Greek Alphabet

Capital	Α	Β	Γ	Δ	Ε	Ζ	Η	Θ	Ι	Κ	Λ	Μ
Lowercase	α	β	γ	δ	ε	ζ	η	θ, ϑ	ι	κ	λ	μ
Name	alpha	beta	gamma	delta	epsilon	zeta	eta	theta	iota	kappa	lambda	mu
Capital	Ν	Ξ	Ο	Π	Ρ	Σ	Τ	Υ	Φ	Χ	Ψ	Ω
Lowercase	ν	ξ	ο	π	ρ	σ	τ	υ	ϕ, φ	χ	ψ	ω
Name	nu	xi	omicron	pi	rho	sigma	tau	upsilon	phi	chi	psi	omega

Fundamental constants:

Speed of light: $c = 2.9979 \cdot 10^8$ m/s (roughly a foot per nanosecond)
Planck constant: $h = 6.626 \cdot 10^{-34}$ J s; $\hbar = h / 2\pi$
Fundamental charge unit: $e = 1.602 \cdot 10^{-19}$ C
Electron mass: $m_e = 9.109 \cdot 10^{-31}$ kg
Coulomb's Law constant: $k = 1/ 4\pi\varepsilon_0 = 8.988 \cdot 10^9$ Nm^2/C^2
Gravitational constant: $G = 6.674 \cdot 10^{-11}$ Nm^2/kg^2
Avogadro constant: $N_A = 6.022 \cdot 10^{23}$ particles per mol
Boltzmann constant: $k = 1.38 \cdot 10^{-23}$ J/K $= 8.617 \cdot 10^{-5}$ eV/K; $R = N_A \cdot k = 8.314$ J/K/mol

Useful conversions:

1 fm (= 1 "Fermi") = 10^{-15} m, 1 nm = 10^{-9} m = 10 Å; 1 PHz = 10^{15} Hz ("Petahertz")
1 eV = $e \cdot 1$V = $1.602 \cdot 10^{-19}$ J (Energy of elementary charge after 1 V potential difference)
1 keV = 1000 eV, 1 MeV= 10^6 eV, GeV = 10^9 eV, 1 TeV = 10^{12} eV ("Tera-electronvolt")
New unit of mass m: 1 eV/c^2 = mass equivalent of 1 eV (Relativity!) = $1.78 \cdot 10^{-36}$ kg
Momentum p: 1 eV/c = $5.34 \cdot 10^{-28}$ kg m/s; p in eV/c = mass in eV/c^2 times velocity in units of c
Planck contant: $\hbar = h/2\pi = 197.33$ eV/$c \cdot$ nm $= 6.582 \cdot 10^{-16}$ eV $\cdot$ s = 0.658 eV/PHz
Fine-structure constant: $\alpha = e^2 / 4\pi\varepsilon_0\hbar c = 1/137.036$
Electron mass: $m_e = 510{,}999$ eV/$c^2 \approx 0.511$ MeV/c^2
Muon mass: $m_\mu = 105.658$ MeV/$c^2 \approx 207 \cdot m_e$
Proton mass: $m_p = 938.272$ MeV/$c^2 \approx 1836 \cdot m_e$
Neutron mass: $m_n = 939.565$ MeV/$c^2 \approx 1839 \cdot m_e$
Atomic mass unit (1/12 of the mass of a ^{12}C atom): $u = 931.494$ MeV/$c^2 \approx 1823 \cdot m_e$
Rydberg constant: $Ry = m_e c^2 \alpha^2 / 2 = 13.606$ eV
Bohr Radius: $a_0 = \hbar c / (m_e c^2 \alpha) = 0.0529$ nm (roughly ½ Å).

Special Relativity:

For an inertial system S' moving along the x-axis of S with constant velocity $v < c$, and with all axes aligned and the same origin ($x = y = z = ct = 0 \Leftrightarrow x' = y' = z' = ct' = 0$):

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}};\ x' = \gamma\left(x - \frac{v}{c}ct\right);\ ct' = \gamma\left(ct - \frac{v}{c}x\right);\ y = y';\ z = z'$$

Clocks in S' appear to S as if they were going slow by factor 1/γ, and vice versa.
Length of object at rest in S' appears contracted by factor 1/γ in S.

Velocity addition: $\frac{u_x}{c} = \frac{\frac{u_x'}{c} + \frac{v}{c}}{1 + \frac{u_x'}{c}\frac{v}{c}};\ \frac{u_y}{c} = \frac{\frac{1}{\gamma}\frac{u_y'}{c}}{1 + \frac{u_x'}{c}\frac{v}{c}}$.

Four-vectors: $x^\mu = (ct, x, y, z);\ x_\mu = (ct, -x, -y, -z)$ (μ=0,1,2,3 for ct,x,y,z).

Invariant interval between two events (=points in 4-dim. space-time):
$\Delta x^\mu = (\Delta ct, \Delta x, \Delta y, \Delta z) \Rightarrow \Delta s^2 = \Delta x^\mu \Delta x_\mu = \Delta ct^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$ (same in all inertial systems.)

Positive Δs^2: "time-like separation", Δs^2 = square of elapsed eigentime $c\tau$ in a system that travels from the start point (event) to the end point (event) of the interval.

Negative Δs^2: "space-like separation", $-\Delta s^2$ = square of distance between the two events in a system (which always exists!) where they occur simultaneously.

$\Delta s^2 = 0$: "light-like separation"; a light ray could travel from one event to the other.

Four-momentum: $P^\mu = (E/c, P_x, P_y, P_z) = (\Gamma mc, \Gamma m\vec{u});\ \Gamma = \frac{1}{\sqrt{1-\vec{u}^2/c^2}}$. Sum of all momenta is conserved in collisions, separately for each component. 0^{th} component times c is total energy, including kinetic and rest mass energy ($E_{rest} = mc^2$). Transformation of P^μ to coordinate system S' is analog to x^μ (see above).

Invariant: $P^\mu P_\mu = \frac{E^2}{c^2} - \vec{P}^2 = m^2c^2 \Rightarrow E = \sqrt{m^2c^4 + \vec{P}^2c^2}$; $\frac{\vec{u}}{c} = \frac{\vec{P}c}{E}$.

Quantum Mechanics:

Formal/abstract: All *possible* knowledge about a system is encoded in its state vector $|\psi\rangle$ - often only probabilities can be predicted. State vectors are members of a (complex) Hilbert space: they can be added, multiplied by a complex number (scalar), and we can define a scalar product $\langle\psi_1|\psi_2\rangle$ (= some complex number c, with $\langle\psi_2|\psi_1\rangle$=$c^*$). All state vectors must be normalizable and by convention are normalized to 1: $\langle\psi|\psi\rangle = 1$.

Example: Motion in Motion in 1D => state vector represented by "wave function" ψ(x). Addition: $[\psi_1 + \psi_2](x) = \psi_1(x) + \psi_2(x)$. Multiplication with scalar: $[c\psi_1](x) = c\psi_1(x)$.

Scalar product: $\langle \psi_1 | \psi_2 \rangle = \int_{-\infty}^{\infty} \psi_1^*(x)\psi_2(x)\,dx$. Normalizable: $\langle \psi | \psi \rangle \overset{!}{=} \int_{-\infty}^{\infty} \psi^*(x)\psi(x)\,dx < \infty$.

Probability to find particle in interval $x \ldots x+dx$: $d\,\Pr(x \ldots x+dx) = |\psi(x)|^2\,dx = \psi(x)^*\psi(x)dx$ (assuming normalized state vector, $\langle \psi | \psi \rangle = 1$).

Formal/abstract: Operators are linear functions turning vectors into other vectors: $\mathbf{O}|\psi\rangle = |\varphi\rangle; \mathbf{O}\left[c|\psi\rangle\right] = c|\varphi\rangle; \mathbf{O}\left[|\psi_1\rangle + |\psi_2\rangle\right] = \mathbf{O}|\psi_1\rangle + \mathbf{O}|\psi\rangle_2$. A vector $|\varphi_\omega\rangle$ is called an eigenvector of an operator **O** with eigenvalue ω (=complex number) IF $\mathbf{O}|\varphi_\omega\rangle = \omega|\varphi_\omega\rangle$.

Observables are represented by (Hermitian) operators $\mathbf{\Omega}$ with only **real** eigenvalues ω_i. Any measurement of the observable must give one of these eigenvalues as result. After we measure ω_i, the system will be in the state described by vector $|\varphi_{\omega_i}\rangle$ ("collapse of the wave function"). The probability to measure this particular eigenvalue for a state described by $|\psi\rangle$ is given by $\Pr(\omega_i) = \left|\langle \varphi_{\omega_i} | \psi \rangle\right|^2$. The average (expectation value) for the observable over many independent trials with the same initial state $|\psi\rangle$ is $\langle \Omega \rangle_\psi = \langle \psi | \mathbf{\Omega} | \psi \rangle$ with standard deviation $\sigma_\Omega = \sqrt{\langle \Omega^2 \rangle - \langle \Omega \rangle^2}$.

Example: Motion in Motion in 1D => Important observables:
Position $\mathbf{X}\psi(x) = x \cdot \psi(x) \rightarrow$ eigenvectors $\psi_{x_0}(x) = \delta(x - x_0)$ w/ eigenvalue x_0; Momentum $\mathbf{P}\psi(x) = \frac{\hbar}{i}\frac{\partial}{\partial x}\psi(x) \rightarrow$ eigenvectors $\psi_{p_0}(x) = e^{ip_0 x/\hbar}$ w/ eigenvalue p_0; Hamiltonian (= total energy, kinetic plus potential): $\mathbf{H}\psi(x) = \left(\frac{\mathbf{P}^2}{2m} + V(X)\right)\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x)$.

Heisenberg's uncertainty principle: Position x and momentum p cannot be predicted with arbitrary precision simultaneously; $\sigma_x\sigma_p \geq \hbar/2$.

Formal/abstract: Time evolution (Schrödinger Equation): State vector becomes function of time: $|\psi\rangle(t)$; $\frac{\partial}{\partial t}|\psi\rangle(t) = \frac{1}{i\hbar}\mathbf{H}|\psi\rangle(t)$ where **H** is the Hamiltonian operator.

Eigenstates of **H**: $\mathbf{H}|\varphi_E\rangle = E|\varphi_E\rangle$ => "stationary" solutions of Schrödinger Equation: $|\psi_E(t)\rangle = |\varphi_E\rangle e^{-iEt/\hbar}$ (no time dependence for any observable).

Example: Motion in 1D => Eigenvalue equation: $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$.

Solution: "Stationary States". Eigenstates of the free Hamiltonian ($V(x) = 0$):

$\psi_p(x,t) = Ae^{\frac{i}{\hbar}px} e^{-\frac{i}{\hbar}\frac{p^2}{2m}t}$ (simultaneously eigenstates of momentum operator)

Gaussian Wave Package:

= Linear combination of "free Hamiltonian eigenstates" (but not an eigenstate itself), with Gaussian weighting over a range of momenta. At time $t = 0$:

$$\psi_{GWP}(x,t=0) = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_p}}\int_{-\infty}^{\infty} e^{-\frac{(p-p_0)^2}{4\sigma_p^2}} e^{\frac{i}{\hbar}px}\,dp = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_x}} e^{\frac{i}{\hbar}p_0 x} e^{-\frac{x^2}{4\sigma_x^2}};\ \sigma_x = \frac{\hbar}{2\sigma_p}$$

Average momentum p_0, with standard deviation σ_p. Average position $x = 0$; standard deviation for position is $\sigma_x = \frac{\hbar}{2\sigma_p}$ which is the smallest possible given Heisenberg's Uncertainty Relation. However, σ_x will increase with time while σ_p is constant.

Eigenstates of a 1-dim. square well potential ($V(x) = 0$ for $0 \le x \le L$, infinite elsewhere):

$$\varphi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right);\ E_n = \frac{n^2\pi^2\hbar^2}{2mL^2},\ n = 1, 2, \ldots$$

Eigenstates of Harmonic Oscillator:

$$\mathbf{H} = \frac{\mathbf{P}^2}{2m} + \frac{m\omega^2}{2}\mathbf{X}^2$$

$$\varphi_n(x) = AH_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)e^{-\frac{m\omega}{2\hbar}x^2};\ E_n = (n+\tfrac{1}{2})\hbar\omega,\ n = 0,1,\ldots$$

$$H_0(y) = 1,\ H_1(y) = 2y,\ H_2(y) = 4y^2 - 2;$$

$$A_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4},\ A_1 = \frac{1}{\sqrt{2}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4},\ A_2 = \frac{1}{\sqrt{8}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}.$$

Quantum Mechanics in 3D:

Cartesian coordinates: (x,y,z)

$$\psi(x,y,z);\ \mathbf{H} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V(x,y,z);\ \Delta\Pr(\vec{r},\Delta\tau) = |\psi(x,y,z)|^2\,\Delta\tau$$

(Small volume $\Delta\tau = \Delta x\Delta y\Delta z$ located at position (x,y,z)).

Separation of variables: Look for solutions for the eigenvalue equation of the type

$$\psi(x,y,z) = \psi_1(x)\psi_2(y)\psi_3(z)$$

Example: Infinite square well in 3D:

$$\varphi_{njk}(x,y,z) = \sqrt{\frac{8}{L^3}}\sin\frac{n\pi x}{L}\sin\frac{j\pi y}{L}\sin\frac{k\pi z}{L};\ E_{njk} = (n^2 + j^2 + k^2)\frac{\hbar^2\pi^2}{2mL^2}$$

Spherical coordinates: r, θ, ϕ

Small volume for probability: $\Delta\tau = r^2\Delta r\,\sin\theta\Delta\theta\,\Delta\phi$

Hamiltonian in spherical coordinates:

$$\mathbf{H} = -\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta} + \frac{1}{r^2}\frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}\right) + V(r)$$

$$= -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{2mr^2}\vec{\mathbf{L}}^2 + V(r)$$

Here, $\vec{\mathbf{L}}^2$ is the squared orbital angular momentum operator with eigenfunctions $Y_{\ell m}(\vartheta,\varphi)$; $\vec{\mathbf{L}}^2 Y_{\ell m} = \hbar^2\ell(\ell+1)Y_{\ell m}$; $\ell = 0,1,2\ldots$; $\mathbf{L}_z Y_{\ell m} = \hbar m Y_{\ell m}$; $m = -\ell, -\ell+1, \ldots, \ell$

($\mathbf{L}_z$ is the z-component of the angular momentum operator). Examples:

$$Y_{00}(\vartheta,\varphi) = \sqrt{\frac{1}{4\pi}};\quad Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta$$

$$Y_1^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\cos\theta \qquad =$$

$$Y_1^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta$$

Separation of variables: Look for eigenstates of the Hamiltonian of form $\psi_{E\ell m}(r,\vartheta,\varphi) = R_{E,\ell}(r)Y_{\ell m}(\vartheta,\varphi)$ with

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}R_{E,\ell}(r) + \frac{\hbar^2\ell(\ell+1)}{2mr^2}R_{E,\ell}(r) + V(r)R_{E,\ell}(r) = E\cdot R_{E,\ell}(r)$$

Probability to find particle in volume $\Delta\tau$ at position (r, θ, ϕ): $\left|\psi_{E\ell m}(r,\vartheta,\varphi)\right|^2\Delta\tau$

Radial probability distribution: $\Delta\Pr(r\ldots r+\Delta r) = |R_{E,\ell}(r)|^2 r^2\Delta r$

Hydrogen-like atoms:

(Nucleus of mass m_2 and charge Ze, bound particle of mass m_1 and charge $-e$)

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Z\alpha\hbar c}{r} \quad \alpha = e^2/4\pi\varepsilon_0\hbar c$$

Mass must be replaced by "reduced mass" of 2-body system with masses m_1 and m_2:

$$\mu_r = \frac{m_1 m_2}{m_1 + m_2}$$

Energy Eigenvalues:

$E_{n\ell} = -\frac{\mu_r}{m_e}\frac{Z^2}{n^2}Ry$ ($n = 1, 2, \ldots$; $Ry = m_e c^2\alpha^2/2 = 13.6$ eV). Degenerate in ℓ and m; $\ell = 0, 1, \ldots, n-1$, $m_\ell = -\ell\ldots+\ell$; also degenerate in electron spin $m_s = \pm 1/2$ => total degeneracy $2n^2$.

Characteristic radius: $a = \frac{m_e}{\mu_r Z}a_0 \quad a_0 = \hbar c/(m_e c^2\alpha) = 0.53$ Å $= 0.053$ nm.

Eigenstates: $\psi_{n,\ell,m}(r,\vartheta,\varphi) = R_{n,\ell}(r)Y_{\ell m}(\vartheta,\varphi)$. $R_{n,\ell}(r)$ (examples):

$$R_{1,0}(r) = \frac{2}{a^{3/2}}e^{-r/a};\ R_{2,0}(r) = \frac{2-r/a}{\sqrt{8}a^{3/2}}e^{-r/2a};\ R_{2,1}(r) = \frac{r/a}{\sqrt{24}a^{3/2}}e^{-r/2a}$$

Energy of a photon: $E_\gamma = hf = hc/\lambda$
Momentum of a photon: $p_\gamma = h/\lambda$
Light emitted or absorbed in transition with energy difference $\Delta E = E_{\text{init}} - E_{\text{final}}$:
$f = \Delta E/h$, $\lambda = hc/\Delta E = 2\pi\hbar c/\Delta E$
Pauli principle: No two identical Fermions (spin-1/2, 3/2, … particles) can be in the same exact quantum state. (-> See Fermi-Dirac statistics)

Nuclear Physics

Mass-energy of an atom: (Z protons, N neutrons, $A = Z+N$):
$M_A c^2 = Z\, M_p c^2 + N\, M_n c^2 + Z\, m_e c^2 - BE$ (Binding energy)
typical binding energies BE = 7-8 MeV·A with a maximum for nuclei around iron (A=56).
Light nuclei have significantly lower BE per nucleon; beyond iron, the BE per nucleon decreases slowly with A (due to Coulomb repulsion).
Energy liberated during a nuclear fusion reaction 1 + 2 -> 3: $\Delta E = M_1c^2 + M_2c^2 - M_3c^2$
Energy liberated during a nuclear decay 1 -> 2 + 3: $\Delta E = M_1c^2 - M_2c^2 - M_3c^2$
Density: roughly constant $\rho = 0.16$ Nucleons / fm^3 = 2×10^{17} kg/m^3
Radioactive nuclei:

alpha-decay: $(Z,A) \rightarrow (Z\text{-}2, A\text{-}2) + {}^4\text{He}$ + energy
beta-plus decay: $(Z,A) \rightarrow (Z\text{-}1, A) + e^+ + \nu_e$
beta-minus decay: $(Z,A) \rightarrow (Z+1, A) + e^- + \bar{\nu}_e$
Decay probability in time Δt: $\Delta\text{Pr}(\Delta t) = \Delta t/\tau$ (τ = lifetime = $T_{1/2}$ / ln 2)
Number of undecayed nuclei at time t (starting with N_0): $N(\text{t}) = N_0\, e^{-t/\tau}$

Particle Physics

Fundamental Fermions (spin-1/2 particles obeying Pauli Exclusion Principle):
quarks (up, down, charm, strange, top, bottom) and leptons (electron, muon, tau, electron-neutrino, muon-neutrino, tau-neutrino) and their antiparticles:

Name	Symbol	Mass (MeV/c^2)*	J	B	Q (e)
Up	u	$2.3^{+0.7}_{-0.5}$	½	+⅓	+⅔
Down	d	$4.8^{+0.5}_{-0.3}$	½	+⅓	−⅓
Charm	c	1275 ±25	½	+⅓	+⅔
Strange	s	95 ±5	½	+⅓	−⅓
Top	t	173 210 ±510 ± 710	½	+⅓	+⅔
Bottom	b	4180 ±30	½	+⅓	−⅓

Particle/antiparticle name	Symbol	Q (e)
Electron / Positron[18]	e^- / e^+	−1 / +1
Muon / Antimuon[19]	μ^- / μ^+	−1 / +1
Tau / Antitau[21]	τ^- / τ^+	−1 / +1
Electron neutrino / Electron antineutrino[34]	ν_e / $\bar{\nu}_e$	0
Muon neutrino / Muon antineutrino[34]	ν_μ / $\bar{\nu}_\mu$	0
Tau neutrino / Tau antineutrino[34]	ν_τ / $\bar{\nu}_\tau$	0

Force-mediating Gauge Bosons (spin-1 particles obeying Bose-Einstein statistics): Photon γ (electromagnetic interaction), W^+, W^-, Z^0 (weak interaction), gluons (strong interaction) [graviton (gravity) only conjectured]. All except weak interaction bosons are massless; the latter gain mass (80-91 GeV/c^2) through interaction with the Higgs field. All interactions proceed via gauge bosons coupling to various charges:
- electromagnetic interaction: electric charge (+ or -) (all Fermions except neutrinos, plus *W* bosons)
- weak interaction: weak charges ("weak isospin and weak hypercharge") – all particles except photons, gluons
- strong interaction: color charges ("red", "green", "blue") – all quarks and gluons.

Molecules and Condensed Matter

Ionic Bond: One atom gives up 1 (or more) electron(s), the other picks it (them) up; binding through electrostatic attraction.
Covalent Bond: Electron(s) shared between two atoms. Example: Let $\psi_1(\vec{r}_e)$ = wave function for hydrogen ground state with proton at position 1, and $\psi_2(\vec{r}_e)$ for proton at position 2. Symmetric superposition $\psi_S(\vec{r}_e) = \frac{1}{\sqrt{2}}\psi_1(\vec{r}_e) + \frac{1}{\sqrt{2}}\psi_2(\vec{r}_e)$ is attractive (net charge between protons), antisymmetric superposition $\psi_A(\vec{r}_e) = \frac{1}{\sqrt{2}}\psi_1(\vec{r}_e) - \frac{1}{\sqrt{2}}\psi_2(\vec{r}_e)$ is non-binding (zero net charge between protons).
Metallic Bond: Many electrons (one or more per atom) shared between a large number *N* of atoms -> positively charged "rest atoms" in "Fermi gas" of electrons. Electron energy eigenstates are clustered in "bands"; highest (partially or totally unoccupied) band = conduction band, next lower (filled) band = valence band. Each band contains of order *N* eigenstates. Interaction between electron gas and oscillation modes (=phonons) of the "rest atoms" gives rise to conductive heating, $V = RI$, and superconductivity (Bose-Einstein condensation of "Cooper pairs" of electrons).
Conductors: partially filled conduction band and/or overlapping conduction and valence bands.
Isolators: Completely empty conduction band, completely filled valence band, large band gap.
Semi-conductors: Similar to isolators, but with smaller band gap. Can conduct at finite temperatures (see Fermi-Dirac distribution below). Conductivity increased through electron donor (n-doped) or electron acceptor (p-doped) impurities. pn-junction = diode.

Thermal/Statistical Physics

Boltzmann Distribution: number $n(E)$ of atoms (molecules, …) out of an ensemble with a total of N atoms (…) with given energy E in a system with absolute temperature T (in K).

Discrete energy levels E_i (*e.g.*, quantum systems) with degeneracy g_i (= number of eigenstates of the Hamiltonian with energy eigenvalue E_i):

$$n(E_i) = Cg_i e^{-E_i/kT} = \frac{g_i}{e^{(E_i-\mu)/kT}}; C = e^{\mu/kT} = N / \sum g_i e^{-E_i/kT}$$

(C is a normalization constant; μ is the "chemical potential")

Continuous energy levels E (classical system, e.g. monatomic gas) with state density $g(E)\mathrm{d}E$ (= volume in "phase space" between energy E and energy E + dE):

$$dn(E...E+dE) = C\, g(E)dE\, e^{-E/kT}; C = N / \int g(E)\, dE\, e^{-E/kT}$$

State density for simple monatomic gas:

$$g(E)dE = 4\pi p^2 dp = 4\pi m\sqrt{2mE}dE$$

Consequences: Ideal gas law $PV = nRT = nN_A kT$, (n = number of mols; $N = nN_A$); average energy per degree of freedom (dimension of motion) = ½ kT => total kinetic energy of a monatomic gas = 3/2 kT per atom or $E_{tot} = {}^3/_2\, nN_A kT = {}^3/_2\, nRT$

Fermi-Dirac Distribution (for a system of indistinguishable Fermions):

$$n(E_i) = N\frac{g_i}{e^{(E_i-\mu)/kT}+1}$$

; μ here is right above the Fermi energy = the highest filled energy level necessary to accommodate all N fermions, where all lower energy levels are filled with as many Fermions as the Pauli principle allows (= the state of a (degenerate) Fermi gas at (close to) zero temperature).

Bose-Einstein Distribution (for a system of indistinguishable bosons):

$$n(E_i) = N\frac{g_i}{e^{(E_i-\mu)/kT}-1}$$

; μ here is right below the ground state energy (the lowest available energy level). If T goes to zero, all levels but that lowest energy level are empty = Bose-Einstein condensation.

Photon density for black-body radiation: $\frac{dn_\gamma(\lambda\ldots\lambda+d\lambda)}{dV} = \frac{8\pi}{\lambda^4}\frac{d\lambda}{e^{hc/\lambda kT}-1} = 8\pi\frac{f^2}{c^3}\frac{df}{e^{hf/kT}-1}$

Energy density (= energy contained in electromagnetic radiation of frequency f or wave length λ, per unit volume V) for black-body radiation (i.e., Bose-Einstein Distribution for a photon gas - Planck's Law):

$$\frac{dE}{V} = 8\pi h\frac{f^3}{c^3}\frac{df}{e^{hf/kT}-1} = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$$

; Energy flux/surface area $\frac{dE}{dAdt} = \frac{2\pi hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$