PHYS101

Week 3

Reminder: Acceleration

- $$a = \frac{\text{change in velocity during time } \Delta t}{\text{elapsed time interval } \Delta t} = \frac{\Delta \mathbf{v}}{\Delta t}$$

- Can be specified by giving magnitude $a = |\Delta \mathbf{v}| / \Delta t$ and sign.
- Positive velocity, increasing speed => positive acceleration $a > 0$
- Positive velocity, decreasing speed (slowing down) => negative acceleration (deceleration) $a < 0$
- Negative velocity, increasing speed => negative acceleration $a < 0$
- Negative velocity, slowing down => positive acceleration $a > 0$
- NOTE: Acceleration in an inertial system must have a cause! (Force... See later)

Examples for accelerated motion

- Constantly accelerating car
- Police catching up with speeder
- Car going around a corner
- Objects falling down
- Objects thrown upwards
- Objects gliding down ramps
- Objects pulled by a falling weight

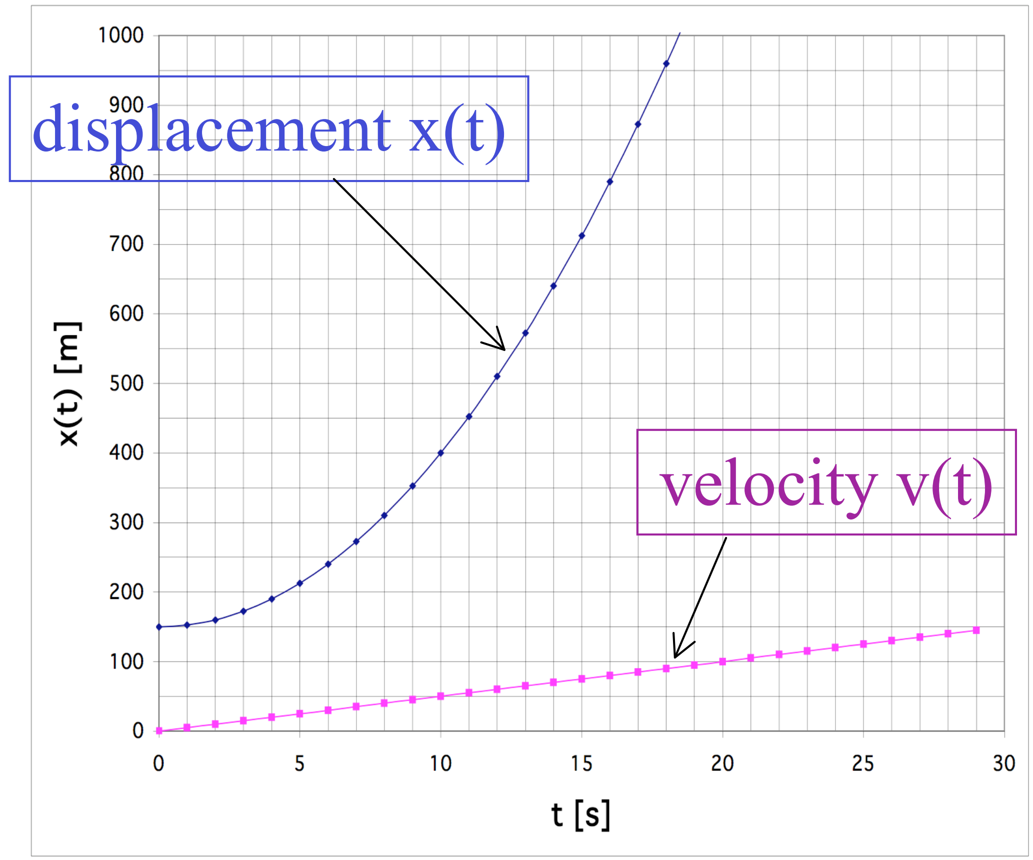
Motion with constant Acceleration

- $a(t) = a_{av} = a_0 = a(t{=}0) = \text{const.}$
  $a = a_{av} = (v(t) - v_0)/(t - 0)$ [$v_0 = v(t{=}0)$]
- $\Rightarrow \quad v(t) = v_0 + a\,t$
- If initial velocity $v_0 = 0$: $v(t) = a\,t$
- In that case:
  Average velocity during the time interval
  $t = 0 \ldots t$ is given by
  $v_{av}\ (0 \ldots t) = {}^1/_2\,[0 + v(t)]$
  $= {}^1/_2\,[0 + a\,t] = {}^1/_2\,a\,t$

Constant Acceleration Cont'd

- Plugging it into expression for position:
  $^1/_2\, a\, t = v_{av} = (x(t) - x_0)/(t - 0)$
  $\Rightarrow x(t) = x_0 + {}^1/_2\, a\, t^2$

- Typical graph $x(t)$:



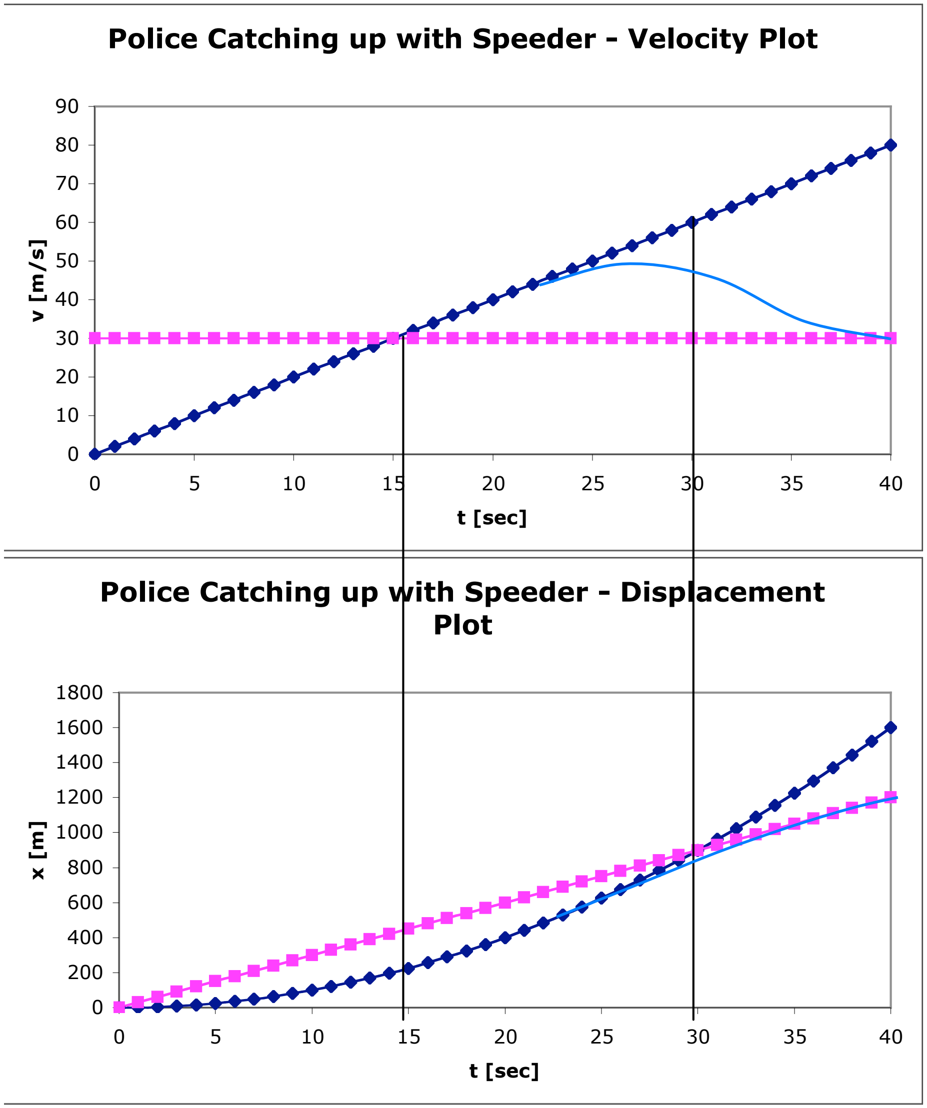
- General case:
  $v(t) = v_0 + a\, t$
  $x(t) = x_0 + v_0 t + {}^1/_2\, a\, t^2$

Police catching up with speeder

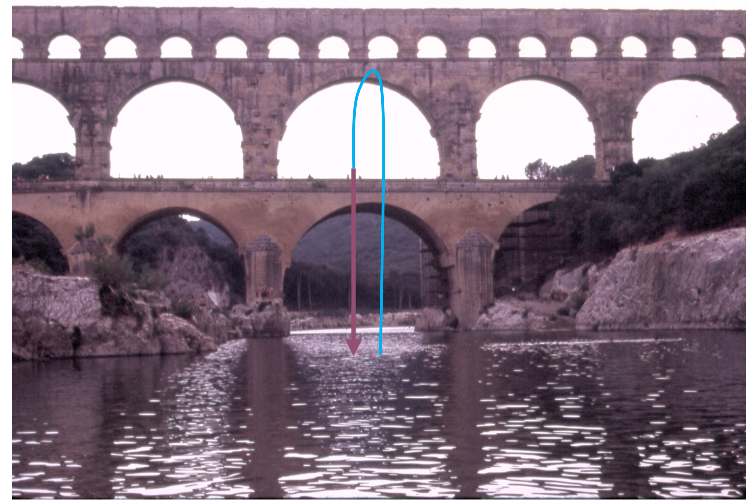
- Police:
  $a_P = 2.0\ \mathrm{m/s^2}$,
  $v_P(t) = a_P t$,
  $x_P(t) = {}^1/_2\, a_P t^2$

- Speeder:
  $a_S = 0$,
  $v_S(t) = 30\ \mathrm{m/s}^*$ = const.,
  $x_S(t) = v_S t$

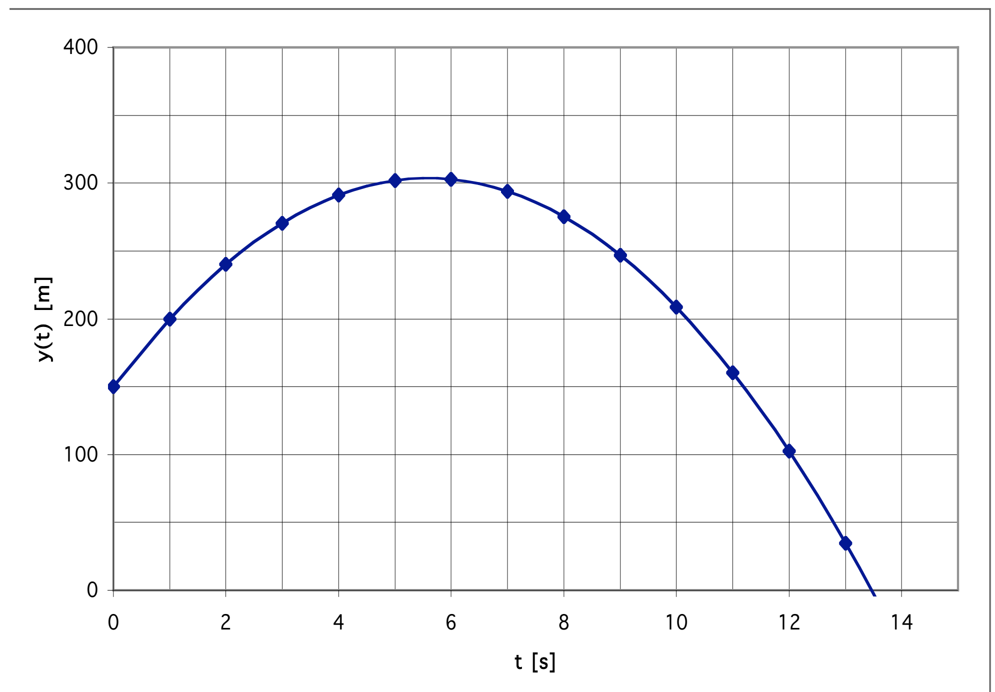
** 67 miles/hour*



Free Fall



- $a = -g = -\ 9.81\ \mathrm{m/s^2}$ *)
- $v(t) = -\ g\ t$
- $x(t) = x_0 - {}^1/_2\, g\, t^2$
- Example 1: Fall from the Pont du Gard (45 m above water) => t, v_{final}?
- Example 2: Throwing a ball upwards from the top of a building.



*) in the absence of air resistance (see demo, NASA movie)

Important Rules for Problem Solving

- Make sure you are very clear about whether you are dealing with velocity (really a vector, and a signed quantity in 1 dimension) or speed (a scalar).
- Distinguish carefully between average and instantaneous quantities (velocity, acceleration).
- Distinguish carefully between position (displacement), velocity, and acceleration.
- Don’t mistake x(t) plots (or v(t) plots) for representations of 2D motion.

From linear to 3D motion

- In general, displacement is a vector
  - Give size and direction ("10 miles north", "4 m along z" etc.)
- In general, velocity is a vector
  - Change in displacement = difference between 2 vectors
  - Can point in a totally different direction than displacement
  - Give size (magnitude = speed) and direction
- In general, acceleration is a vector
  - Similarly, difference between 2 (velocity) vectors divided by elapsed time.

…back to: **Forces**

- Push or pull on an object (mass point) due to its interaction with “something else”
- Cause of **changes** in motional state (**acceleration**)
- Has both a magnitude (strength - “how hard do we push/pull”) and a direction (“which way do we push/pull”)
- -> Force is a **vector**

Newton's First Law

- IF the net force (Σ $\mathbf{F}_i$) acting on an object is zero, its velocity will not change:
  - If it is at rest, it will remain at rest.
  - If it is moving with velocity **v**, it will continue to move with constant velocity **v**.
- => IF the velocity changes, there must be a net force acting!
  - Examples: Car on Freeway, Puck on Ice, Spaceship,…
- Remember: Always add up **all** forces to get net force!
- You don't need any net force to keep on moving - that's the "default" behavior!

Newton's Second Law

- What if there **is** a net force acting?
  => The object will accelerate!
- How much?
  |**a** | ~ |**F** | ; |**a** | ~ 1/m
  (for given |**a** | need |**F** | ~ mass)
- Which direction?
  a points in the direction of **F**

mass = inertia = resistance to change of motion

$$\Rightarrow \mathbf{a} = (\Sigma \mathbf{F})/m$$

$a = F/m$

- Predict acceleration from net force and mass
- Explain observed acceleration
- Newton's First Law follows: if net force is zero, acceleration will be zero => constant velocity
- Valid only in Inertial Frames of Reference
- Include **all** forces (including friction, normal force, weight, ropes and sticks,…)
- **Only** include external forces
- **Only** include forces actually acting on the body (mass point) under consideration
- Explains why all objects fall with same acceleration *g*

F = m **a**

- Operational definition of "Force"
  - Unit must be kg m/s^2 = N (Newton)
- "How much net force do I need to accelerate a known mass *m* with acceleration **a** ?" Example: roller coaster
- If I observe a known mass *m* accelerate with acceleration **a** , how much force can I infer to be acting on it?
  - All bodies fall with acceleration *g* in Earth's gravity field => Gravity Force must be $|\mathbf{F}_{grav}| = mg$. This is the **weight** of mass *m* .
- **Warning:** The expression "m **a** " is **not** a force itself. It is **equal** to the net force.

$m = |F| / |a|$

- Inertia = net force applied / acceleration achieved
- Can be used to determine mass:
  - Use known force and measure acceleration
  - Compare ratio of accelerations for 2 different masses and same net force:
    $m_1 / m_2 = a_2 / a_1$
  - Use gravity to determine mass: Measure weight (in Newton!), divide by known *g* (automatically done by most scales). Depends on location!
- **Note:** this is not the **definition** of mass - that is given by comparison with standard 1 kg mass. **But:** can be used for that comparison.

Important Hints for Problem Solving

- Take **all** external forces into consideration. Take their **directions** into account.
- $m\mathbf{a}$ is ***not*** a force!
- Don’t confuse mass and weight!
- Newton’s 2nd Law is only valid in Inertial Frames of Reference
  - In an accelerating car, there is **no** force pushing you into the seat - instead, the seat is exerting an accelerating force on you
  - In a falling elevator, there **is** a force (weight) acting on you, even if you don’t feel it
  - You don’t actually feel gravitational force pulling on you - you feel the normal force holding you up!

Summary: Newton's 2^{nd} Law + 1^{st} Law

- $$\vec{\mathbf{F}}_{\text{resultant}} = \underbrace{\sum \vec{\mathbf{F}}_{\text{i}}}_{\substack{\text{all forces acting \textbf{on} object} \\ \text{due to \textbf{other} objects}}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \ldots + \vec{\mathbf{F}}_N = m\vec{\mathbf{a}}_{\text{Object}}$$

  if we measure acceleration in an inertial coordinate system
- Examples:
  - block on incline with and without friction (dynamic and static)
    - the double role played by the normal force
  - cart on incline with and without friction (rolling)
  - friction through air resistance:
    - feather vs. hammer
    - terminal velocity (ant vs. human)
    - Parachutes