PHYSICS 313 - Winter/Spring Semester 2017 - ODU

<u>Astrophysics - Problem Set 7 – Solution</u>

<u>Problem 1</u>

Mark each of the following statements with "Y" or "T" if they are correct, and with "F" or "N" if they are incorrect:

1a) In General Relativity, coordinate systems that are at absolute rest have preferential status ("Inertial System"). **F** [There is no such things as "absolute rest"

1b) In General Relativity, freely falling systems (on which no force other than gravity acts) have a preferential status ("Inertial System"), even if they accelerate according to Newtonian interpretation. **T**

1c) In General Relativity, the concept of "straight line" has no meaningful interpretation. **F** [See next question]

1d) In General Relativity, the concept of "straight line" is generalized to a "geodesic" which is the path followed by a freely falling object. **T**

1e) Two observers that start at the same point in space and time and then meet again at a later time may experience different elapsed proper ("eigen") times in their own rest frames. **T**

1f) In the scenario under 1e) above, the observer who traveled while being accelerated by a non-gravitational force tends to age less than the observer in a freely falling rest frame. **T**

1g) The statement in 1e) can even be true if both observers are freely falling (i.e., follow "straight lines"). In other words, two straight lines can intersect twice, with different "lengths" between the intersection points. **T** [Example: The supernova which was observed several times in slightly different directions]

<u>Problem 2</u>

The following is a set of multiple choice questions. Answer each with one single digit:

2a) Which of the following examples is **not** a direct consequence of space-time curvature, according to the General Theory of Relativity? **2**

1 – If you see two images of the same object (e.g., a quasar) at slightly different spots in the sky
2 – The bending (refraction) of the light from a setting sun in Earth's atmosphere
3 – The existence of black holes with event horizons
4 – The motion of a weight that is dropped from 1m height above the surface of Earth.

2b) How can we observe stellar sized black holes in the universe, in spite of their "blackness"? **4**

1 – Through the gamma rays emitted from their event horizons.
2 – Through their Hawkins radiation
3 – Through a regular train of radio frequency pulses they emit
4 – Through the gravitational waves emitted when two of them coalesce [This is only true since September last year!]

Problem 3

Calculate the time that appears to elapse at the surface of

a) Earth **0.9999999993 s** (0.7 ns less)

b) Sun **0.9999979 s** (2.1 μs less – that's the lifetime of a muon!)

c) a neutron star of 2 solar masses and 9 km radius **0.586 s** (only little more than half!)

during one second, according to an observer very far away from all these objects. (Numerical results only required, but you can show your work for partial credit).

[*Note: we use the equations* $R_s = \frac{2GM}{c^2} \Rightarrow \quad \Delta t_{local} = \sqrt{1 - \frac{R_s}{r}} \Delta t_\infty$ *from the formula sheet.]*

Problem 4

Calculate the Schwarzschild radius R_s of a hypothetical particle with mass $m = 1.088 \cdot 10^{-8}$ kg (one-half of the so-called Planck mass). R = $\mathbf{1.616 \cdot 10^{-35}}$ **m**

Now calculate the minimum momentum uncertainty Δp, according to the Heisenberg uncertainty principle, that an object must have if we know its position with a precision of at least the value of R_s you calculated. $\Delta p = \frac{\hbar}{2R} = 3.26$ kgm/s

Show that this momentum uncertainty is equal to the mass of the particle times c.

***mc* = 3.26 kgm/s** as advertised. The meaning of this is that we can imagine that if we somehow managed to squeeze a massless object inside a radius R, its momentum (uncertainty) will be so large that its kinetic energy will suffice to create a microscopic black hole with just the Schwarzschild radius R – the smallest black hole possible! Whether you think this is really happening or not – it is clear that Quantum Mechanics **must** somehow affect (modify) General Relativity at this size scale. But how? Today's greatest minds are fighting over that… In any case, you can tell from this example the origin and meaning of the so-called "Planck units" (see our formula sheet).