Some kinematics – in the lab system

$k^\mu = (E, 0, 0, E) = (E, \vec{k})$

$k'^\mu = (E', E'\sin\theta, 0, E'\cos\theta) = (E', \vec{k}')$

$q^\mu = (E-E', -E'\sin\theta, 0, E-E'\cos\theta) = (\nu, \vec{q})$

$Q^2 = -q^\mu q_\mu = -\nu^2 + \vec{q}^{\,2} = (E-E'\cos\theta)^2 + E'^2\sin^2\theta - (E-E')$

$= E^2 - 2EE'\cos\theta + E'^2 - E^2 - E'^2 + 2EE'$

$= 2EE'(1-\cos\theta) = 4EE'\sin^2\frac{\theta}{2}$

$p^\mu = (M, 0, 0, 0) = (M, \vec{0})$

$p'^\mu = p^\mu + q^\mu = (M+\nu, \vec{q})$

$p'^\mu p'_\mu =: W^2 = M^2 + 2M\nu + \nu^2 - \vec{q}^{\,2}$

$= M^2 + 2M\nu - Q^2$

Elastic Scattering: $W^2 \overset{!}{=} M^2$

$\Rightarrow \nu_{el} \overset{!}{=} \frac{Q^2}{2M}$ or $x_{el} = \frac{Q^2}{2M\nu_{el}} \overset{!}{=} 1$

Elastic cross section – final form

$$\Delta\sigma = \frac{4z^2\alpha^2(\hbar c)^2}{Q^4}\cos^2\frac{\theta}{2}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau\tan^2\frac{\theta}{2}\,G_M^2\right] \cdot E'^2\,\Delta\Omega\,\frac{E'}{E}$$

$\frac{E'}{E}$: Recoil

$\frac{G_E^2 + \tau G_M^2}{1+\tau}$: LONGITUDINAL (charge)

$2\tau\tan^2\frac{\theta}{2}\,G_M^2$: TRANSVERSE (magnetic)

$\tau = \frac{\nu^2}{Q^2}$, $G_E(Q^2)$, $G_M(Q^2)$: Form Factors

Dirac Particle: $G_E = G_M = 1$ (const.)

Anomalous magnetic moment: $G_M \approx (1+\kappa)G_E$

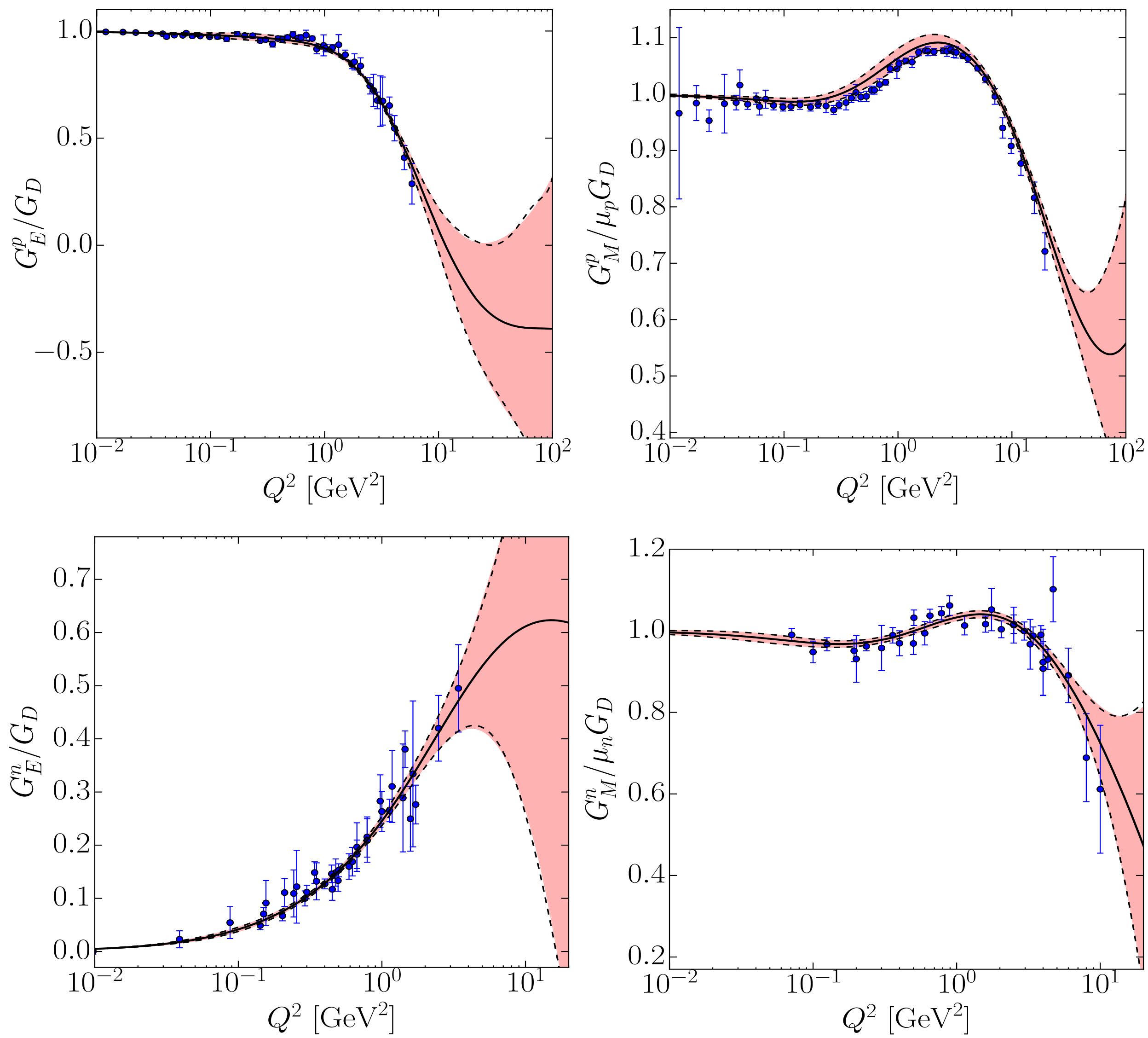
Extended Charge distribution:

$G_E(Q^2) \simeq$ Fourier transform of $\rho(r)$

Ex: $\rho(r) \simeq \frac{a^3}{8\pi}e^{-ar} \Rightarrow G_E(Q^2) \simeq \left(\frac{1}{1+\frac{Q^2}{a^2}}\right)^2$

(Dipole Form). p: $a^2 = 0.71\ \mathrm{GeV}^2$

Electromagnetic Form Factors



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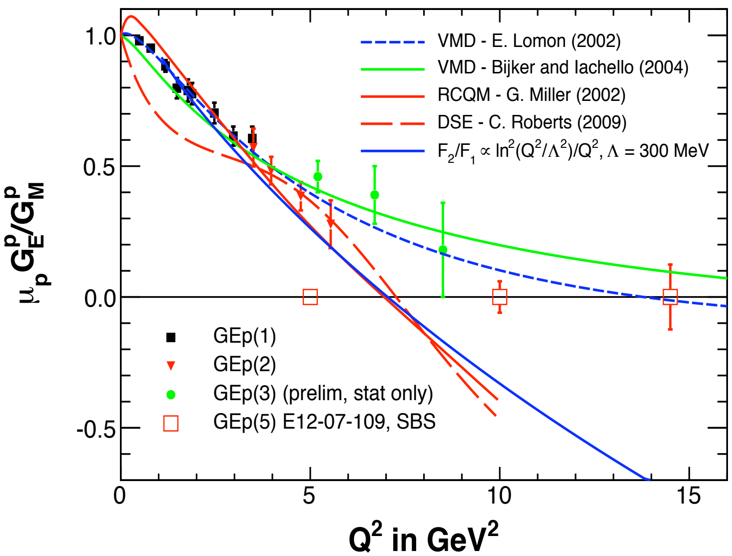
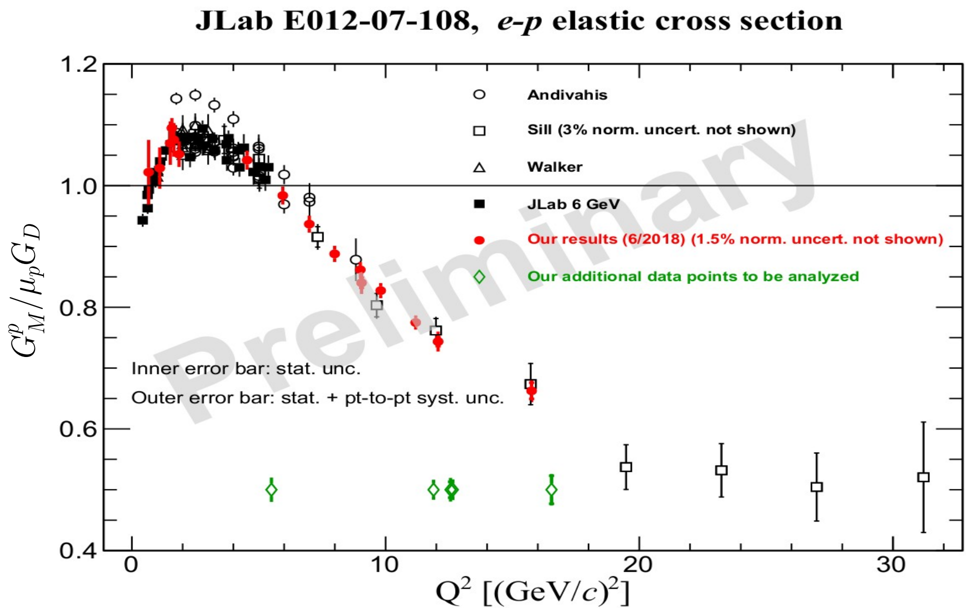
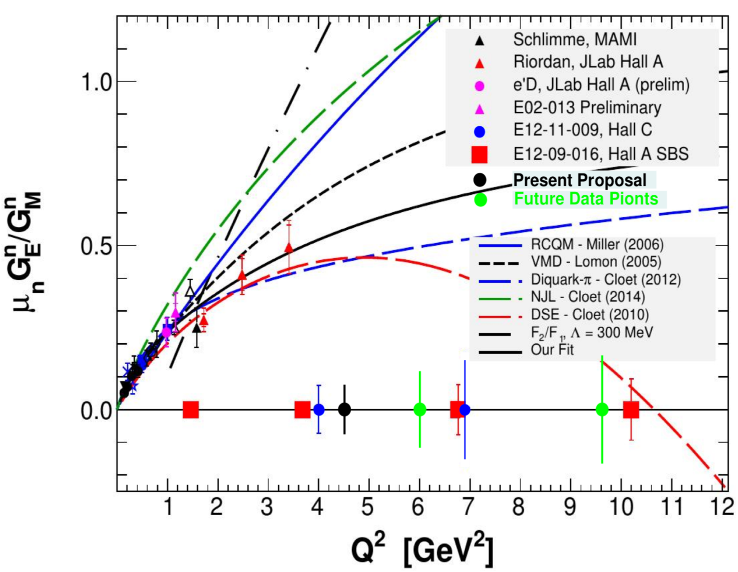
$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\epsilon (G_E^N)^2 + \tau (G_M^N)^2}{\epsilon(1+\tau)}$$

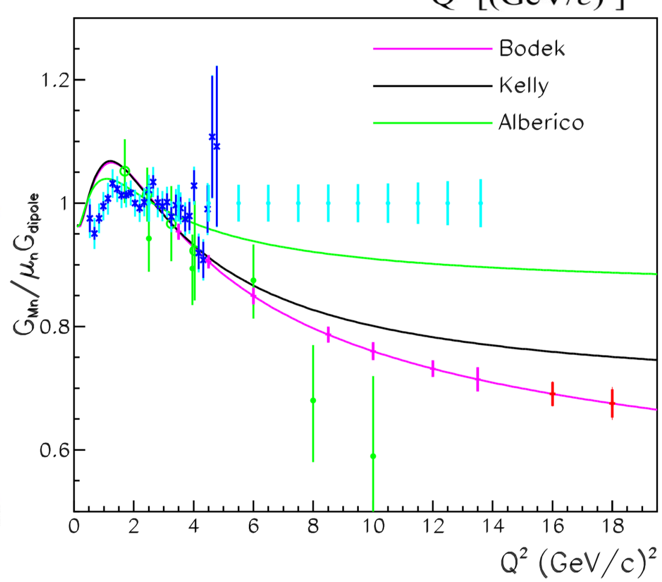
$$\tau = \frac{Q^2}{4m_N^2}, \quad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

Electromagnetic Form Factors









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