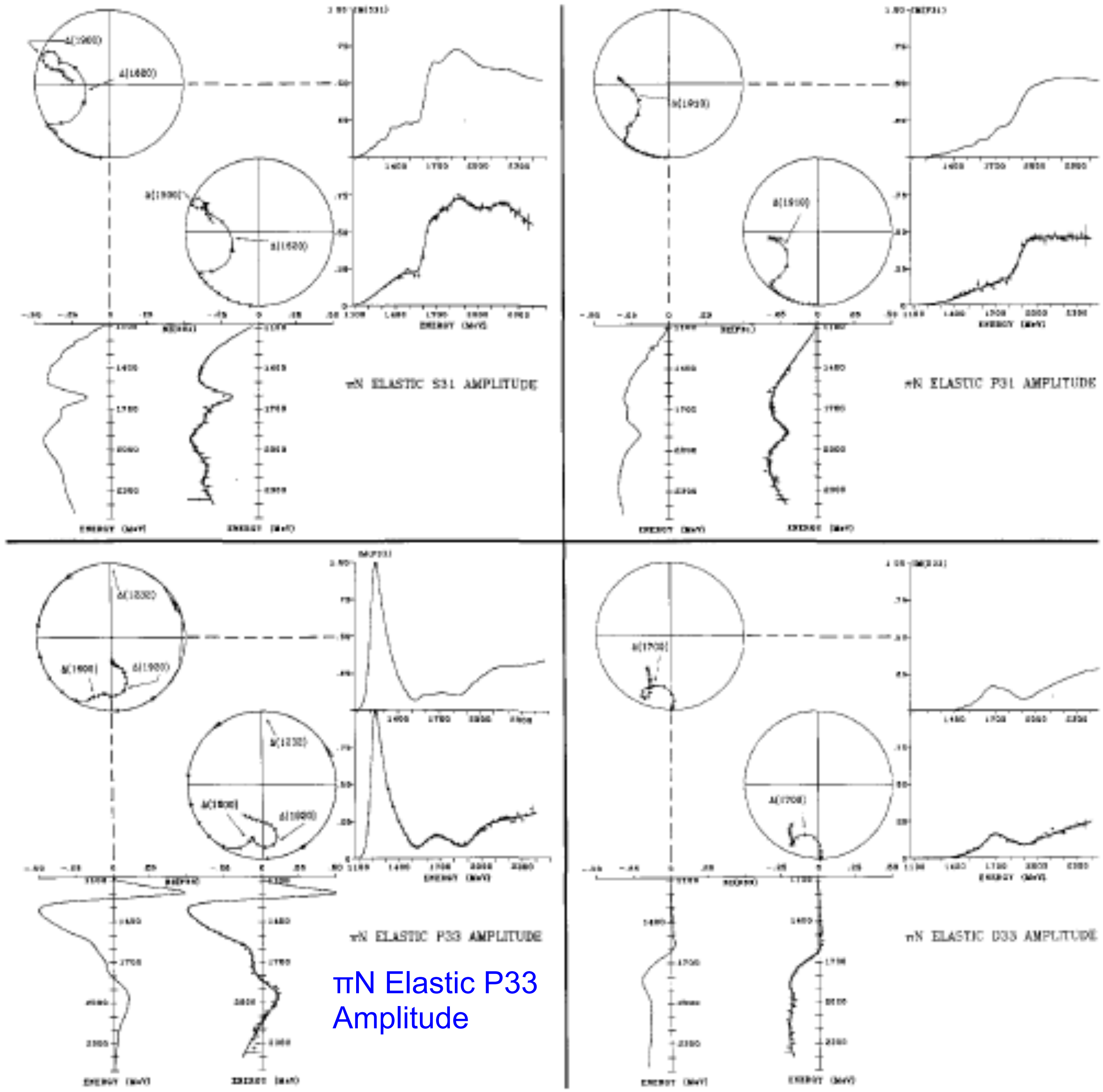
Argand Diagram



Peak of the resonance is when Re(z) changes sign
Usually use steps of 50 MeV to trace out the scattering amplitude

PDG 1986

Argand diagrams for all the major resonances



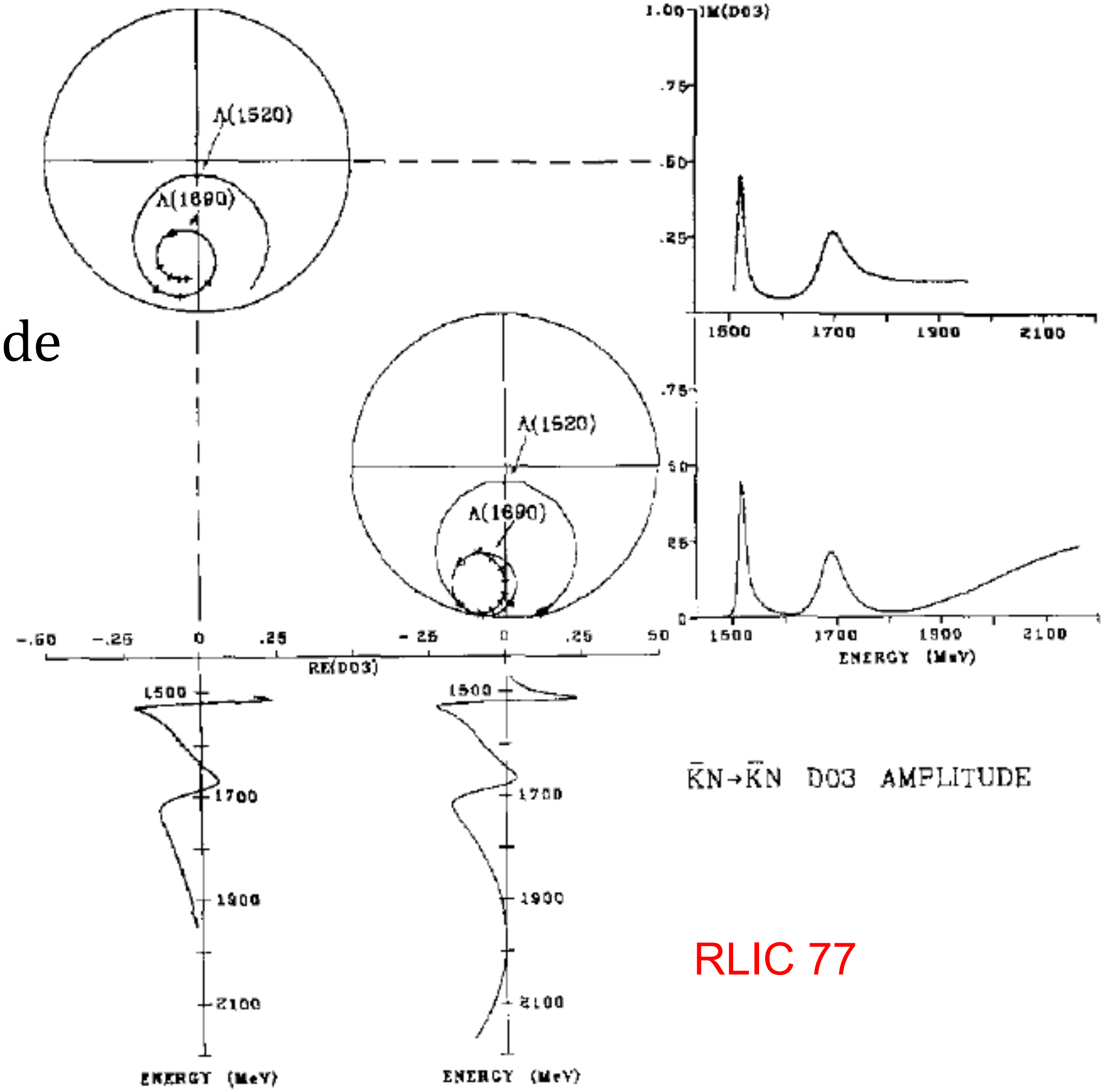
πN Elastic P33 Amplitude

Y

More Argand – D_{03}

$\bar{K}N \rightarrow \bar{K}N$ D03 Amplitude

Alston 78



Quark Model

Table 15.1: Additive quantum numbers of the quarks.

	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	−1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	−1	0
T – topness	0	0	0	0	0	+1

Each quark has baryon number B = 1/3; Anti-quarks have opposite quantum numbers

$$Y = \mathcal{B} + S - \frac{C - B + T}{3} \qquad Q = I_z + \frac{\mathcal{B} + S + C + B + T}{2}$$

PDG

Multiplets

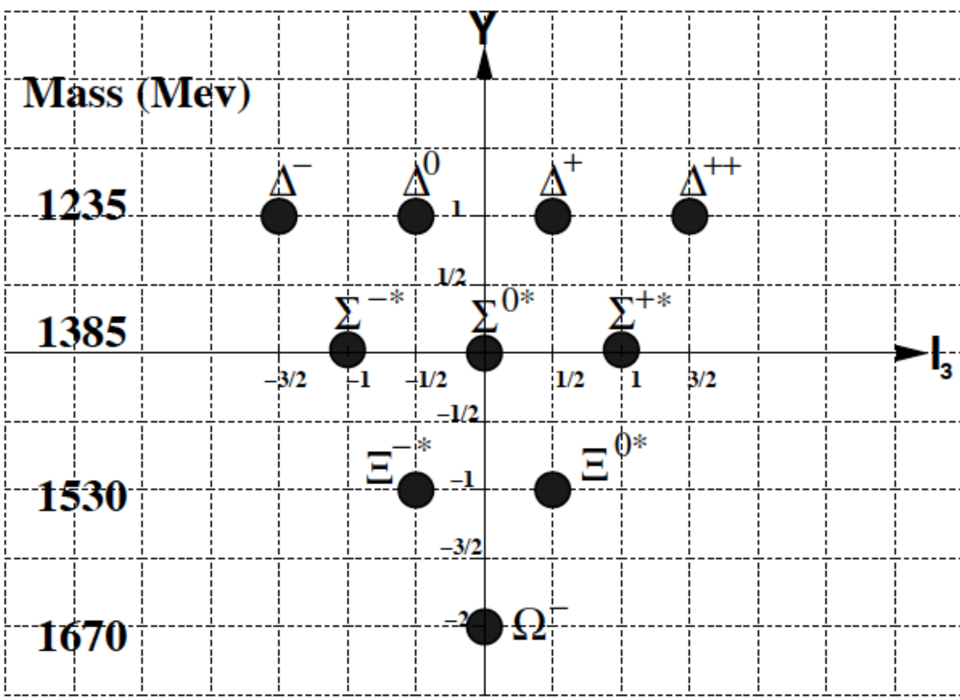
$$|\,qqq\,\rangle_A = |\,\text{color}\,\rangle_A \times |\,\text{space, spin, flavor}\,\rangle_S$$

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_M \oplus \mathbf{8}_M \oplus \mathbf{1}_A$$

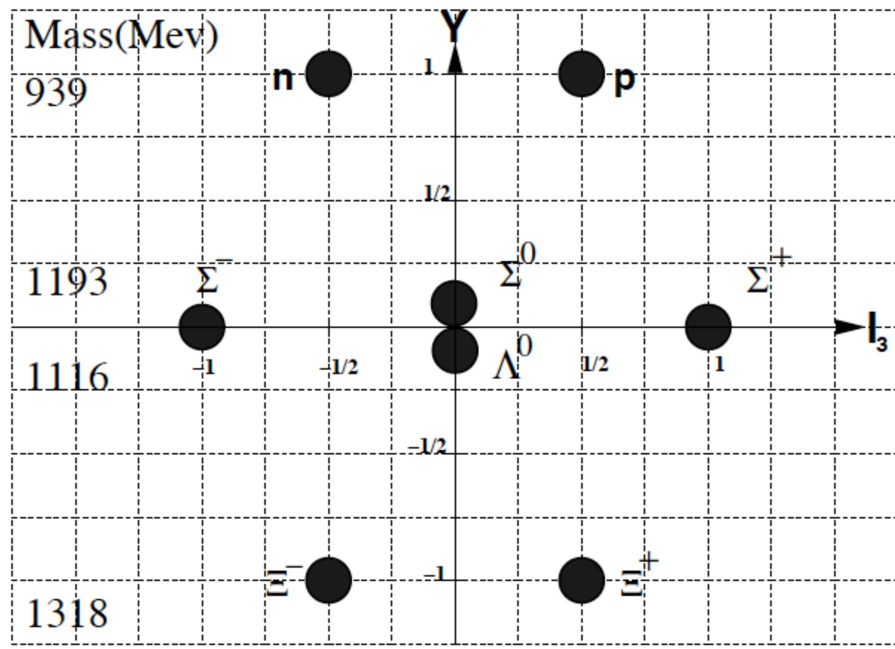
Λ_1 = uds Λ_8 = uds

Observed Λ is a mixture of Λ_1 and Λ_8.
States of same spin and parity can mix.

$J^P = 3/2^+$



$J^P = 1/2^+$



SU(3)Notes.pdf

Adding Spin

3 flavors times 2 spins – like 6 independent particles

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_M \oplus \mathbf{70}_M \oplus \mathbf{20}_A$$

spatial symmetry

$$\mathbf{56} = {}^4\mathbf{10} \oplus {}^2\mathbf{8}$$

L=0 "ground state"; has Δ decaplet (times 4 spin states) and nucleon octet (times 2 spin states)

$$\mathbf{70} = {}^2\mathbf{10} \oplus {}^4\mathbf{8} \oplus {}^2\mathbf{8} \oplus {}^2\mathbf{1}$$

The superscript is 2S + 1

$$\mathbf{20} = {}^2\mathbf{8} \oplus {}^4\mathbf{1} \ ,$$

N denotes excitation bands
N = 0 has 56-plet with nucleon and Δ

Notation (D,L_N^P)

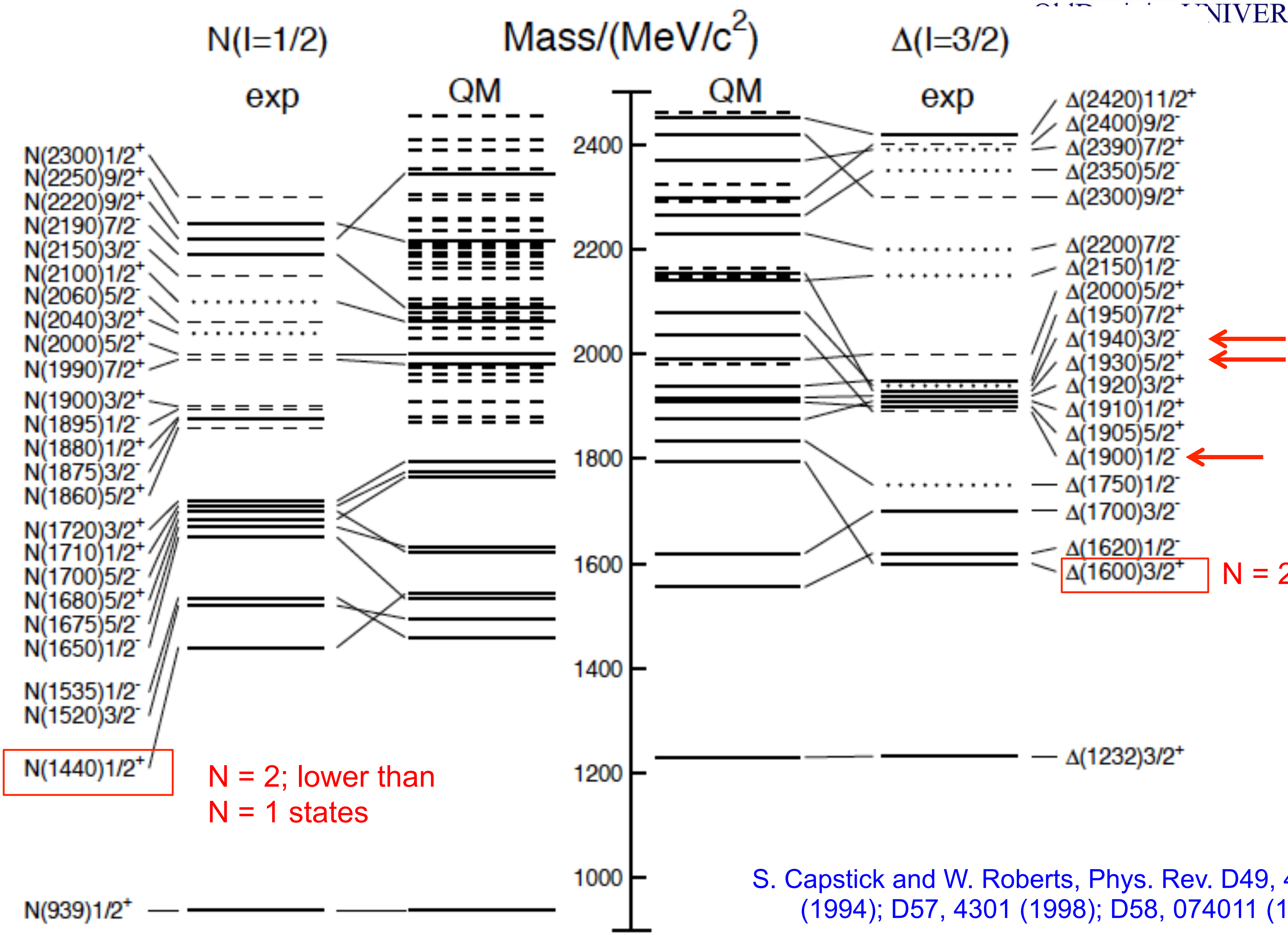
$(56,0_0^+)$ N=0; Nucleon and Δ
$(70,1_1^-)$ N=1; negative parity states below 1.9 GeV

Quark Model Assignments

Table 15.5: N and Δ states in the N=0,1,2 harmonic oscillator bands. L^P denotes angular momentum and parity, S the three-quark spin and 'sym'=A,S,M the symmetry of the spatial wave function. Only dominant components indicated. Assignments in the N=2 band are partly tentative.

N	sym	L^P	S	$N(I=1/2)$				$\Delta(I=3/2)$			
2	A	1^+	1/2	$1/2^+$	$3/2^+$						
2	M	2^+	3/2	$1/2^+$	$3/2^+$	$5/2^+$	$7/2^+$				
2	M	2^+	1/2		$3/2^+$	$5/2^+$			$3/2^+$	$5/2^+$	
2	M	0^+	3/2		$3/2^+$						
2	M	0^+	1/2	$1/2^+$				$1/2^+$			
				$N(1710)$				$\Delta(1750)$			
2	S	2^+	3/2					$1/2^+$	$3/2^+$	$5/2^+$	$7/2^+$
								$\Delta(1910)$	$\Delta(1920)$	$\Delta(1905)$	$\Delta(1950)$
2	S	2^+	1/2		$3/2^+$	$5/2^+$					
					$N(1720)$	$N(1680)$					
2	S	0^+	3/2						$3/2^+$		
									$\Delta(1600)$		
2	S	0^+	1/2	$1/2^+$							
				$N(1440)$							
1	M	1^-	3/2	$1/2^-$	$3/2^-$	$5/2^-$					
				$N(1650)$	$N(1700)$	$N(1675)$					
1	M	1^-	1/2	$1/2^-$	$3/2^-$			$1/2^-$	$3/2^-$		
				$N(1535)$	$N(1520)$			$\Delta(1620)$	$\Delta(1700)$		
0	S	0^+	3/2						$3/2^+$		
									$\Delta(1232)$		
0	S	0^+	1/2	$1/2^+$							
				$N(938)$							

Comparison to Reality – from PDG



S. Capstick and W. Roberts, Phys. Rev. D49, 4570 (1994); D57, 4301 (1998); D58, 074011 (1998)

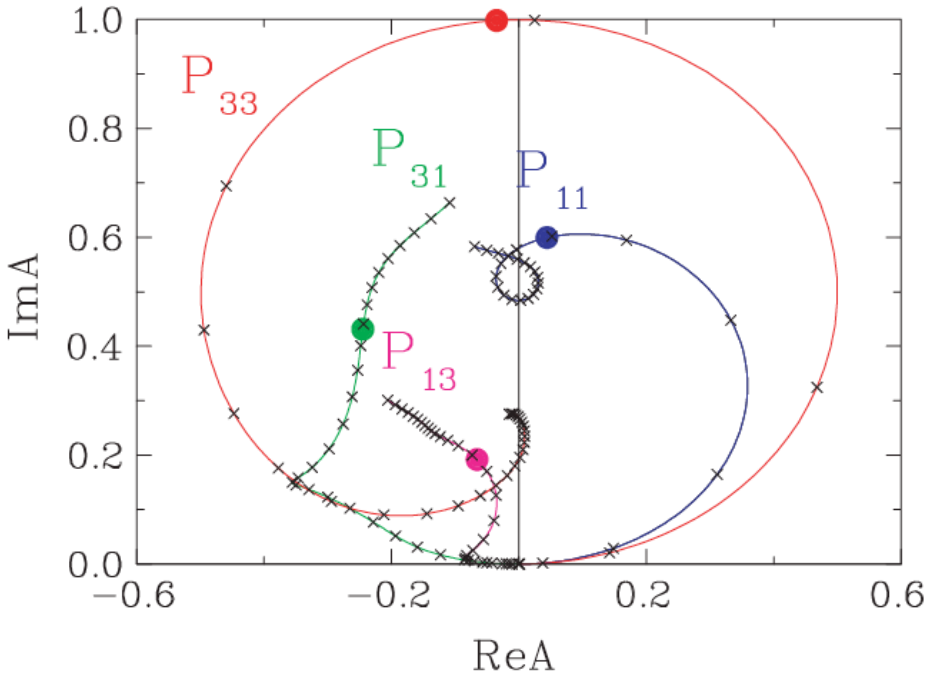
Missing Resonances

Physics
Old Dominion UNIVERSITY

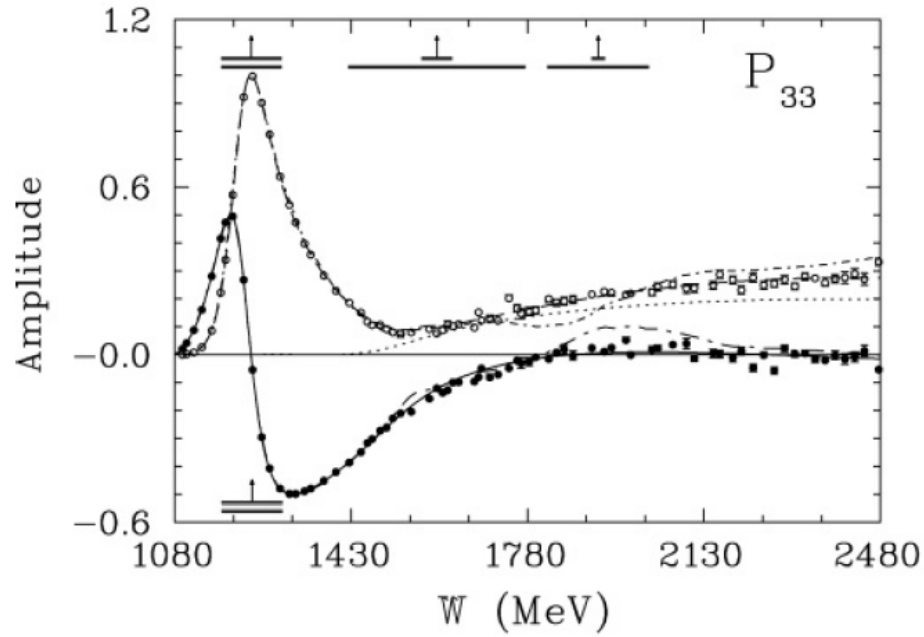
- Clearly there are many resonances that have not been observed.
- GWU 2006 fit finds fewer resonances than have been reported previously. No evidence for almost half of the states listed on previous page. But resonances which do not couple strongly to πN may not be seen here.
- Photo and electro production data are relatively recent and just now being analyzed as part of global fits.
- Jlab has reported evidence for a few missing resonances. See, e.g., Burkert: Mesons 2012
- Di-quark model postulates that two of the three quarks are coupled, which then limits the available degrees of freedom and would predict fewer resonances. Even the di-quark model predicts more states than we see. So far lattice QCD does not favor di-quark type models.

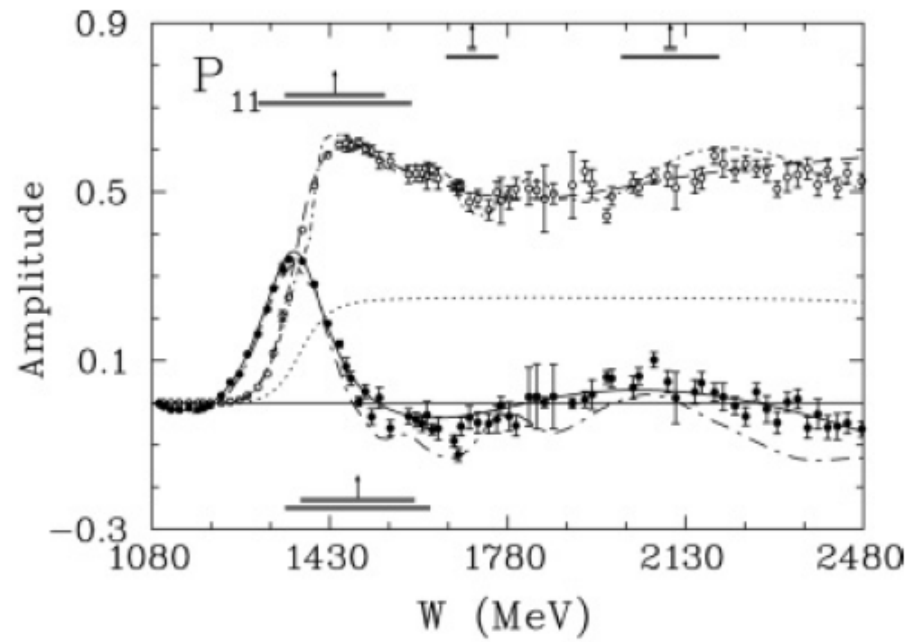
GWU fit to data 2006

PRC 74, 045205 (2006)



Solid dot gives BW mass from fit





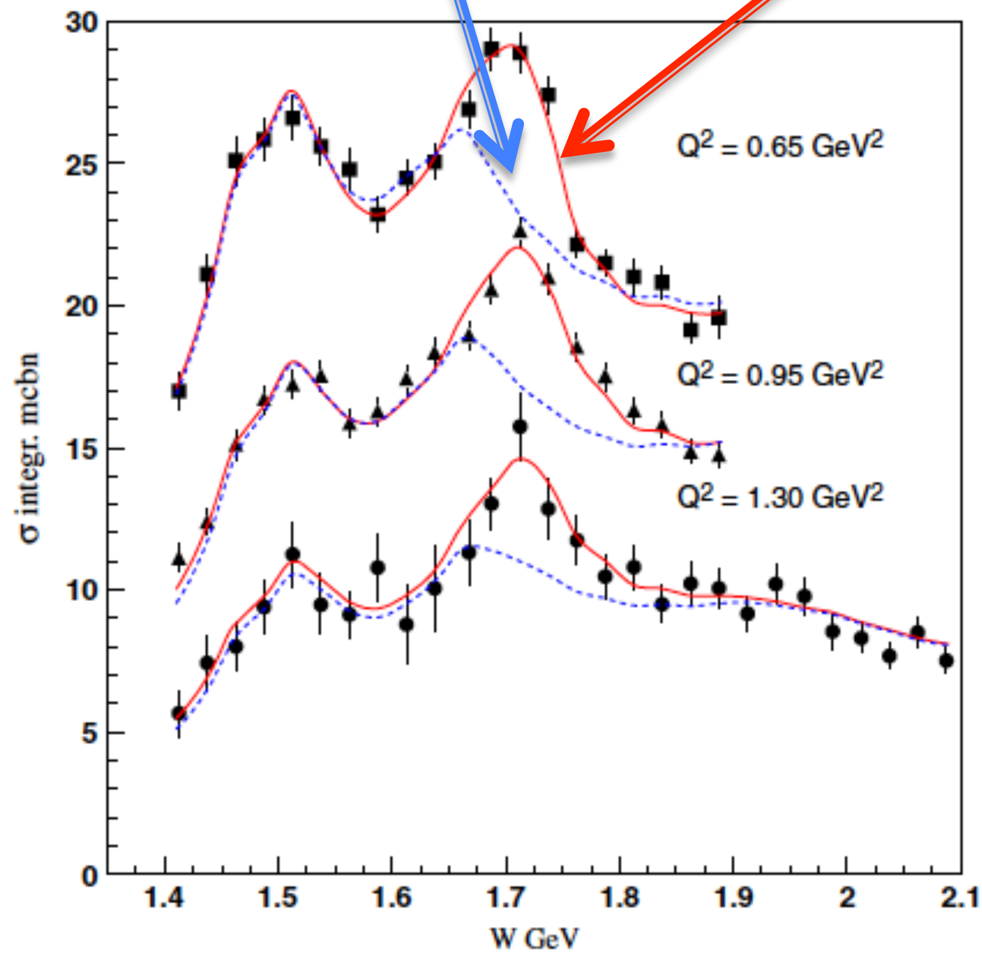
A new resonance?

Electroproduction to $\pi^+\pi^-p$ channel

Data: Ripani, PRL 91, 022002
Analysis: Mokeev, PRC 80 045212

Conventional N*'s only

New $3/2^+$ state



Aznauryan and Burkert, Prog. Part. Nucl. Phys. **67, 1** (2012)

Electro and photo-production data are starting to have an impact

Physics
Old Dominion UNIVERSITY

Bonn-Gatchina analysis

N(mass)J^P	PDG 2012	$K\Lambda$	$K\Sigma$	γ p	pω	pη'
$N(1710)1/2^+$	***	***	**	***		
$N(1880)1/2^+$	**	**		**		
$N(2100)1/2^+$	*					2130
$N(1895)1/2^-$	**	**	*	**		1920
$N(1900)3/2^+$	***	***	**	***		
$N(2040)3/2^+$	*					2050
$N(1875)3/2^-$	***	***	**	***		
$N(2150)3/2^-$	**	**		**		
$N(2000)5/2^+$	**	**	*	**	1950	
$N(2060)5/2^-$	**		**	**		2080

N(1710) ½+ was not observed by GWU in the πN data.

Burkert, Mesons 2012

The “complete” experiment

If you consider electro-production, you have three amplitudes, each of which is complex:

$$\sigma_T(\nu_R, Q^2) = \frac{2M}{\Gamma_R M_R}\left[|A_{\frac{1}{2}}|^2 + |A_{\frac{3}{2}}|^2\right],$$

$$\sigma_L(\nu_R, Q^2) = \frac{4M}{\Gamma_R M_R}\left[|S_{\frac{1}{2}}|^2\right],$$

There are 12 amplitudes (3 photon spin × 2 N × 2 N’)
6 amplitudes are independent (parity)

6 Helicity amplitudes, each of which are complex
Must have 12 observables to extract reliably

For real photons you need 8 observables.

Detailed formalism in many reviews, for example:
Aznauryan and Burkert, Prog. Part. Nucl. Phys. **67, 1** (2012)

Observables: production of pseudoscalar mesons

Physics
Old Dominion UNIVERSITY

In real photon experiments, there are 16 observables:

- Unpolarized cross section (1)
- Beam, Target and Recoil single spin asymmetries (Σ, T, P) (3)
- Beam-Target, Beam-Recoil, Target-Recoil asymmetries (each with 4 combinations) (12)

For electron scattering there are 20 additional observables!!! (longitudinal photon + LT interference)

Beam		Target			Recoil			Target-Recoil								
		x	y	z	x'	y'	z'	x x'	y x'	z x'	x y'	y y'	z y'	x z'	y z'	z z'
	σ_0		T			P		$T_{x'}$		$L_{x'}$		$\hat{\Sigma}$		$T_{z'}$		$L_{z'}$
P_T	Σ	H	$\hat{P}$	G	$O_{x'}$	$\hat{T}$	$O_{z'}$	$\tilde{L}_{z'}$	$\tilde{C}_{z'}$	$\tilde{T}_{z'}$	$\tilde{E}$		$\tilde{F}$	$\tilde{L}_{x'}$	$\tilde{C}_{x'}$	$\tilde{T}_{x'}$
$P_\odot$		F		E	$C_{x'}$		$C_{z'}$		$\tilde{O}_{z'}$		$\tilde{G}$		$\tilde{H}$		$\tilde{O}_{x'}$	

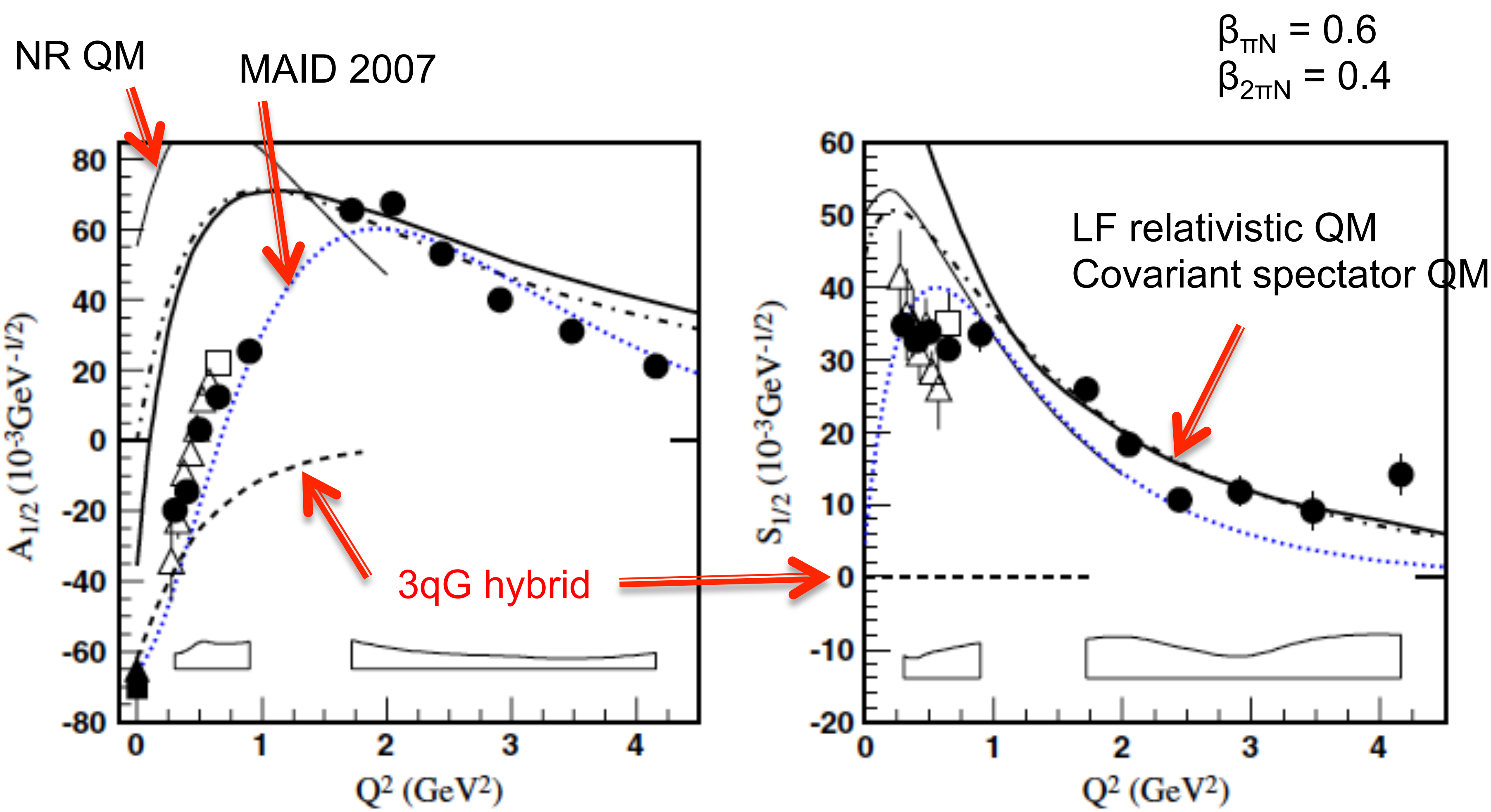
Roberts and Crede, 2013

Roper Resonance N(1440) P_{11}

Physics
Old Dominion UNIVERSITY

Data from CLAS π and 2π production

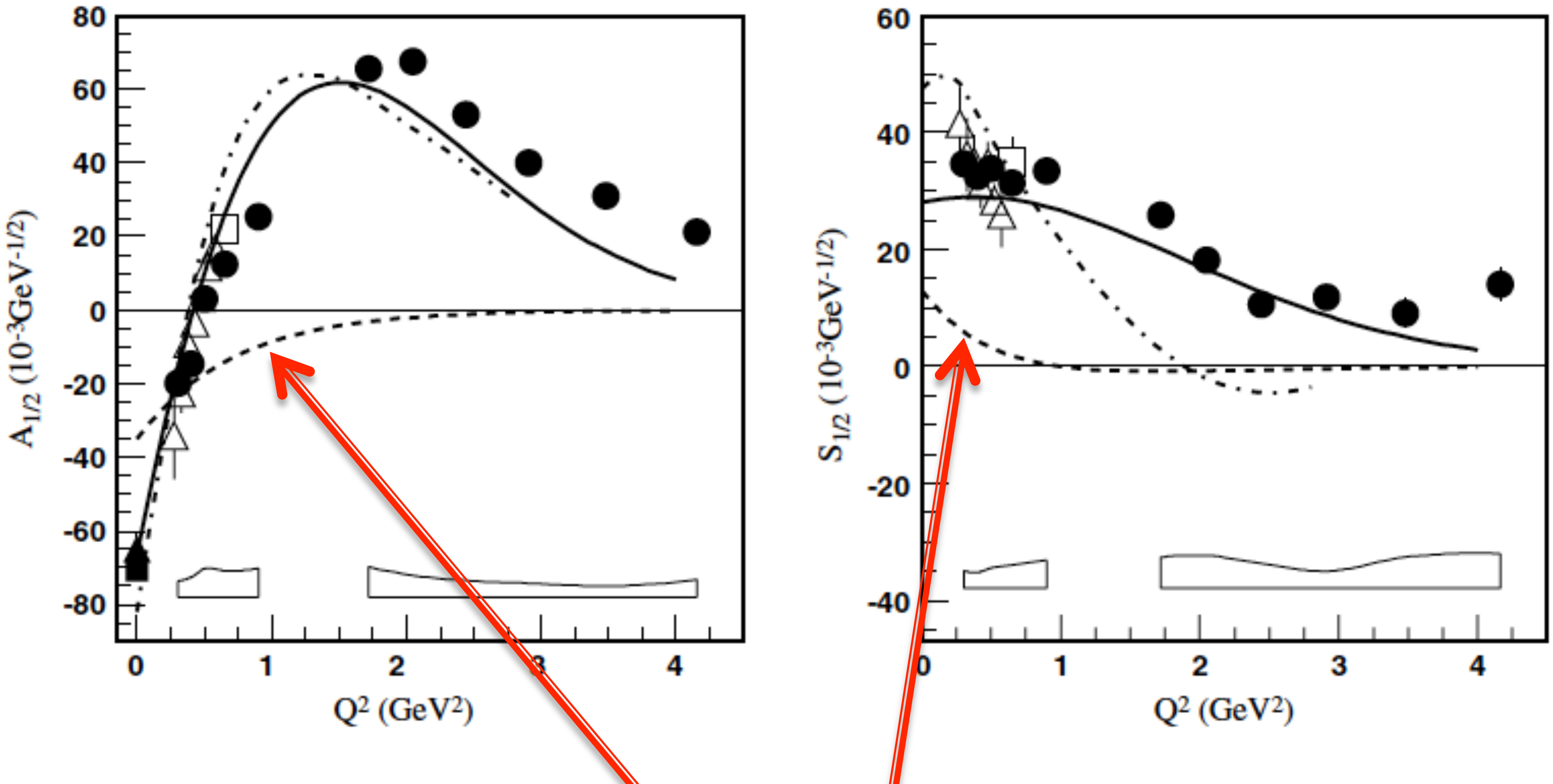
Evidence suggests a 3q radial excitation. Hybrid 3qG solutions is ruled out.



Aznauryan and Burkert, Prog. Part. Nucl. Phys. **67, 1** (2012)

Roper, cont.

Full curve (from Obukhovsky 2011)
includes meson-cloud contribution



meson-cloud contribution separately

Aznauryan and Burkert, Prog. Part. Nucl. Phys. **67, 1** (2012)

Partial Wave Analyses

Physics
Old Dominion UNIVERSITY

- SAID/MAID – includes data for pseudoscalar meson production; uses BW parametrization; websites have predictions for various quantities; does not include 2π decay channels
- EBAC – dynamical coupled channel of world data including 2π channels
- Giessen - coupled-channel analysis; Bethe-Salpeter equation with K-matrix for overlapping resonances
- Bonn-Gatchina – largest experimental database, including multiparticle final states; K-matrix, phenomenological background

Roberts and Crede, 2013

Physics
Old Dominion UNIVERSITY

In Conclusion…

This is an exciting time for nucleon resonances as new data enables discovery of missing resonances through rigorous PWIA fits.

SU(3), Group Theory & the Quark Model

Physics
Old Dominion UNIVERSITY

- PDG
- http://vietsciences1.free.fr/vietscience/giaokhoa/vatly/vatlyluongtu/PhamXuanYem/QYbookCHAPT07.PDF
- https://workspace.imperial.ac.uk/theoreticalphysics/public/MSc/PartSymm/SU(3)Notes.pdf
- http://hepwww.rl.ac.uk/Haywood/Group_Theory_Lectures/Lecture_4.pdf

Breit-Wigner

Physics
Old Dominion UNIVERSITY

For an isolated resonance (i.e. Δ), the cross section can be expressed as

$$\sigma_{BW}(E) = \frac{(2J+1)}{(2S_1+1)(2S_2+1)} \frac{\pi}{k^2} \frac{B_{in} B_{out} \Gamma_{tot}^2}{(E-E_R)^2 + \Gamma_{tot}^2/4}$$

Non-relativistic

S_1 and S_2 are the spins of the collision particles
J is the total angular momentum of the resonance
k is the c.m. momentum
E is the c.m. energy
Bin and Bout are the branching fractions of the resonance into the entrance and exit channel
Γ is the width of the resonance

The lifetime of the resonance/state is $\tau = \frac{\hbar}{\Gamma}$

Nucleon resonance widths are large: ~100 MeV and the resonances overlap