Fundamental constants:

Speed of light: $c = 2.9979 \cdot 10^8$ m/s (roughly a foot per nanosecond)
Planck constant: $h = 6.626 \cdot 10^{-34}$ J s; $\hbar = h / 2\pi = 197.33$ eV/$c \cdot$ nm = 0.658 eV/PHz
Fundamental charge unit: $e = 1.602 \cdot 10^{-19}$ C
Electron mass: $m_e = 9.109 \cdot 10^{-31}$ kg
Hydrogen atom (^{1}H) mass: $m_H = 1.6735 \cdot 10^{-27}$ kg ($A = 1.0078$)
Helium atom (^{4}He) mass: $m_{^4He} = 6.6465 \cdot 10^{-27}$ kg ($A = 4.0026$)
Coulomb's Law constant: $k = 1/4\pi\varepsilon_0 = 8.988 \cdot 10^9$ Nm^2/C^2
Gravitational constant: $G = 6.674 \cdot 10^{-11}$ Nm^2/kg^2
Avogadro constant: $N_A = 6.022 \cdot 10^{23}$ particles per mol

Special Relativity:

Lorentz Transformation from S to S':

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$; $x' = \gamma\left(x - \frac{v}{c}ct\right)$; $ct' = \gamma\left(ct - \frac{v}{c}x\right)$; $y = y'$; $z = z'$, replace v with $-v$ for S' -> S

Velocity Addition:

$$\frac{u_x}{c} = \frac{\frac{u_x'}{c} + \frac{v}{c}}{1 + \frac{u_x'}{c}\frac{v}{c}}; \frac{u_y}{c} = \frac{\frac{1}{\gamma}\frac{u_y'}{c}}{1 + \frac{u_x'}{c}\frac{v}{c}}$$

Doppler Shift: $\frac{\lambda_{obs}}{\lambda_{emitted}} = (z+1) = \frac{1 + v_{\parallel}/c}{\sqrt{1 - v^2/c^2}}$

Four-vectors: $x^\mu = (ct, x, y, z)$; $x_\mu = (ct, -x, -y, -z)$ (μ=0,1,2,3 for ct,x,y,z).

Invariant (squared) Interval:

$\Delta x^\mu = (\Delta ct, \Delta x, \Delta y, \Delta z) \Rightarrow \Delta s^2 = \Delta x^\mu \Delta x_\mu = \Delta ct^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$

Positive Δs^2: "time-like separation", Δs^2 = square of elapsed time $c\tau$ in a system that travels from the start point (event) to the end point (event) of the interval.
Negative Δs^2: "space-like separation", $-\Delta s^2$ = square of distance between the two events in a system (which always exists!) where they occur simultaneously.
$\Delta s^2 = 0$: "light-like separation"; a light ray could travel from one event to the other.

Four-momentum:

$P^\mu = (E/c, P_x, P_y, P_z) = (\Gamma mc, \Gamma m\vec{u})$; $\Gamma = \frac{1}{\sqrt{1 - \vec{u}^2/c^2}}$. u = velocity.

Transformation of P^μ to coordinate system S' is analog to x^μ (see above).

$E = P^0 c$, $E_{rest} = mc^2$, $T_{kin} = (\Gamma - 1) * mc^2$ ($\approx \frac{m}{2} u^2$ **only** if $u \ll c$); **Photons**: $u = c$, $E = |P|c$.

Invariant Interval: $(P^0)^2 - \vec{P}^2 = \left(\frac{E}{c}\right)^2 - P_x^2 - P_y^2 - P_z^2 = m^2c^2 \Rightarrow E = c\sqrt{m^2c^2 + \vec{P}^2}$; $\frac{\vec{u}}{c} = \frac{\vec{P}c}{E}$

Quantum Mechanics – Motion in 1D:

Scalar Product: $\langle \psi_1 | \psi_2 \rangle = \int_{-\infty}^{\infty} \psi_1^*(x)\psi_2(x)\,dx$.

Normalization: $\langle \psi | \psi \rangle \overset{!}{=} \int_{-\infty}^{\infty} \psi^*(x)\psi(x)\,dx < \infty$ => $|\psi_{normalized}\rangle = |\psi\rangle / (\langle \psi | \psi \rangle)^{1/2}$.

Probability of locating a particle in an interval $x \ldots x+dx$:

$$d\Pr(x...x+dx) = |\psi(x)|^2\,dx = \psi(x)^*\,\psi(x)dx$$

Operator O with eigenvalue ω and eigenvector $|\varphi_\omega\rangle$: $\mathbf{O}|\varphi_\omega\rangle = \omega|\varphi_\omega\rangle$.

Position Operator X: $\mathbf{X}\psi(x) = x \cdot \psi(x) \rightarrow$ eigenvectors $\psi_{x_0}(x) = \delta(x - x_0)$, eigenvalue x_0

Momentum Operator: $\mathbf{P}\psi(x) = \frac{\hbar}{i}\frac{\partial}{\partial x}\psi(x) \rightarrow$ eigenvectors $\psi_{p_0}(x) = e^{ip_0 x/\hbar}$, eigenvalue p_0

Hamiltonian: $\mathbf{H}\psi(x) = \left(\frac{\mathbf{P}^2}{2m} + V(X)\right)\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x)$.

Heisenberg: $\sigma_x \sigma_p \geq \hbar/2$.

Schrödinger Equation: $|\psi\rangle(t)$; $\frac{\partial}{\partial t}|\psi\rangle(t) = \frac{1}{i\hbar}\mathbf{H}|\psi\rangle(t)$

Eigenstates of Hamiltonian: $\mathbf{H}|\varphi_E\rangle = E|\varphi_E\rangle$ => $|\psi_E(t)\rangle = |\varphi_E\rangle e^{-iEt/\hbar}$

Motion in 1-D, eigenstates of the Hamiltonian: $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$

Eigenstate of the **free Hamiltonian** (V(x)=0): $\psi_p(x,t) = Ae^{\frac{i}{\hbar}px}e^{-\frac{i}{\hbar}\frac{p^2}{2m}t}$

Gaussian Wave Package: $\psi_{GWP}(x,t=0) = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_p}}\int_{-\infty}^{\infty} e^{-\frac{(p-p_0)^2}{4\sigma_p^2}} e^{\frac{i}{\hbar}px}\,dp = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_x}} e^{\frac{i}{\hbar}p_0 x} e^{-\frac{x^2}{4\sigma_x^2}}$; $\sigma_x = \frac{\hbar}{2\sigma_p}$

Eigenstates for a **1-dim square well**: ($V(x)$=0, $0 \leq x \leq L$, ∞ else)

$\varphi_n(x) = 0$ for $x < 0$, $x > L$; else $\varphi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$; $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$, $n = 1, 2, \ldots$

Eigenstates of Harmonic Oscillator: $\mathbf{H} = \frac{\mathbf{P}^2}{2m} + \frac{m\omega^2}{2}\mathbf{X}^2$ =>

$$\varphi_n(x) = AH_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)e^{-\frac{m\omega}{2\hbar}x^2};\ E_n = (n+\tfrac{1}{2})\hbar\omega\ ,\ n = 0,1,\ldots$$

$$H_0(y) = 1,\ H_1(y) = 2y,\ H_2(y) = 4y^2 - 2;$$

$$A_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4},\ A_1 = \frac{1}{\sqrt{2}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4},\ A_2 = \frac{1}{\sqrt{8}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}.$$

Quantum Mechanics in 3D:

Cartesian coordinates: (x,y,z); $\Delta\tau = \Delta x \Delta y \Delta z$

$$\psi(x,y,z);\ \mathbf{H} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(x,y,z);\ \Delta \Pr(\vec{r},\Delta\tau) = \left|\psi(x,y,z)\right|^2 \Delta\tau$$

Infinite square well: $\varphi_{njk}(x,y,z) = \sqrt{\frac{8}{L^3}}\sin\frac{n\pi x}{L}\sin\frac{j\pi y}{L}\sin\frac{k\pi z}{L};\ E_{njk} = (n^2+j^2+k^2)\frac{\hbar^2\pi^2}{2mL^2}$

Spherical coordinates: r, θ, φ**;** $\Delta\tau = r^2\Delta r \sin\theta\Delta\theta\,\Delta\varphi$

$$\mathbf{H} = -\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta}+\frac{1}{r^2}\frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}\right)+V(r)$$

$$= -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}+\frac{1}{2mr^2}\vec{\mathbf{L}}^2+V(r)$$

$\vec{\mathbf{L}}^2$ is the squared orbital angular momentum operator with eigenfunctions

$Y_{\ell m}(\vartheta,\varphi);\ \vec{\mathbf{L}}^2 Y_{\ell m} = \hbar^2\ell(\ell+1)Y_{\ell m};\ \ell = 0,1,2\ldots;\ \mathbf{L}_z Y_{\ell m} = \hbar m Y_{\ell m};\ m = -\ell,-\ell+1,\ldots,\ell$

Examples:

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta$$

$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\cos\theta \qquad =$$

$$Y_{00}(\vartheta,\varphi) = \sqrt{\frac{1}{4\pi}};\quad Y_1^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta$$

Separation of Variables: $\psi_{E\ell m}(r,\vartheta,\varphi) = R_{E,\ell}(r)Y_{\ell m}(\vartheta,\varphi)$ with

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}R_{E,\ell}(r)+\frac{\hbar^2\ell(\ell+1)}{2mr^2}R_{E,\ell}(r)+V(r)R_{E,\ell}(r) = E\cdot R_{E,\ell}(r)$$

Probability to find particle in volume $\Delta\tau$ at position (r, θ, ϕ): $\left|\psi_{E\ell m}(r,\vartheta,\varphi)\right|^2\Delta\tau$

Radial probability distribution: $\Delta\Pr(r\ldots r+\Delta r) = |R_{E,\ell}(r)|^2 r^2\Delta r$

Hydrogen-like atoms:

(Nucleus of mass m_2 and charge Ze, bound particle of mass m_1 and charge $-e$)

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Z\alpha\hbar c}{r} \qquad \alpha = e^2/4\pi\varepsilon_0\hbar c$$

Reduced mass of 2-body system with masses m_1 and m_2: $\mu_r = \frac{m_1 m_2}{m_1+m_2}$

Energy Eigenvalues: $E_{n\ell} = -\frac{\mu_r}{m_e}\frac{Z^2}{n^2}Ry$ (n = 1, 2, … ; $Ry = m_e c^2 \alpha^2/2$ = 13.6 eV).

Degenerate in ℓ and m; ℓ =0, 1,…, n-1, m_ℓ = -ℓ…+ℓ;
also degenerate in electron spin m_s = ±1/2 => total degeneracy $2n^2$.

Characteristic radius: $a = \frac{m_e}{\mu_r Z}a_0$ $\quad a_0 = \hbar c / (m_e c^2 \alpha)$ = 0.53 Å = 0.053 nm.

Eigenstates: $\psi_{n,\ell,m}(r,\vartheta,\varphi) = R_{n,\ell}(r)Y_{\ell m}(\vartheta,\varphi)$. $R_{n,\ell}(r)$ (examples):

$$R_{1,0}(r) = \frac{2}{a^{3/2}}e^{-r/a};\ R_{2,0}(r) = \frac{2-r/a}{\sqrt{8}a^{3/2}}e^{-r/2a};\ R_{2,1}(r) = \frac{r/a}{\sqrt{24}a^{3/2}}e^{-r/2a}$$

Energy of a photon: $E_\gamma = hf = hc/\lambda$
Momentum of a photon: $p_\gamma = h/\lambda$
Light emitted or absorbed in transition with energy difference $\Delta E = E_{\text{init}} - E_{\text{final}}$:
$f = \Delta E/h$, $\lambda = hc/\Delta E = 2\pi\hbar c/\Delta E$
Pauli principle: No two identical Fermions (spin-1/2, 3/2, … particles) can be in the same exact quantum state.

Molecules and Condensed Matter

Ionic Bond: One atom gives up 1 (or more) electron(s), the other picks it (them) up; binding through electrostatic attraction.
Covalent Bond: Electron(s) shared between two atoms. Example: Let $\psi_1(\vec{r}_e)$ = wave function for hydrogen ground state with proton at position 1, and $\psi_2(\vec{r}_e)$ for proton at position 2. Symmetric superposition $\psi_S(\vec{r}_e) = \frac{1}{\sqrt{2}}\psi_1(\vec{r}_e) + \frac{1}{\sqrt{2}}\psi_2(\vec{r}_e)$ is attractive (net charge between protons), antisymmetric superposition $\psi_A(\vec{r}_e) = \frac{1}{\sqrt{2}}\psi_1(\vec{r}_e) - \frac{1}{\sqrt{2}}\psi_2(\vec{r}_e)$ is non-binding (zero net charge between protons).
Metallic Bond: Many electrons (one or more per atom) shared between a large number N of atoms -> positively charged "rest atoms" in "Fermi gas" of electrons. Electron energy eigenstates are clustered in "bands"; highest (partially or totally unoccupied) band = conduction band, next lower (filled) band = valence band. Each band contains of order N eigenstates. Interaction between electron gas and oscillation modes (=phonons) of the "rest atoms" gives rise to conductive heating, $V = RI$, and superconductivity (Bose-Einstein condensation of "Cooper pairs" of electrons).
Conductors: partially filled conduction band and/or overlapping conduction and valence bands.
Isolators: Completely empty conduction band, completely filled valence band, large band gap.
Semi-conductors: Similar to isolators, but with smaller band gap. Can conduct at finite temperatures (see Fermi-Dirac distribution below). Conductivity increased through electron donor (n-doped) or electron acceptor (p-doped) impurities. pn-junction = diode.

Particle Physics

Fundamental Fermions (spin-1/2 particles obeying Pauli Exclusion Principle): quarks (up, down, charm, strange, top, bottom) and leptons (electron, muon, tau, electron-neutrino, muon-neutrino, tau-neutrino) and their antiparticles.

Force-mediating Gauge Bosons (spin-1 particles obeying Bose-Einstein statistics): Photon γ (electromagnetic interaction), W^+, W^-, Z^0 (weak interaction), gluons (strong interaction) [graviton (gravity) only conjectured]. All except weak interaction bosons are massless; the latter gain mass (80-91 GeV/c^2) through interaction with the Higgs field.

Name	Symbol	Mass (MeV/c^2)*	J	B	Q (e)
Up	u	$2.3\,^{+0.7}_{-0.5}$	½	+⅓	+⅔
Down	d	$4.8\,^{+0.5}_{-0.3}$	½	+⅓	−⅓
Charm	c	1275 ±25	½	+⅓	+⅔
Strange	s	95 ±5	½	+⅓	−⅓
Top	t	173 210 ±510 ± 710	½	+⅓	+⅔
Bottom	b	4180 ±30	½	+⅓	−⅓

Particle/antiparticle name	Symbol	Q (e)
Electron / Positron[18]	e^- / e^+	−1 / +1
Muon / Antimuon[19]	μ^- / μ^+	−1 / +1
Tau / Antitau[21]	τ^- / τ^+	−1 / +1
Electron neutrino / Electron antineutrino[34]	$\nu_e / \bar{\nu}_e$	0
Muon neutrino / Muon antineutrino[34]	$\nu_\mu / \bar{\nu}_\mu$	0
Tau neutrino / Tau antineutrino[34]	$\nu_\tau / \bar{\nu}_\tau$	0

All interactions proceed via gauge bosons coupling to various charges:
- electromagnetic interaction: electric charge (+ or -) (all charged Fermions plus W bosons)
- weak interaction: weak charges ("weak isospin and weak hypercharge") – all particles except gluons
- strong interaction: color charges ("red", "green", "blue") – all quarks and gluons.

Nuclear Physics

Mass-energy of an atom: (Z protons, N neutrons, $A = Z+N$):

$M_A c^2 = Z\, M_p c^2 + N\, M_n c^2 + Z\, m_e c^2 - BE$ (Binding energy)

typical binding energies BE = 7-8 MeV·A with a maximum for nuclei around iron (A=56). Light nuclei have significantly lower BE per nucleon; beyond iron, the BE per nucleon decreases slowly with A (due to Coulomb repulsion).

Energy liberated during a nuclear fusion reaction 1 + 2 -> 3: $\Delta E = M_1c^2 + M_2c^2 - M_3c^2$

Energy liberated during a nuclear decay 1 -> 2 + 3: $\Delta E = M_1c^2 - M_2c^2 - M_3c^2$

Density: roughly constant $\rho = 0.16$ Nucleons / fm^3 = 2×10^{17} kg/m^3

Radioactive nuclei:

alpha-decay: $(Z,A) \rightarrow (Z-2, A-2) + {}^4\text{He}$ + energy

beta-plus decay: $(Z,A) \rightarrow (Z-1, A) + e^+ + \nu_e$

beta-minus decay: $(Z,A) \rightarrow (Z+1, A) + e^- + \bar{\nu}_e$

Decay probability in time Δt: $\Delta\text{Pr}(\Delta t) = \Delta t/\tau$ (τ = lifetime = $T_{1/2}$ / ln 2)

Number of undecayed nuclei at time t (starting with N_0): $N(t) = N_0\, e^{-t/\tau}$