Physics Notes 10-25-16

Separation of variables

1D → 3D

$\Psi(x,t)$ → $\Psi(x,y,z,t)$

Prob: $(x....x+\Delta x)$ → Prob. $(x.... x+\Delta x, y.....y+\Delta y, z....z+\Delta z)$

$|\Psi(x,t)|^2 * \Delta x$ (inside a 3-D volume $\Delta\tau=\Delta x\Delta y\Delta z$)

$|\Psi(x,y,z,t)|^2 * \Delta x * \Delta y * \Delta z$

Require: $\int |\Psi(x,t)|^2\, dx=1$ → $\int\int\int|\Psi(x,y,z,t)|^2\, dxdydz=1$

3 position operators X,Y,Z

$(X_\psi)(x,y,z) = x * \phi(x,y,z)$

$(Y_\psi)(x,y,z) = y * \phi(x,y,z)$

$(Z_\psi)(x,y,z) = z * \phi(x,y,z)$

Momentum P vector form $\tilde{P}= (\hbar/i)$

P_x (h/i) d/dx

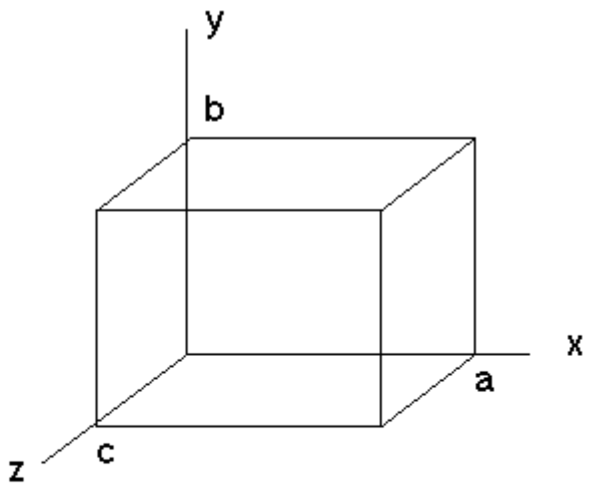
P_y (h/i) d/dy

P_z (h/i) d/dz

$H=(P_x^2 + P_y^2 + P_z^2)/(2m) + V(x,y,z) = (-\hbar^2/2m)(d^2/dx^2{}_\psi + d^2/dy^2{}_\psi + d^2/dz^2{}_\psi) + V(x,y,z)* \Psi(x,y,z) = H_\psi$

To Do:

1. Find eigenstates $H\, \phi_E (x,y,z) = E\, \phi_E(x,y,z)$
2. Write full solution to Schrödinger equation



$V=(x,y,z) \rightarrow$ 0 if $0 < x < L$ and $0 < y < L$ and $0 < z < L$ and infinity else

Separation of Variables: $\phi_E(x,y,z)= \phi_1(x)\, \phi_2(y)\, \phi_3(z)$

$[-\hbar^2/2m][(d^2 \phi_1/dx^2)\, \phi_2\, \phi_3 + (d^2 \phi_2 /dy^2)\, \phi_1\, \phi_3 + (d^2\phi_3/dz^2)\, \phi_1\, \phi_2) + V^*\phi_1\, \phi_2\, \phi_3= E^*\, \phi_1\, \phi_2\, \phi_3$

$[-\hbar^2/2m][(1/\phi_1(x))(d^2\phi_1/dx^2) + (1/\phi_2)(d^2\phi_2/dy^2) + (1/\phi_3)(d^2\phi_3/dz^2)] + V = E$

$F(x)=E_1$ $\qquad$ $G(y)=E_2$ $\qquad$ $H(z)=E_3$

3 Equations

$(-\hbar^2/2m)(d^2\, \Psi(x)/dx^2)=E\phi_1(x)$ $\qquad$ $\phi_1(x)= A\sin(n\, \pi x/L)$, $E_1=(n^2\, \pi^2\, \hbar^2/2mL^2)$ $\quad 0 < x < L$, 0 else

$(-\hbar^2/2m)(d^2\, \Psi(y)/dy^2)=E\phi_2\, (y)$ $\qquad$ $\phi_2(x)= A\sin(m\, \pi y/L)$,$E_2=(m^2\, \pi^2\, \hbar^2/2mL^2)$

$(-\hbar^2/2m)(d^2\, \Psi(z)/dz^2)=E\phi_3\, (z)$ $\qquad$ $\phi_3(x)= A\sin(k\, \pi z/L)$, $E_3=(k^2\, \pi^2\, \hbar^2/2mL^2)$

Solution: $\phi_{n,m,k}(x,y,z) = A\sin(n\, \pi x/L)\, \sin(m\, \pi y/L)\sin(k\, \pi z/L)$

Quantum Numbers n,m,k=1,2,3....

$E_{n,m,k} = (\pi^2\, \hbar^2/2mL^2)(n^2+m^2+k^2)$

Lowest (ground state) energy $E_{1,1,1} = 3(\pi^2\, \hbar^2/2mL^2)$

Degeneracy: Same energy for several different states, e.g. $E_{1,1,2} = E_{1,2,1} = E_{2,1,1}$