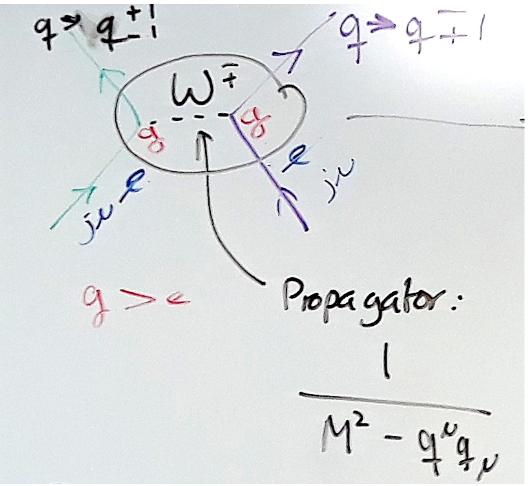
Weak Decays

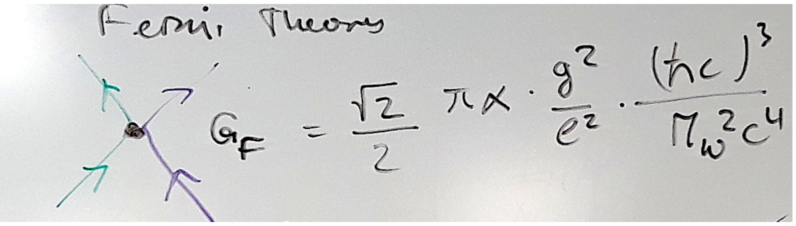


To understand the decay of baryons and mesons, we need to understand their weak interaction. The weak interaction not only couples particles, but it also changes their charge. In this way, the weak interaction is similar to the strong interaction. While the weak interaction changes EM charge, the strong interaction changes the color of quarks.

Much like EM’s charge e, the weak interaction has weak charge g. While one would think g is small, in reality, g is slightly larger than e. So why is weak interaction weak? The answer lies in the propagator W:

$$W = 1/\left(M^2 - q^\mu q_\mu\right)$$

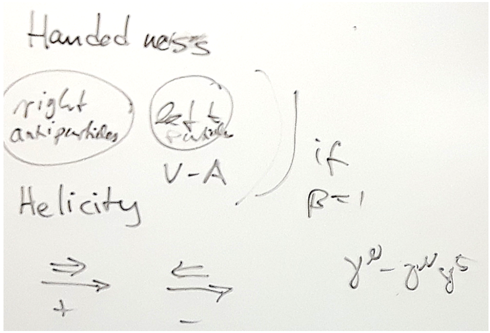
Now for EM, M=0; therefore $W = -1/q^\mu q_\mu = 1/Q^2$. However, the weak interaction does not have a nonzero value for M. In fact, M=80.4 GeV. Since $q^\mu q_\mu$ is of the order of a few to a few 100 MeV for weak decays, W is much smaller for the weak interaction than for the electromagnetic one. One can approximate the weak interaction as point like. One then finds a single coupling constant (the Fermi constant):



$$G_F = \frac{\sqrt{2}}{2} * \pi\alpha * \frac{g^2}{e^2} * \frac{(hc)^3}{M^2c^4}$$

One question one may ask is what is weak interaction coupling to? It cannot be charge since neutrinos have no charge and they undergo the weak interaction. It turns out that the weak interaction couples to something called "weak isospin" and "weak hypercharge".

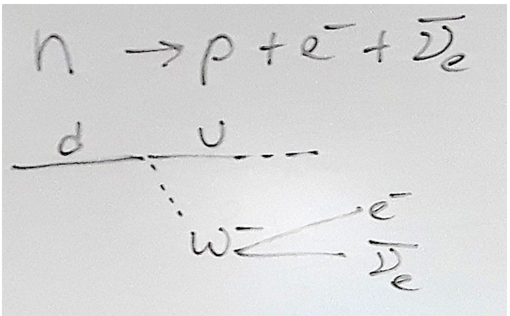
In EM, there isn't just charge q, but also current j. The current can be seen in the EM interaction $j^\mu*A_\mu$which comes from the Hamiltonian, $H = (p-eA)^2/2m + e*\Phi$ whose interaction term is $-v*A+e*\ \Phi$. EM interaction couples to EM current, so Fermi thought the weak interaction would couple to a current as well. In fact, it couples to two, the vector current (momentum) and the axial current (spin). The vector current j^μ changes sign under the parity operator while the axial current a^μ does not. This means the weak interaction violates parity. This affects the weak interactions ability to couple.



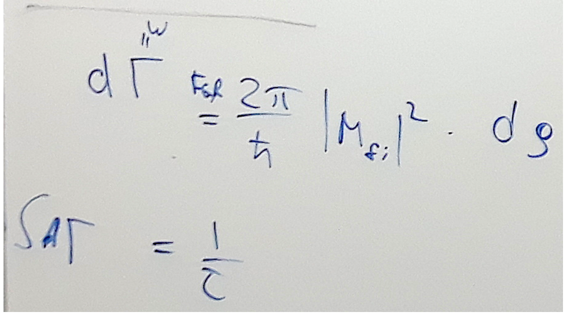
For instance, if the spin of a (nearly) massless particle like the neutrino is in the same direction as its momentum ("right-handed", which equals positive helicity for *v -> c)*, it does not couple. However, if the spin is in the opposite direction ("right-handed" or negative helicity), it does couple. In fact, it undergoes the maximum interaction possible. Parity violation affects which handed particles or antiparticles the weak interaction couples to. Left-handed particles undergo weak interaction, but right-handed do not. Conversely, only right-handed anti particles undergo weak interaction.

Composite particles have complicated internal structures. This does not affect the vector current. For the quark case, the vector transition is $<u|\tau_+|d>=1$ (τ_+ is the isospin 3-component raising ladder operator). In the hadronic case, the vector transition is $<p|T_+|n>=1=gv/gv$. Both are one because of the conservation of charge; this is called the conserved vector current hypothesis (which is more a standard tenet of physics than a hypothesis). But while the vector transition is conserved, the axial transition is not. In the quark case, it is < u↑|τ₊**σ**|d↑>=1, much like the vector current. However, in the hadronic case it is <p↑|T₊**Σ**|n↑>=ga/gv which does not equal 1. This complicates the interpretation of weak reactions involving hadrons. On the other hand, the weak interaction even at low energies is more similar to DIS in that both couple directly to quarks.

The formula for weak decay rates , for example for the neutron, is



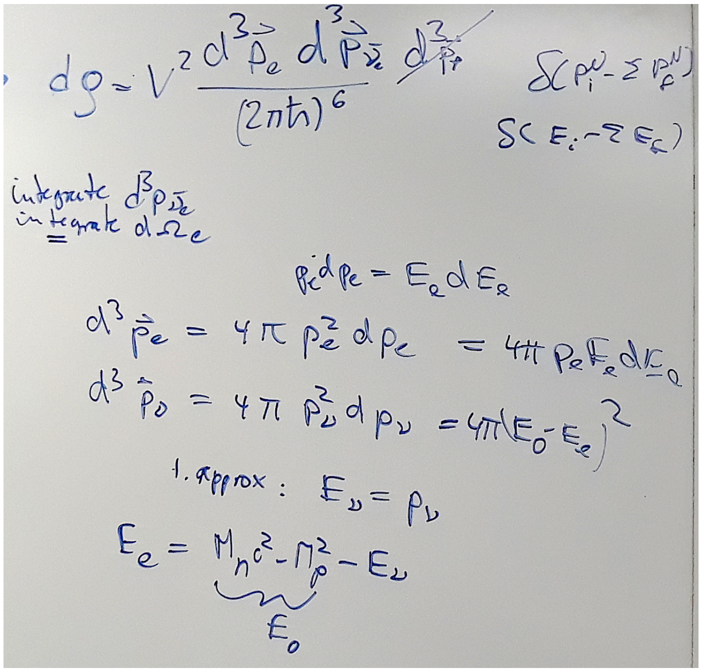
$n \to p + e^- + \bar{\nu}_e$



$$d\Gamma^{W} \overset{FGR}{=} \frac{2\pi}{\hbar} |M_{fi}|^2 \cdot d\rho$$

$$\int A\Gamma = \frac{1}{\tau}$$

where



$$d\rho = V^2 \frac{d^3\vec{p}_e \, d^3\vec{p}_{\bar{\nu}_e}}{(2\pi\hbar)^6} \cancel{d^3\vec{p}_p} \qquad \delta(p_i^\mu - \sum p_f^\mu) \quad \delta(E_i - \sum E_f)$$

integrate $d^3p_{\bar{\nu}_e}$

integrate $d\Omega_e$

$$p_e \, dp_e = E_e \, dE_e$$

$$d^3\vec{p}_e = 4\pi p_e^2 \, dp_e = 4\pi p_e E_e \, dE_e$$

$$d^3\vec{p}_\nu = 4\pi p_\nu^2 \, dp_\nu = 4\pi (E_0 - E_e)^2$$

1. approx : $E_\nu = p_\nu$

$$E_e = \underbrace{M_n c^2 - M_p^2}_{E_0} - E_\nu$$