Scattering Experiments (how to interpret them)

- Experimental Overview
- NN scattering
- Partial Waves and Phase Shifts
- Optical Theorem
- S-Matrix

Next Week: Elastic Scattering, Feynman Rules, Form Factors

Scattering: an Experimental Overview

A + B $\rightarrow$ C + D

Detector at θ, φ with area da at distance r

$$d\Omega = da/r^2 = sin\theta \, d\theta \, d\varphi$$

We measure # counts N detected in detector at θ, φ with solid angle $d\Omega$:

$$\frac{d\sigma}{d\Omega} = \frac{N^*}{(\#\,\text{target particles})(\#\,\text{beam particles})}$$

Units are area per solid angle

#/unit area

Integrated current/*e*

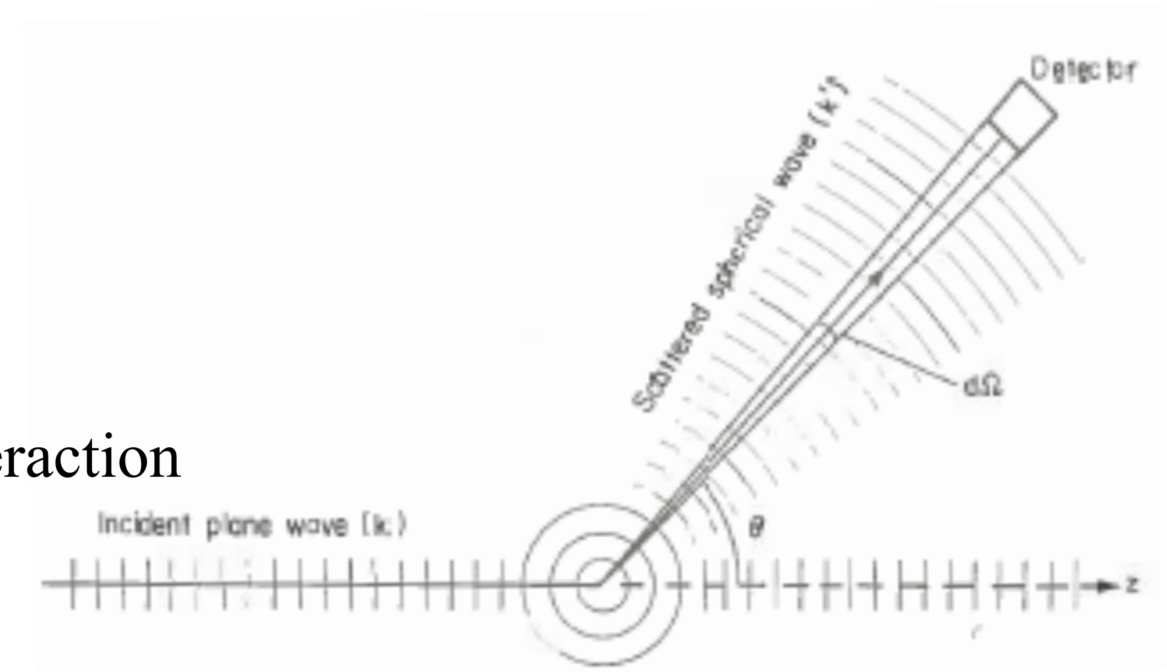
Units: 1 barn = 10^{-28} m^2

Hadronic processes ~ 1 millibarn
Electromagnetic ~ 1 nano barn
Weak ~ 1 femtobarn

NN Scattering

Short range nuclear force, potential V(r)

V(r) = 0 except in the interaction region



$$\psi(r,\theta,\phi) \to e^{ikz} + f(\theta,\phi)\frac{e^{ikr}}{r}$$

$k = \sqrt{2\mu E}/\hbar$

μ is the reduced mass

Incoming plane wave + unscattered

Scattering amplitude

Outgoing spherical wave

f(θ,φ) is independent of φ if the beam and target are unpolarized

Cross section

Quantum mechanical probability current density:

$$S(r,t) = \mathrm{Re}\left\{\psi^* \frac{\hbar}{i\mu}\nabla\psi\right\} = \frac{\hbar}{\mu}\mathrm{Im}\left\{\psi^*\nabla\psi\right\}\Big\}$$

$$S_i = \frac{\hbar k}{\mu} = v$$

$$S_r = \frac{v}{r^2}\left|f(\theta)\right|^2 + \vartheta(r^{-3})$$

S_i is the number of incident particles passing through a unit area perpendicular to the z-axis.
There is a normalization volume *V* that cancels in the cross section and can be taken to be 1.

Number of particles detected per unit time:
$N_r = S_r da = S_r r^2 d\Omega$

$$\frac{d\sigma}{d\Omega} = \frac{S_r r^2}{S_i} = \left|f(\theta)\right|^2$$

Ignoring spin so cross section is independent of φ

Partial Waves

For a central potential the relative angular momentum *l* between two scattered particles is conserved.

$$\psi(r,\theta)=\sum a_l Y_{l0} R_l(k,r)$$

Note that $Y_{l0}(\theta)=\sqrt{\frac{2l+1}{4\pi}}P_l(\cos\theta)$

$$R_l(r)\xrightarrow[r\to\infty]{scattering}\frac{1}{kr}\sin(kr-\tfrac{1}{2}l\pi+\delta_l)$$

Elastic scattering; phase change only

The incoming plane wave can also be decomposed into partial waves:

$$e^{ikz}=\sum_l\sqrt{4\pi(2l+1)}i^l j_l(kr)Y_{l0}(\theta)$$

Free particle: V=0

$$j_l(kr)\xrightarrow[r\to\infty]{free}\frac{\sin(kr-\tfrac{1}{2}l\pi)}{kr}$$

Scattering due to potential V(r) causes partial wave l to be shifted by δ_l compared to a free particle.

δ_l is the phase shift.

Elastic Scattering

The scattering amplitude can also be expressed in terms of partial waves:

$$f(\theta) = \frac{\sqrt{4\pi}}{k} \sum_l \sqrt{2l+1}\, e^{i\delta_l} \sin\delta_l Y_{l0}(\theta)$$

$$\sigma = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2 \delta_l$$

Note that δ_l depends on k

This result assumes real potential and phase shifts:
Elastic scattering

Phase shifts can be compared with calculations

More generally we need to consider complex potentials, which lead to complex phase shifts (inelastic scattering).

So far we have ignored spin; we need to consider total angular momentum

J = L + S

Phase Shifts – NN scattering

Wong, pg. 97

Notation: $^{2S+1}L_J$

NN wavefunction must be antisymmetric

pp scattering:

T=1 (sym)

S = 0 (anti sym)

L = 0 or 2 (sym) $^1S_0$ or $^1D_2$

S = 1 and L = 1

J = 0, 1, or 2 $^3P_0$ $^3P_1$ $^3P_2$

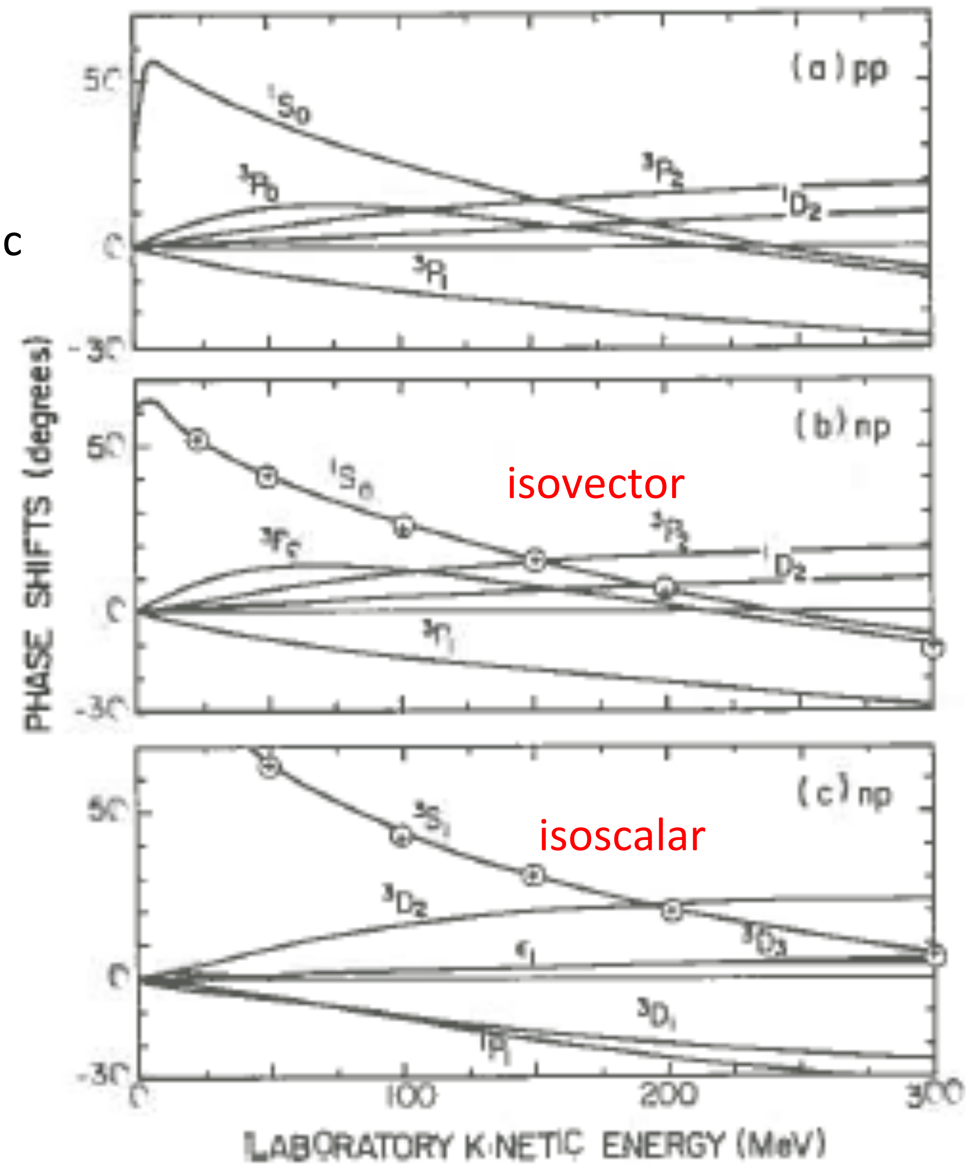
np scattering:

T=0; S=0; L odd $^1P_1$

T=1; S=1; L odd $^3P_0$ $^3P_1$ $^3P_2$

For $\delta > 0$ force is attractive

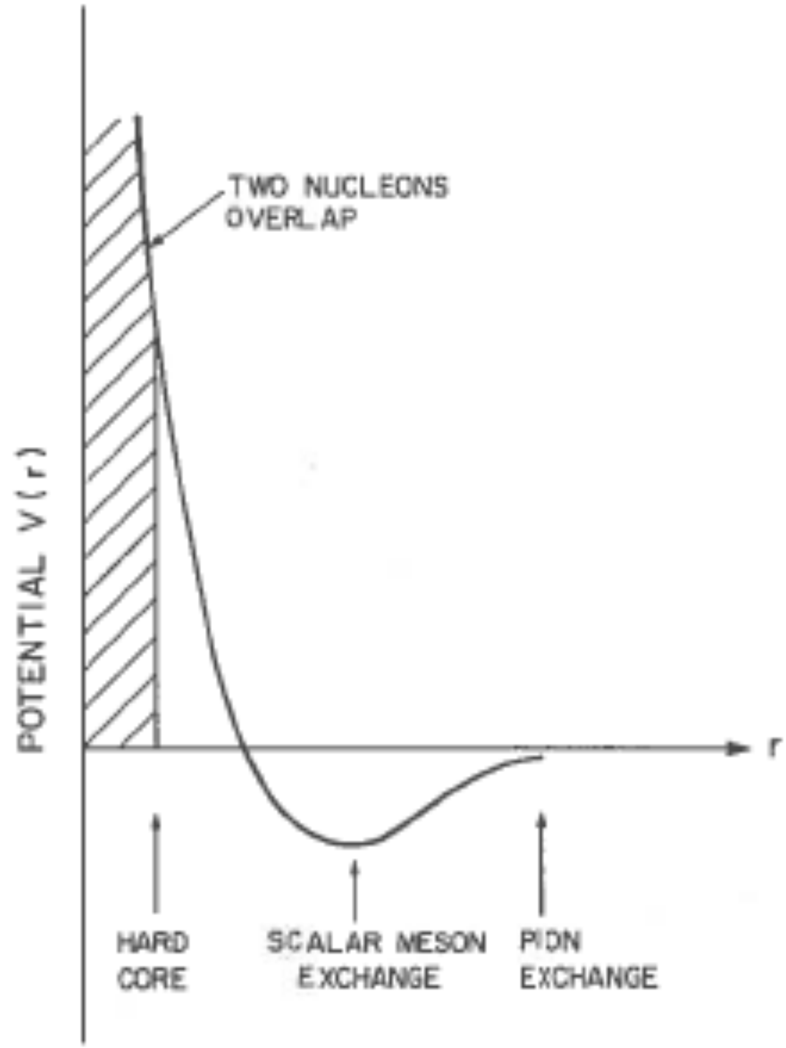
For $\delta < 0$ force is repulsive



R is decreasing as energy increases

NN Potential

Hard Repulsive Core for r ~ 0.4 fm
Attractive at r >~ 1 fm
Long range part starts at ~ 2 fm



Low Energy Scattering

Relative angular momentum causes an effective barrier to the incoming particle

$$\tilde{V}(r) = V(r) + \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}$$

Low energy scattering is dominated by low order partial waves

$$\delta_l \to 0 \text{ for } l \gg kr_0$$

r_0 is the range of the potential
For E = 1 MeV, kr_0 ~ 0.2

At very low energies, we define the scattering length *a*

$$\lim_{E\to 0} \sigma = 4\pi a^2$$

Scattering from a Complex Potential

Scattering amplitude and phase shift are complex

$$u_l(r) \equiv rR_l(r) \xrightarrow[r\to\infty]{} I_l(r) - \eta_l O(r)$$

Inelasticity parameter

$$\sigma^{el} = \frac{\pi}{k^2}\sum_l (2l+1)\left|1-\eta_l\right|^2$$

$$\sigma^{re} = \frac{\pi}{k^2}\sum_l (2l+1)\left(1-\left|\eta_l\right|^2\right)$$

$$\sigma^{tot} = \frac{4\pi}{k}\mathrm{Im}\, f(0)$$

Optical Theorem: total cross section is related to scattering at θ = 0

S Matrix

Define reaction channel c to include all quantum numbers of target and projectile, as well as relative orbital angular momentum, *l* and *m*.

$$S_{cc'} = \left\langle \psi_{c'}^{out}(r) \middle| S \middle| \psi_{c}^{in}(r) \right\rangle$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{cc'} = \frac{\pi}{k^2} \left| \sum_{ll'} \sqrt{(2l+1)} S_{c'c} Y_{lm} \right|^2$$

The S matrix is unitary:

$$\sum_{\text{all states}} \left| S_{c'c} \right|^2 = 1$$

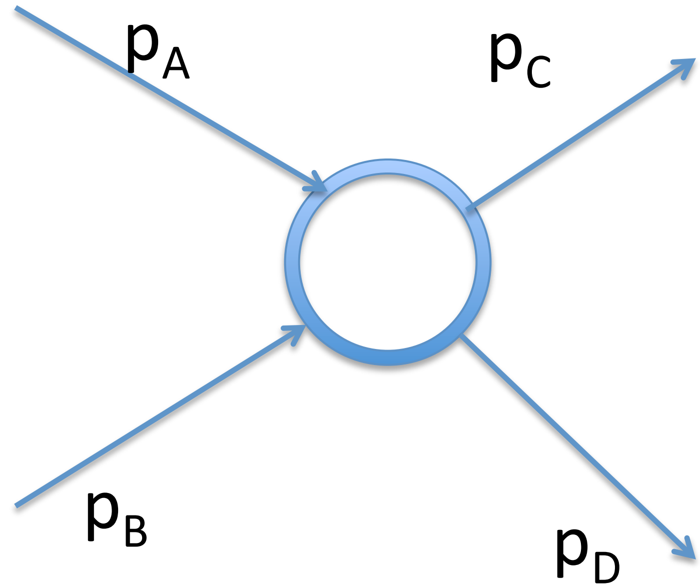
Mandelstam Variables

$A + B \rightarrow C + D$

$s = (p_A + p_B)^2$

$t = (p_A - p_C)^2$

$u = (p_A - p_D)^2$



References

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- F. Halzen and A. Martin, “Quarks & Leptons: An Introductory Course in Modern Particle Physics
- D. Perkins, “Introduction to High Energy Physics,”