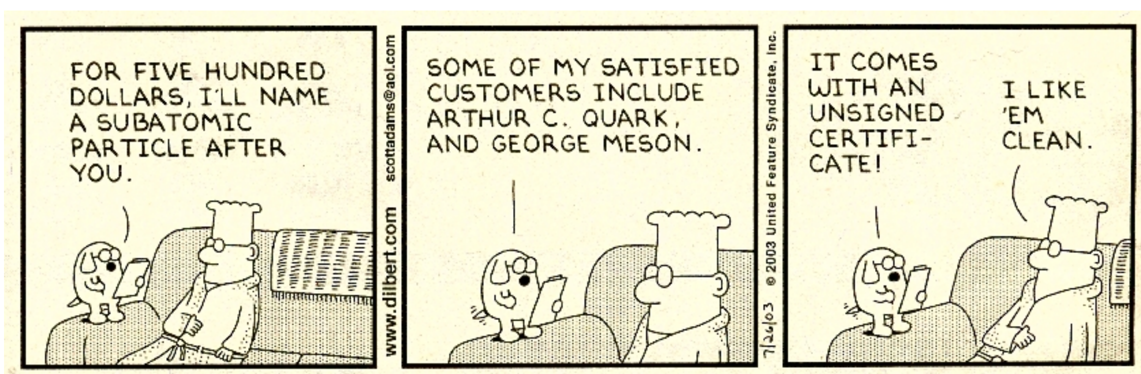
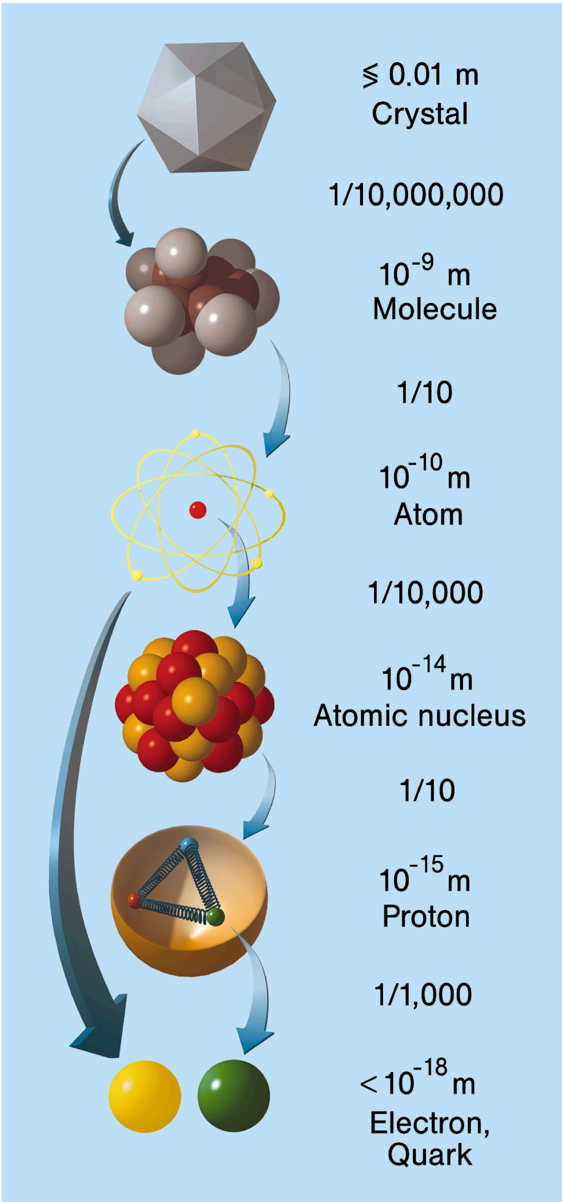
PHYS323:

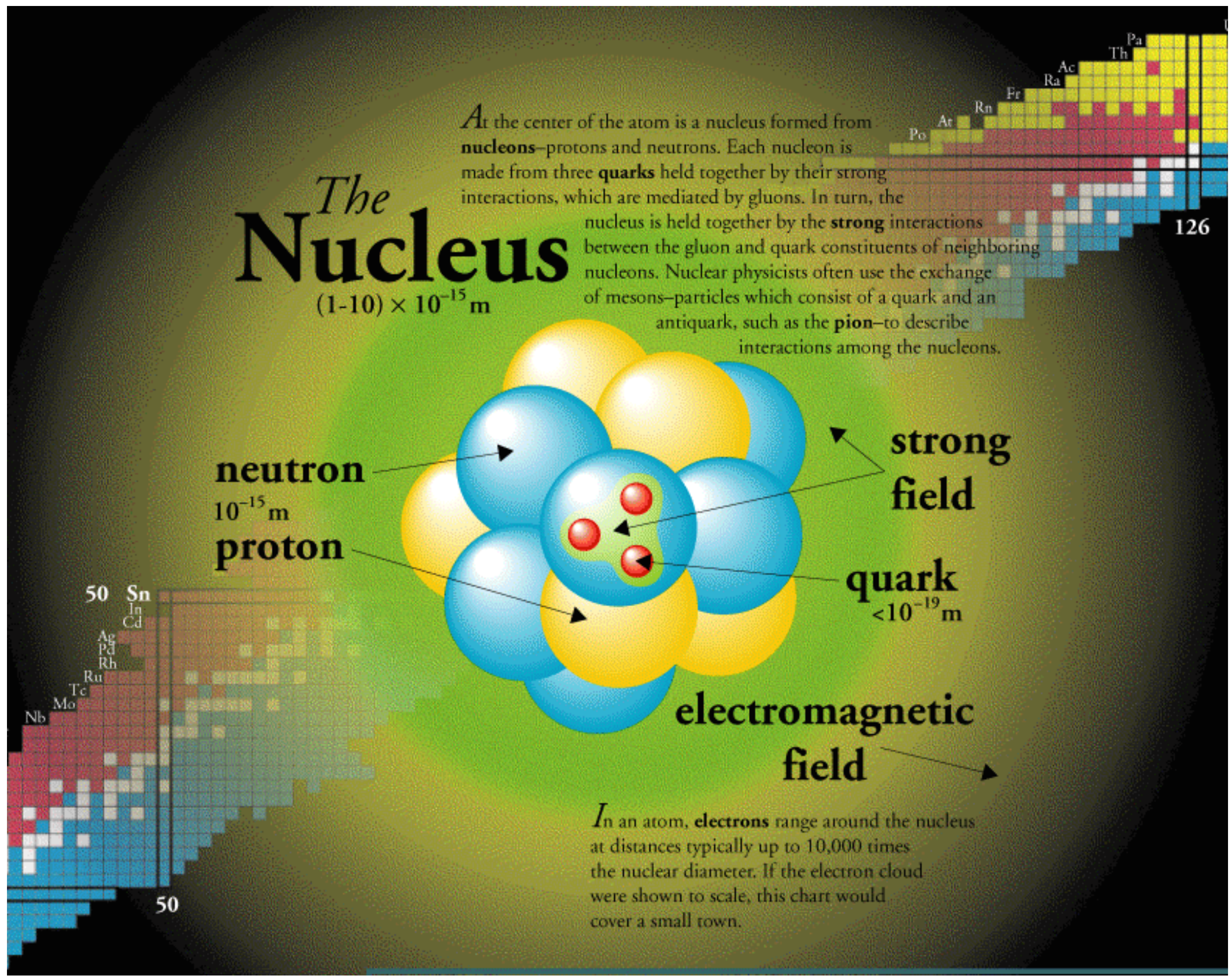
From Particle to Nuclear Physics

Sebastian Kuhn



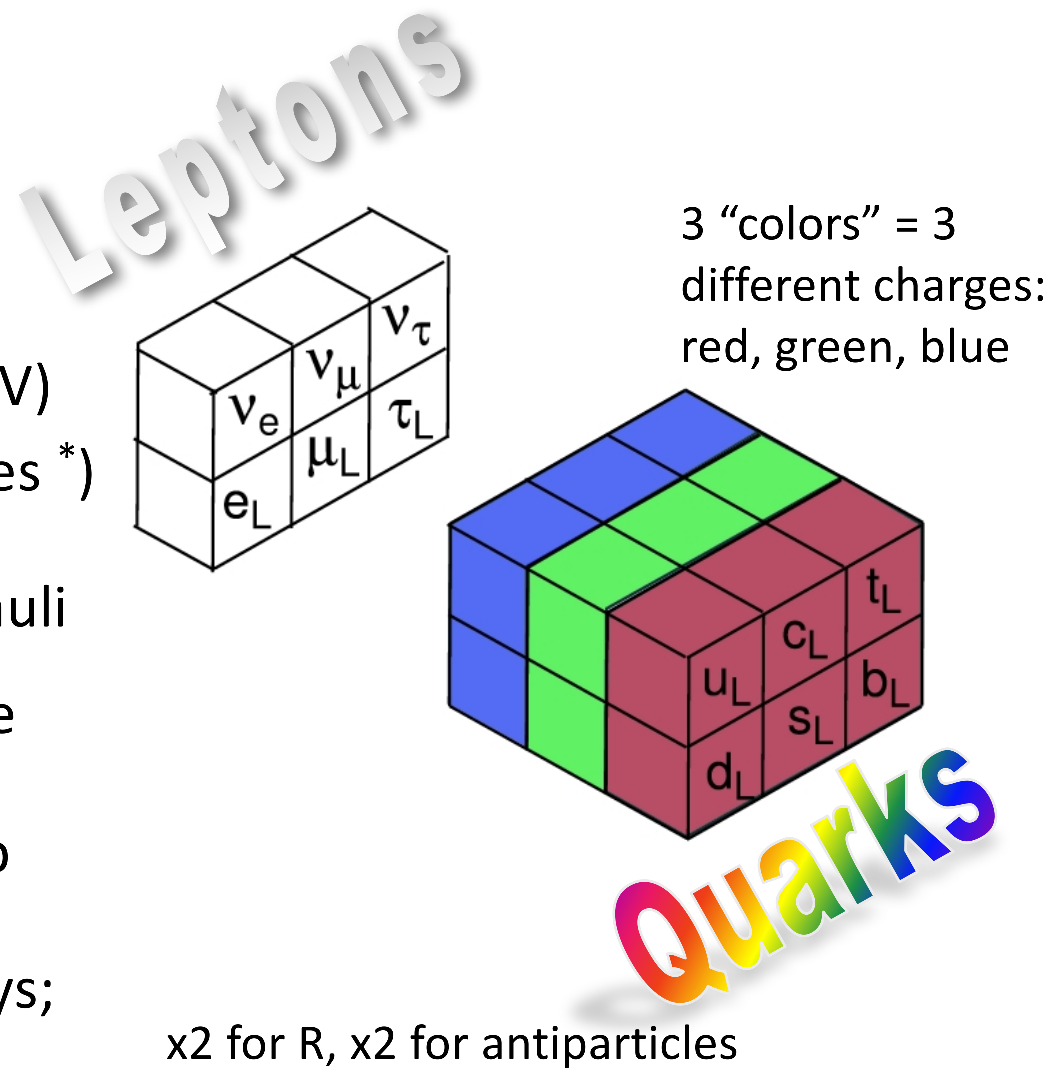
The Structure of Matter





Matter Particles

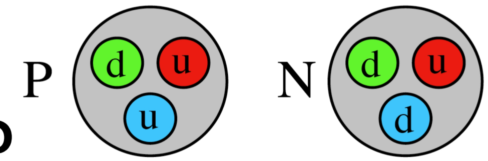
- Make up visible matter
- Pointlike ($<10^{-18}$ m), Fundamental $^{*)}$
- Have mass (from < ½ eV to 178,000,000,000 eV = 178 GeV)
- Distinct from their antiparticles $^{*)}$
- Fermions (Spin ½) ⇒ they "defend" their space (Pauli Principle) and can only be created in particle-antiparticle pairs
- Can be "virtual", but make up matter being (nearly) "real"
- "stable" (against strong decays; lifetimes from ∞ to 10^{-24} s)



$^{*)}$ Until further notice

Hadronic Particle Zoo

- what can one build from quarks?



HADRONS

Family Name	Particle Name	Particle Symbol	Antiparticle Symbol	Composition	Mass	Electric Charge	Lifetime in Seconds
baryon	proton	p or p^+	$\bar{p}$	uud	1,836	+1	stable
	neutron	n or n^0	$\bar{n}$	udd	1,839	0	887
	lambda	Λ^0	$\bar{\Lambda}^0$	uds	2,183	0	2.6×10^{-10}
	lambda-c	Λ^+_c	$\bar{\Lambda}^-_c$	udc	4,471	+1	2.1×10^{-13}
	lambda-b	Λ^0_b	$\bar{\Lambda}^0_b$	udb	11,000	0	1.1×10^{-12}
	sigma	Σ^+	$\bar{\Sigma}^-$	uus	2,328	+1	0.8×10^{-10}
		Σ^0	$\bar{\Sigma}^0$	$\frac{(ud \pm du)s}{\sqrt{2}}$	2,334	0	7.4×10^{-20}
		Σ^-	$\bar{\Sigma}^+$	dds	2,343	−1	1.5×10^{-10}
	xi	Ξ^0	$\bar{\Xi}^0$	uss	2,573	0	2.9×10^{-10}
		Ξ^-	$\bar{\Xi}^+$	dss	2,585	−1	1.6×10^{-10}
	xi-c	Ξ^0_c	$\bar{\Xi}^0_c$	dsc	4,834	0	9.8×10^{-14}
		Ξ^+_c	$\bar{\Xi}^-_c$	usc	4,826	+1	3.5×10^{-13}
	omega	Ω^-	$\bar{\Omega}^+$	sss	3,272	−1	0.8×10^{-10}
	omega-c	Ω^0_c	$\bar{\Omega}^0_c$	ssc	5,292	0	6.4×10^{-14}
meson	pion	π^+	π^-	$u\bar{d}$	273	+1	2.6×10^{-8}
		π^0	π^0	$\frac{(u\bar{u} - d\bar{d})}{\sqrt{2}}$	264	0	8.4×10^{-17}
	kaon*	K^+	K^-	$u\bar{s}$	966	+1	1.2×10^{-8}
		K^0	$\bar{K}^0$	$d\bar{s}$	974	0	8.9×10^{-11} 5.2×10^{-8}
	J/psi	J or Ψ	J or Ψ	$c\bar{c}$	6,060	0	1.0×10^{-20}
	omega	ω	ω	$\frac{(u\bar{u} + d\bar{d})}{\sqrt{2}}$	1,532	0	6.6×10^{-23}
	eta	η	η	$\frac{(u\bar{u} + d\bar{d})}{\sqrt{2}}$	1,071	0	3.5×10^{-22}
	eta-c	η_c	η_c	$c\bar{c}$	5,832	0	3.1×10^{-22}
	B	B^0	$\bar{B}^0$	$d\bar{b}$	10,331	0	1.6×10^{-12}
		B^+	B^-	$u\bar{b}$	10,331	+1	1.6×10^{-12}
	B-s	B^0_s	$\bar{B}^0_s$	$s\bar{b}$	10,507	0	1.6×10^{-12}
	D	D_0	$\bar{D}_0$	$c\bar{u}$	3,649	0	4.2×10^{-13}
		D^+	D^-	$c\bar{d}$	3,658	+1	1.1×10^{-12}
	D-s	D^+_s	D^-_s	$c\bar{s}$	3,852	+1	4.7×10^{-13}
	chi	χ^0_c	χ^0_c	$c\bar{c}$	6,687	0	3.0×10^{-22}
	psi	Ψ^0_c	Ψ^0_c	$c\bar{c}$	7,213	0	1.5×10^{-20}
	upsilon	Υ	Υ	$b\bar{b}$	18,513	0	8.0×10^{-20}

*The neutral kaon is composed of two particles; the average lifetime of each particle is given.

Forces and Force Carriers

- Mediate Interactions (Forces) - form “Waves”
- Pointlike, Fundamental
- Massless *)
- Some are their own antiparticles (photon, Z^0, graviton)
- Spin 1, 2 -> Bosons (tend to cluster together, can be produced in arbitrary numbers)
- Can be real, but carry forces as virtual particles
- Some are absolutely stable (γ, gluons, gravitons)

*) See next slide

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	**Mass GeV/c^2**	**Electric charge**
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

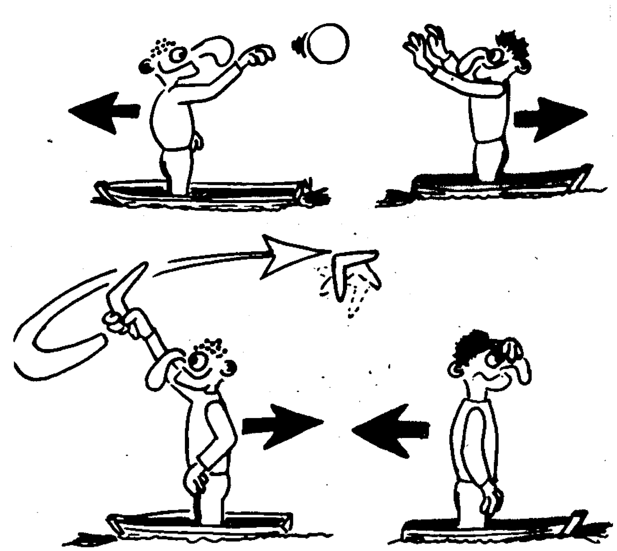
Strong (color) spin = 1		
Name	**Mass GeV/c^2**	**Electric charge**
g gluon	0	0

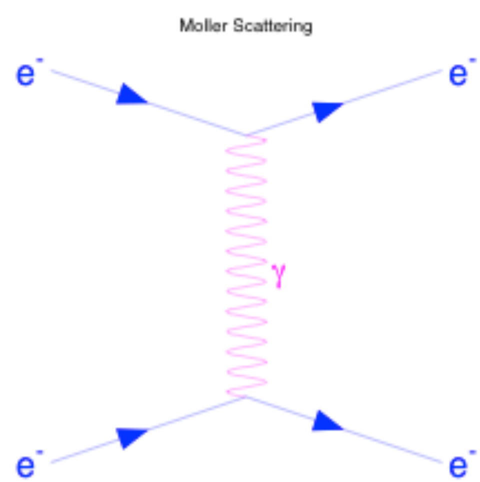
Gravitation spin = 2		
Name	**Mass GeV/c^2**	**Electric charge**
g graviton	0	0

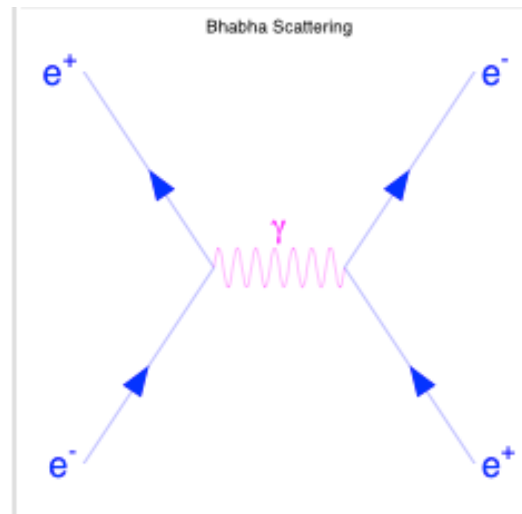
Note: gluons come in 8 possible combinations of color/anticolor (9^{th} is “sterile” – doesn’t exist)

Interactions

- Matter Particles interact with each other by exchanging Gauge Bosons
- Strength of Interaction determined by coupling ("charge": electromagnetic *e*, weak *g*, color α_S)
- Range of interaction determined by mass *) of gauge boson and Heisenberg uncertainty principle
- Examples:
  - $e^- e^{+/-}$ scattering (E&M)
  - neutron beta decay (weak)
  - quark-quark interaction (strong)
    - Confinement
    - Asymptotic freedom
    - Mesons, baryons…
    - Nπ interaction, NN interaction
- ALL interactions MUST conserve energy and charge!



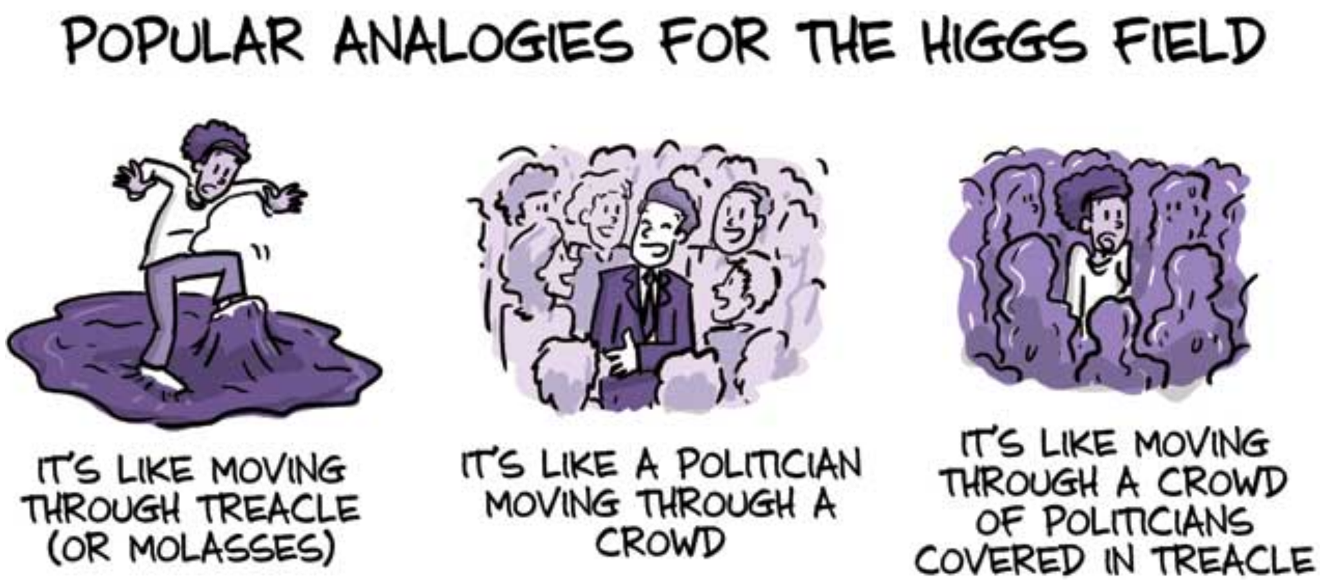


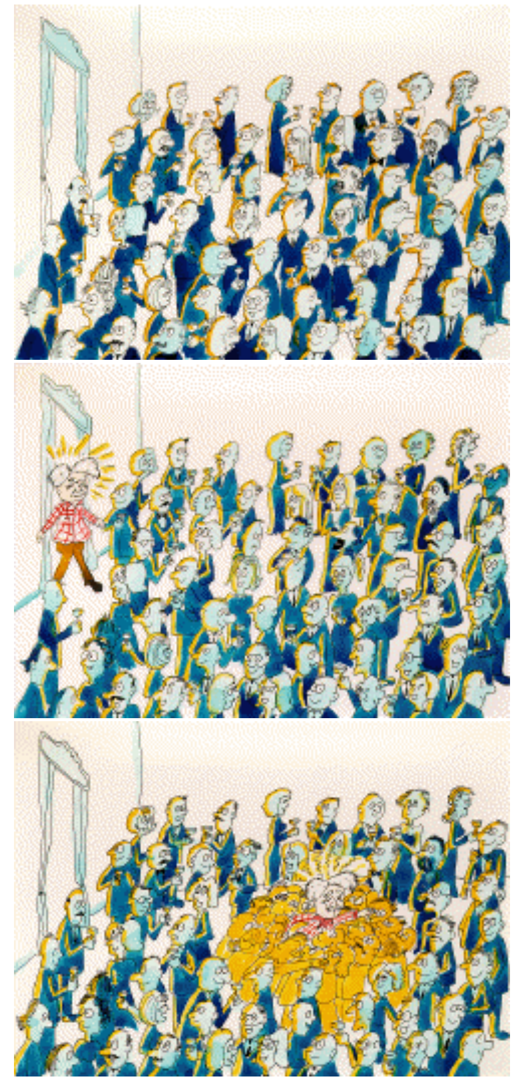


*) Huh? See next slide

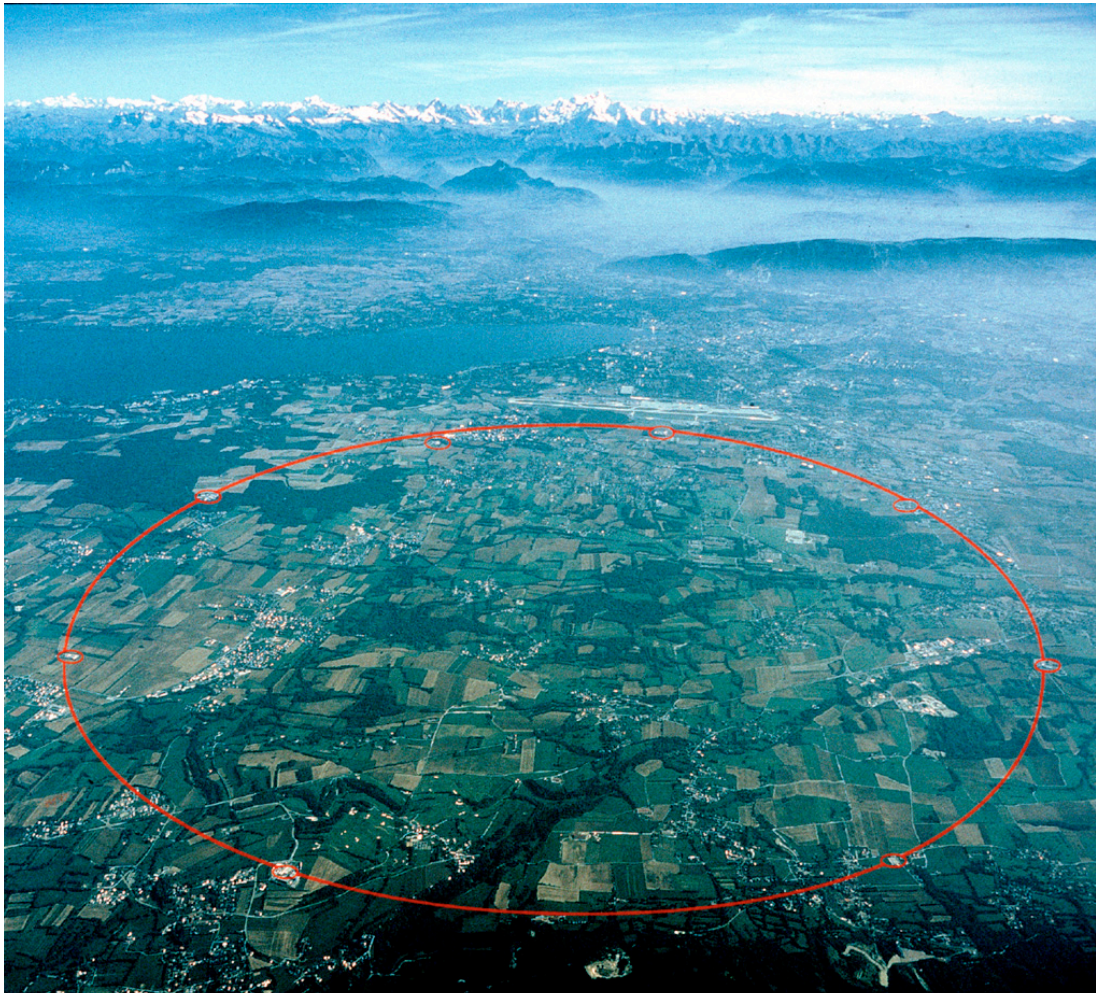
Higgs Field

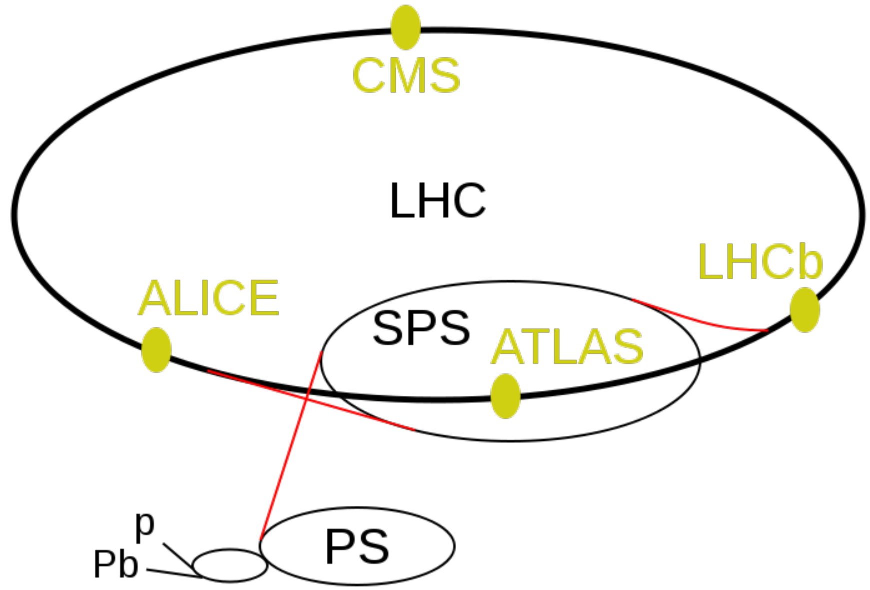
- Create "Drag" on Particles ("Molasses")
- *) Origin of Mass
  Makes some gauge bosons very heavy (W's, Z's) and therefore short-range ("Weak" interaction)
- Origin of electroweak symmetry breaking
- Pointlike, Fundamental

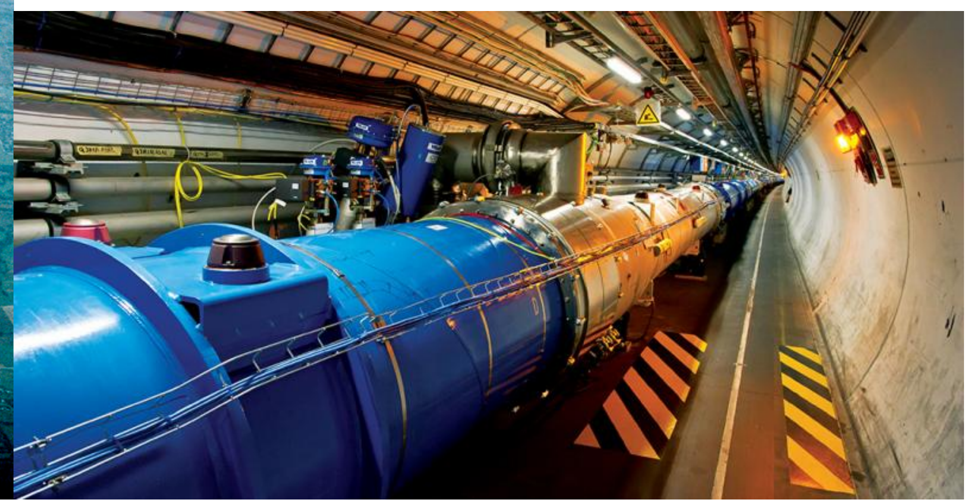




The LHC at CERN

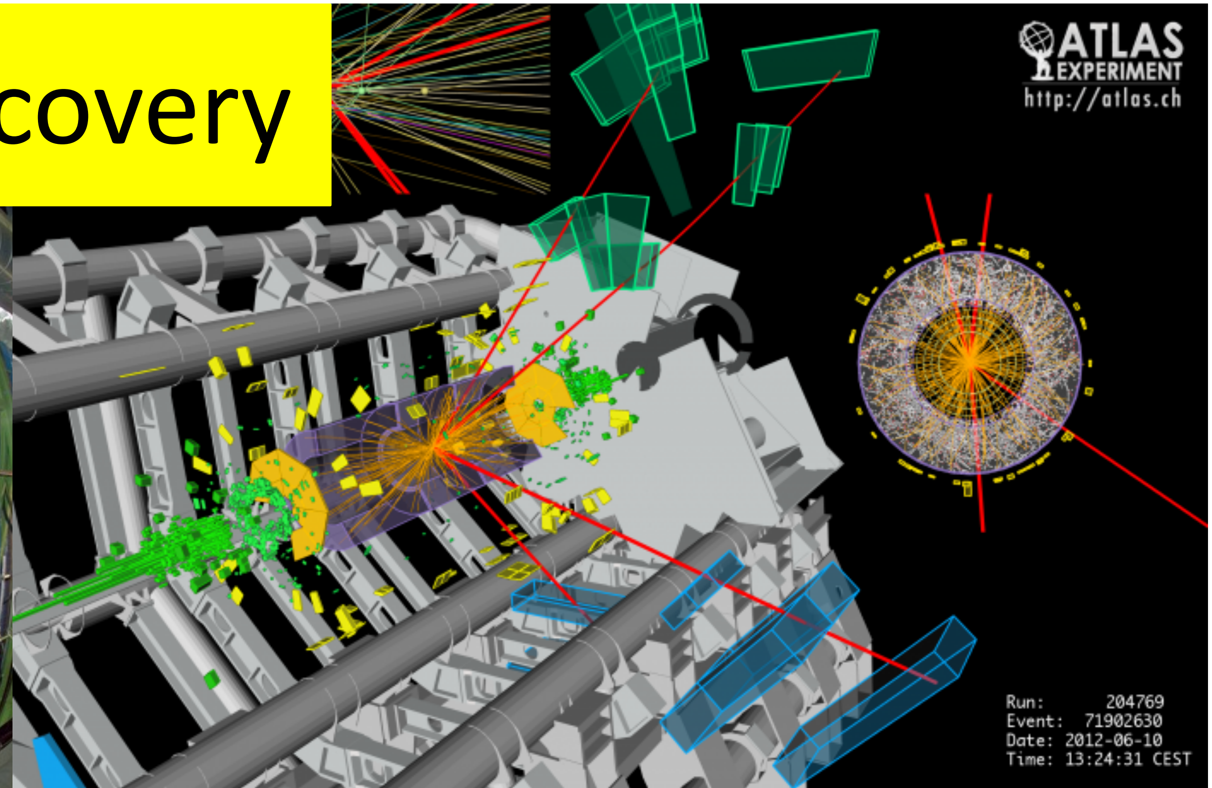


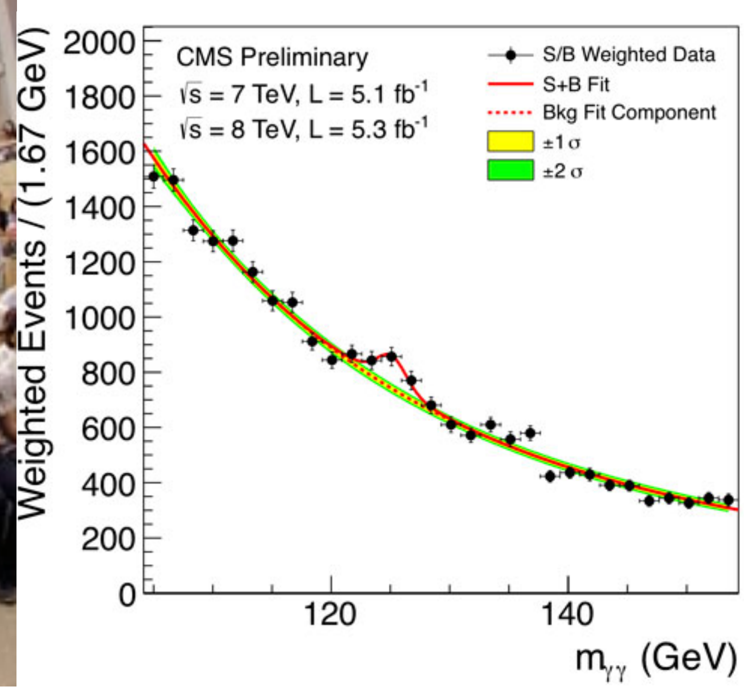




- See also the movie “Particle Fever”

Higgs Discovery





The following are excerpts from

BEYOND THE STANDARD MODEL IN MANY DIRECTIONS*

Chris Quigg
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P.O. Box 500, Batavia, Illinois 60510 USA

arXiv:hep-ph/0404228 v1 26 Apr 2004

and

STRUCTURE OF THE STANDARD MODEL[1]

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Department of Physics, University of Pennsylvania,
Philadelphia, Pennsylvania, USA 19104-6396

arXiv:hep-ph/0304186 v1 19 Apr 2003

The Standard Model (SM) [1,2] is a gauge theory, based on the group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, which describes strong, weak and electromagnetic interactions, via the exchange of the corresponding spin-1 gauge fields: 8 massless gluons and 1 massless photon for the strong and electromagnetic forces, respectively, and 3 massive bosons, $W^\pm$ and Z, for the weak interaction. The gauge symmetry determines the dynamics in terms of the three couplings g_s, g and g', associated with the $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ subgroups. Strong interactions are governed by the first group factor, while the other two provide a unified description of the electroweak forces, their gauge parameters being related through $g \sin\theta_W = g' \cos\theta_W = e$. $\sin^2\theta_W = 0.23113 \pm 0.00021$

The fermionic matter content is given by the known leptons and quarks, which are organized in a 3-fold family structure:

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix}, \qquad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix}, \qquad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}, \tag{1}$$

where (each quark appears in 3 different *colours*)

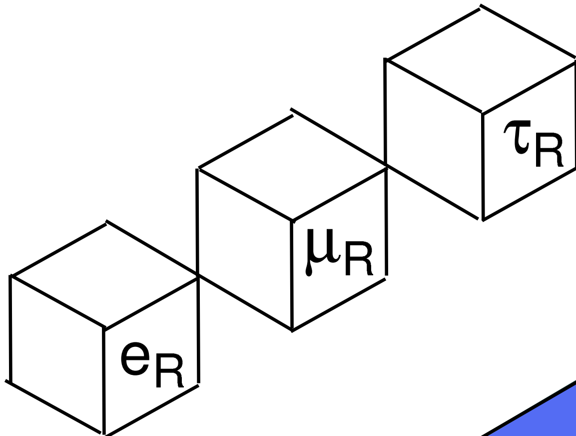
$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L, \quad \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L, \quad l^-_R, \quad (q_u)_R, \quad (q_d)_R, \quad \nu_R \tag{2}$$

Similarly, with L and R interchanged, for antifermons

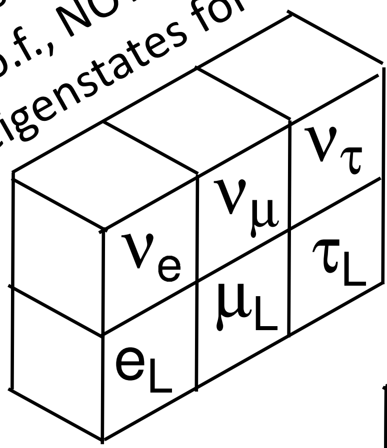
Spontaneous EW symmetry breaking:

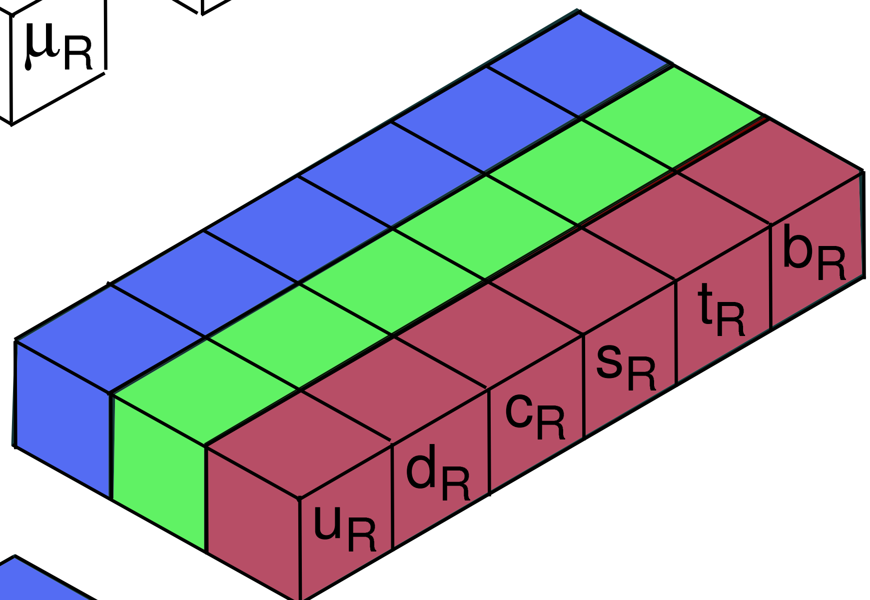
$$\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y \rightarrow \mathrm{U}(1)_\mathrm{em}$$

+ ν_R's

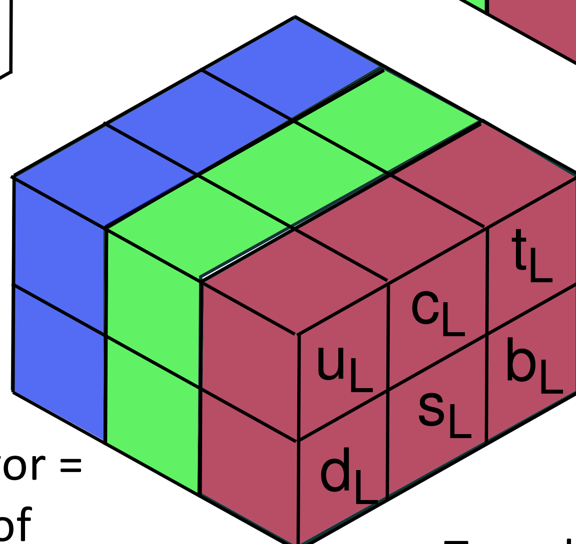


These are the EW d.o.f., NOT the mass eigenstates for ν's





Repeat with antiparticles but exchange R vs. L



These are the Flavor = mass eigenstates of the quarks

True d.o.f. for EW theory: d’, s’, b’

In gauge theories,

the gradient ∂_μ is everwhere replaced by the *gauge-covariant derivative*

$$\mathcal{D}_\mu \equiv \partial_\mu + ieA_\mu, \tag{2.5}$$

where e is the charge in natural units of the particle described by $\psi(x)$ and the field $A_\mu(x)$ transforms under phase rotations (2.3) as

$$A_\mu(x) \to A'_\mu(x) \equiv A_\mu(x) - (1/e)\partial_\mu \alpha(x), \tag{2.6}$$

it is easily verified that under local phase transformations

$$\mathcal{D}_\mu \psi(x) \to e^{i\alpha(x)} \mathcal{D}_\mu \psi(x). \tag{2.7}$$

Only works with massless gauge bosons! -> Symmetry breaking gives massive gauge bosons (see photons in superconductors)

To incorporate electromagnetism into a theory of the weak interactions, we add to the $\mathrm{SU}(2)_\mathrm{L}$ family symmetry suggested by the first two experimental clues a $\mathrm{U}(1)_Y$ weak-hypercharge phase symmetry. We begin by specifying the fermions: a left-handed weak isospin doublet

$$\mathsf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_\mathrm{L} \tag{2.13}$$

with weak hypercharge $Y_\mathrm{L} = -1$, and a right-handed weak isospin singlet

$$\mathsf{R} \equiv e_\mathrm{R} \tag{2.14}$$

with weak hypercharge $Y_\mathrm{R} = -2$.

The electroweak gauge group, $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y$, implies two sets of gauge fields: a weak isovector $\vec{b}_\mu$, with coupling constant g, and a weak isoscalar $\mathcal{A}_\mu$, with coupling constant g'. Corresponding to these gauge fields are the field-strength tensors

In the lecture, we called those W^{+0-} and B^0 instead

$$F^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g\varepsilon_{jk\ell} b^j_\mu b^k_\nu \ , \tag{2.15}$$

for the weak-isospin symmetry, and

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \ , \tag{2.16}$$

for the weak-hypercharge symmetry. We may summarize the interactions by the Lagrangian

$$\mathcal{L} = \mathcal{L}_\mathrm{gauge} + \mathcal{L}_\mathrm{leptons} \ , \tag{2.17}$$

with

$$\mathcal{L}_\mathrm{gauge} = -\tfrac{1}{4} F^\ell_{\mu\nu} F^{\ell\mu\nu} - \tfrac{1}{4} f_{\mu\nu} f^{\mu\nu}, \tag{2.18}$$

$$\mathcal{L}_{\text{leptons}} = \overline{\mathsf{R}}\, i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y\right)\mathsf{R} + \overline{\mathsf{L}}\, i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_\mu\right)\mathsf{L}. \tag{2.19}$$

The $\mathrm{SU}(2)_\mathrm{L}\otimes \mathrm{U}(1)_Y$ gauge symmetry forbids a mass term for the electron in the matter piece (2.19). Moreover, the theory we have described contains four massless electroweak gauge bosons, namely $\mathcal{A}_\mu$, b^1_μ, b^2_μ, and b^3_μ, whereas Nature has but one: the photon. To give masses to the gauge bosons and constituent fermions, we must hide the electroweak symmetry.

To endow the intermediate bosons of the weak interaction with mass, we take advantage of a relativistic generalization of the Ginzburg-Landau phase transition known as the Higgs mechanism [9]. We introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.20}$$

with weak hypercharge $Y_\phi = +1$. Next, we add to the Lagrangian new (gauge-invariant) terms for the interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu\phi)^\dagger(\mathcal{D}_\mu\phi) - V(\phi^\dagger\phi), \tag{2.21}$$

where the gauge-covariant derivative is

$$\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_\mu\ , \tag{2.22}$$

and the potential interaction has the form

$$V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|\,(\phi^\dagger\phi)^2. \tag{2.23}$$

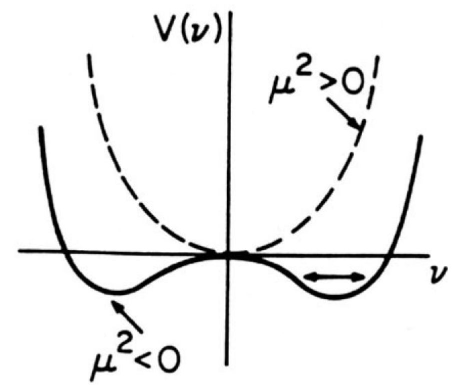
We are also free to add a Yukawa interaction between the scalar fields and the leptons,

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_e\left[\overline{\mathsf{R}}(\phi^\dagger\mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R}\right]. \tag{2.24}$$

We then arrange the scalar self-interactions so that the vacuum state corresponds to a broken-symmetry solution. The electroweak symmetry is spontaneously broken if the parameter $\mu^2 < 0$. The minimum energy, or vacuum state, may then be chosen to correspond to the vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \tag{2.25}$$

where $v = \sqrt{-\mu^2/|\lambda|}$.



$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$$
$$Z = -\sin\theta_W B + \cos\theta_W W^3.$$

$$A = \cos\theta_W B + \sin\theta_W W^3$$
$$M_W = \frac{g\nu}{2}$$

$$M_Z = \sqrt{g^2+g'^2}\frac{\nu}{2} = \frac{M_W}{\cos\theta_W},$$

The mass of fermion f is given by

$$\mathcal{L}_f = -\frac{\zeta_f v}{\sqrt{2}}(\bar{f}_{\mathrm{R}} f_{\mathrm{L}} + \bar{f}_{\mathrm{L}} f_{\mathrm{R}}) = -\frac{\zeta_f v}{\sqrt{2}} \bar{f} f \ , \tag{1.4}$$

where $v/\sqrt{2} = (G_{\mathrm{F}}\sqrt{8})^{-1/2} \approx 174\,\mathrm{GeV}$ is the vacuum expectation value of the Higgs field. The *Yukawa coupling* ζ_f is not predicted by the electroweak theory.

The Yukawa couplings of the charged leptons and quarks range from $\zeta_e \approx 3 \times 10^{-6}$ for the electron to $\zeta_t \approx 1$ for the top quark.

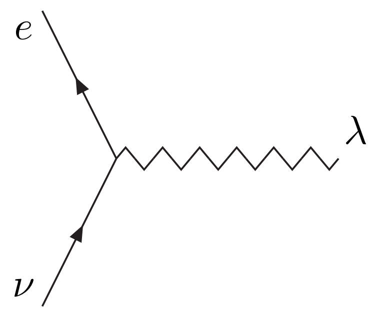
Let us summarize. As a result of spontaneous symmetry breaking, the weak bosons acquire masses, as auxiliary scalars assume the role of the third (longitudinal) degrees of freedom of what had been massless gauge bosons. Specifically, the mediator of the charged-current weak interaction, $W^{\pm} = (b_1 \mp i b_2)/\sqrt{2}$, acquires a mass characterized by $M_W^2 = \pi\alpha/G_{\mathrm{F}}\sqrt{2}\sin^2\theta_W$, where $\sin^2\theta_W = g'^2/(g^2 + g'^2)$ is the weak mixing parameter. The mediator of the neutral-current weak interaction, $Z = b_3 \cos\theta_W - \mathcal{A}\sin\theta_W$, acquires a mass characterized by $M_Z^2 = M_W^2/\cos^2\theta_W$. After spontaneous symmetry breaking, there remains an unbroken $\mathrm{U}(1)_{\mathrm{em}}$ phase symmetry, so that electromagnetism is mediated by a massless photon, $A = \mathcal{A}\cos\theta_W + b_3\sin\theta_W$, coupled to the electric charge $e = gg'/\sqrt{g^2 + g'^2}$. As a vestige of the spontaneous breaking of the symmetry, there remains a massive, spin-zero particle, the Higgs boson. The mass of the Higgs scalar is given symbolically as $M_H^2 = -2\mu^2 > 0$, but we have no prediction for its value.

2.3.1 Charged-current interactions

The interactions of the W-boson with the leptons are given by

$$\mathcal{L}_{W-\mathrm{lep}} = \frac{-g}{2\sqrt{2}} \left[\bar{\nu}_e \gamma^\mu (1-\gamma_5) e W^+_\mu + \bar{e} \gamma^\mu (1-\gamma_5) \nu_e W^-_\mu \right], \text{etc.},$$

so the Feynman rule for the $\nu_e e W$ vertex is



$$\frac{-ig}{2\sqrt{2}} \gamma_\lambda (1-\gamma_5)$$

2.3.2 Neutral Currents

The interactions of the Z-boson with leptons are given by

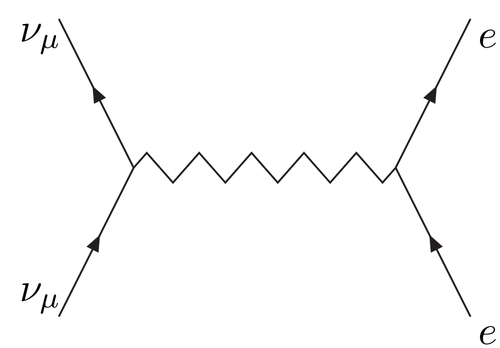
$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu} \gamma^\mu (1-\gamma_5) \nu \, Z_\mu$$

and

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W} \bar{e} \left[L_e \gamma^\mu (1-\gamma_5) + R_e \gamma^\mu (1+\gamma_5) \right] e \, Z_\mu \ ,$$

where the chiral couplings are

$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \ , \\ R_e &= 2\sin^2\theta_W \ . \end{aligned}$$



To extend our theory to include the electroweak interactions of quarks, we observe that each generation consists of a left-handed doublet

$$\begin{array}{lccc} & I_3 & Q & Y = 2(Q - I_3) \\ \mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_{\mathrm{L}} & \begin{array}{c} +\frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} +\frac{2}{3} \\ -\frac{1}{3} \end{array} & \frac{1}{3} \; , \end{array} \tag{2.50}$$

and two right-handed singlets,

$$\begin{array}{lccc} & I_3 & Q & Y = 2(Q - I_3) \\ \mathsf{R}_u = u_{\mathrm{R}} & 0 & +\frac{2}{3} & +\frac{4}{3} \\ \mathsf{R}_d = d_{\mathrm{R}} & 0 & -\frac{1}{3} & -\frac{2}{3} \end{array} \; , \tag{2.51}$$

Proceeding as before, we find the Lagrangian terms for the W-quark charged-current interaction,

$$\mathcal{L}_{W-\mathrm{quark}} = \frac{-g}{2\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1 - \gamma_5) d \, W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u \, W_\mu^- \right] \; , \tag{2.52}$$

which is identical in form to the leptonic charged-current interaction (2.33). Universality is ensured by the fact that the charged-current interaction is determined by the weak isospin of the fermions, and that both quarks and leptons come in doublets.

The neutral-current interaction is also equivalent in form to its leptonic counterpart, (2.44) and (2.45). We may write it compactly as

$$\mathcal{L}_{Z-\mathrm{quark}} = \frac{-g}{4 \cos \theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[L_i (1 - \gamma_5) + R_i (1 + \gamma_5) \right] q_i \, Z_\mu \; , \tag{2.53}$$

where the chiral couplings are

$$\begin{aligned} L_i &= \tau_3 - 2 Q_i \sin^2 \theta_W \; , \\ R_i &= -2 Q_i \sin^2 \theta_W \; . \end{aligned} \tag{2.54}$$

Again we find a quark-lepton universality in the form—but not the values—of the chiral couplings.

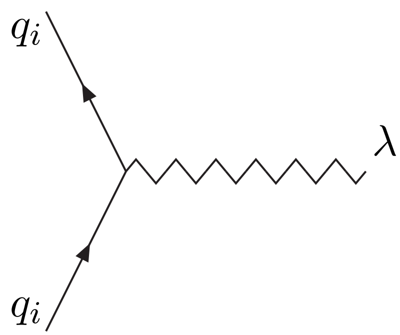
No flavor-changing weak neutral currents -> GIM mechanism -> charmed quark:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_{\mathrm{L}} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_{\mathrm{L}} \qquad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_{\mathrm{L}} \begin{pmatrix} c \\ s_\theta \end{pmatrix}_{\mathrm{L}} , \tag{2.57}$$

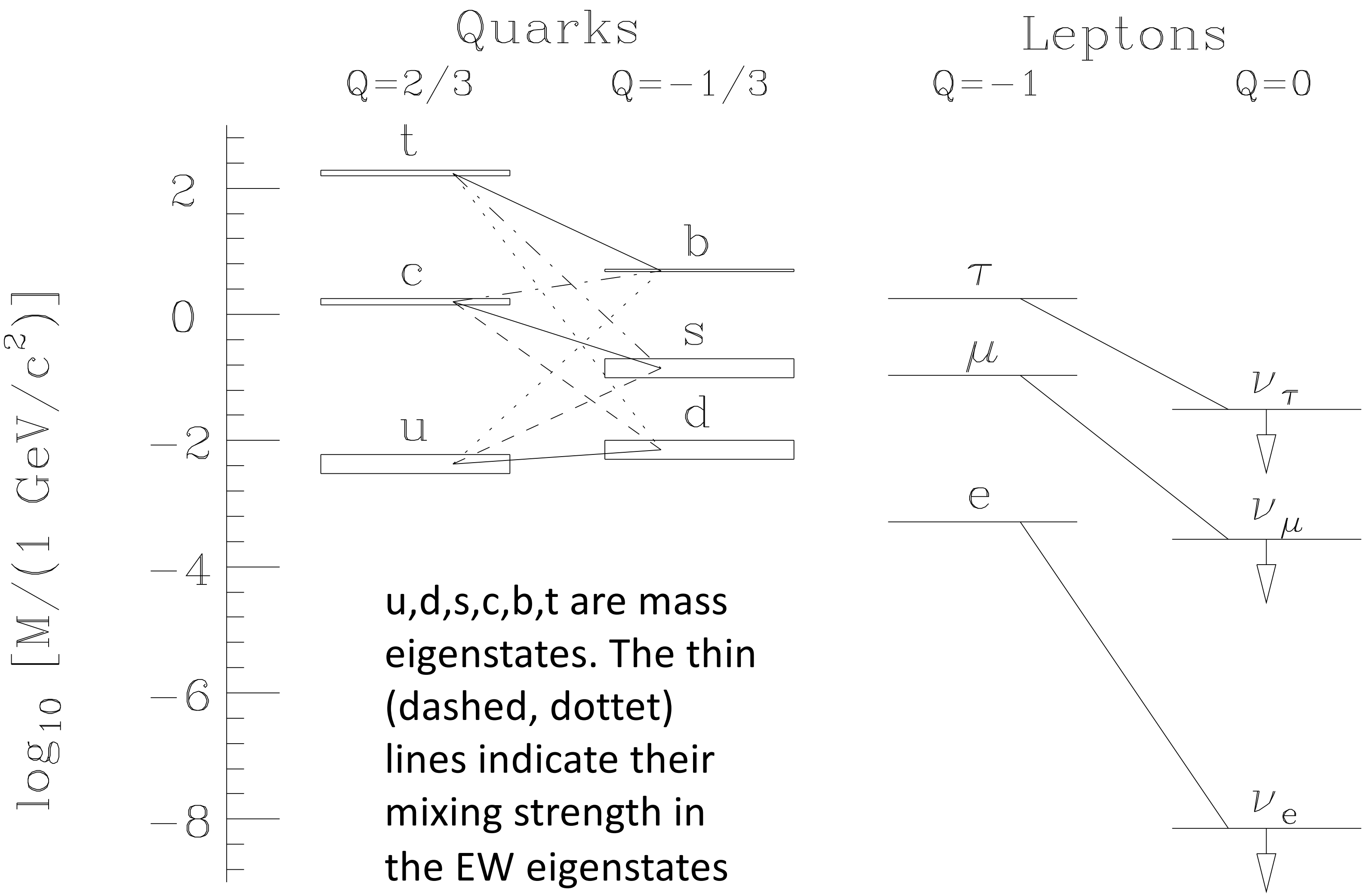
where

$$s_\theta = s \, \cos\theta_C - d \, \sin\theta_C \ , \tag{2.58}$$

plus the corresponding right-handed singlets, e_{R}, μ_{R}, u_{R}, d_{R}, c_{R}, and s_{R}. This required the introduction of the charmed quark, c, which had not yet been observed. By the addition of the second quark generation, the flavor-changing cross terms vanish in the Z-quark interaction, and we are left with:



$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$



u,d,s,c,b,t are mass eigenstates. The thin (dashed, dottet) lines indicate their mixing strength in the EW eigenstates d',s' and b'

The neutrinos are the EW eigenstates. They are NOT mass eigenstates.

In the SM, all mass scales are generated through the Higgs mechanism. After the SSB, the Yukawa couplings to the Higgs scalar doublet give rise to non-diagonal fermionic mass terms. The mass eigenstates are then different from the weak eigenstates, which leads to flavour mixing in the charged-current interaction:

$$\mathcal{L} \;=\; \frac{g}{2\sqrt{2}}\, W_\mu^\dagger \,\sum_{ij}\; \bar{u}_i\,\gamma^\mu(1-\gamma_5)\,\mathbf{V}_{ij}\,d_j \;+\; \text{h.c.}\;. \tag{11}$$

With non-zero neutrino masses, there are analogous mixing effects in the lepton sector, which are covered in [18].

The Cabibbo-Kobayashi-Maskawa [19,20] (CKM) matrix $\mathbf{V}$ is unitary and couples any up-type quark with all down-type quarks. It is a priori unknown, because the gauge symmetry does not fix the Yukawa couplings. The matrix element $\mathbf{V}_{ij}$ can be obtained experimentally from semileptonic weak processes associated with the quark transition $d_j \to u_i l^- \bar{\nu}_l$. The present determinations are summarized in Table 2. The uncertainties are dominated by theoretical errors, related to the strong interaction which binds quarks into hadrons.

The most precisely known CKM matrix element is $\mathbf{V}_{ud}$. The weighted average of the two determinations in Table 2 gives $\mathbf{V}_{ud} = 0.9738 \pm 0.0008$. Taking for $\mathbf{V}_{us}$ the more reliable K_{e3} determination, one obtains

$$|\mathbf{V}_{ud}|^2 \,+\, |\mathbf{V}_{us}|^2 \,+\, |\mathbf{V}_{ub}|^2 \;=\; 0.9965 \pm 0.0019\;. \tag{12}$$

Alternative Summary:

The electroweak theory is based on the $SU_2 \times U_1$ Lagrangian [4]

$$Ł_{SU_2\times U_1} = Ł_{\text{gauge}} + Ł_\varphi + Ł_f + Ł_{\text{Yukawa}}. \tag{5}$$

The gauge part is

$$Ł_{\text{gauge}} = -\frac{1}{4}F^i_{\mu\nu}F^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{6}$$

where W^i_μ, $i = 1, 2, 3$ and B_μ are respectively the SU_2 and U_1 gauge fields, with field strength tensors

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ F_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu, \end{aligned} \tag{7}$$

where $g(g')$ is the SU_2 (U_1) gauge coupling and ϵ_{ijk} is the totally antisymmetric symbol. The SU_2 fields have three and four-point self-interactions. B is a U_1 field associated with the weak hypercharge $Y = Q - T_3$, where Q and T_3 are respectively the electric charge operator and the third component of weak SU_2. It has no self-interactions. The B and W_3 fields will eventually mix to form the photon and Z boson.

The scalar part of the Lagrangian is

$$Ł_\varphi = (D^\mu \varphi)^\dagger D_\mu \varphi - V(\varphi), \tag{8}$$

where $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ is a complex Higgs scalar, which is a doublet under SU_2 with U_1 charge $Y_\varphi = +\frac{1}{2}$. The gauge covariant derivative is

$$D_\mu \varphi = \left(\partial_\mu + ig \frac{\tau^i}{2} W_\mu^i + \frac{ig'}{2} B_\mu \right) \varphi, \tag{9}$$

where the τ^i are the Pauli matrices. The square of the covariant derivative leads to three and four-point interactions between the gauge and scalar fields [1].

$V(\varphi)$ is the Higgs potential. The combination of $SU_2 \times U_1$ invariance and renormalizability restricts V to the form

$$V(\varphi) = +\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2. \tag{10}$$

For $\mu^2 < 0$ there will be spontaneous symmetry breaking. The λ term describes a quartic self-interaction between the scalar fields. Vacuum stability requires $\lambda > 0$.