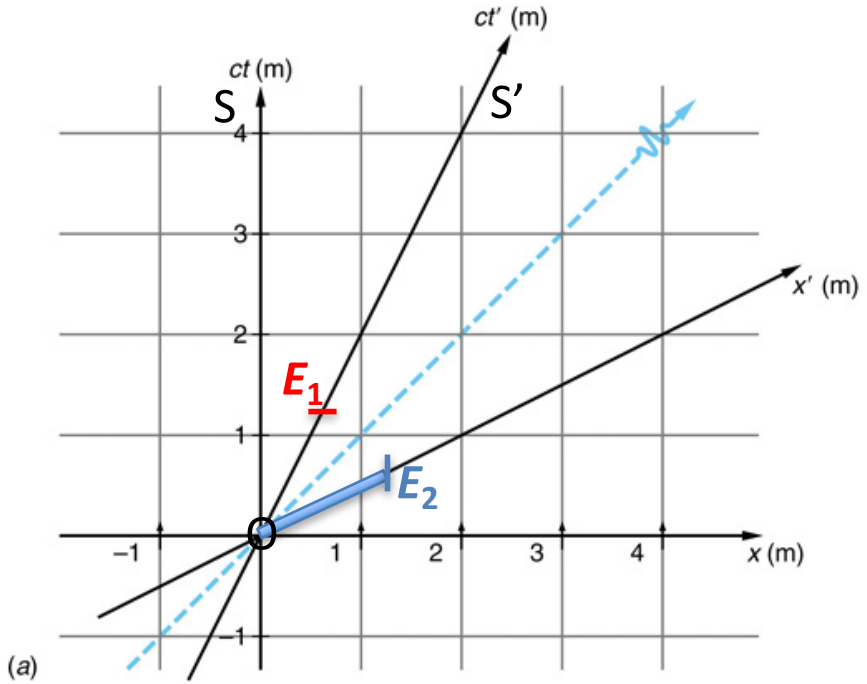
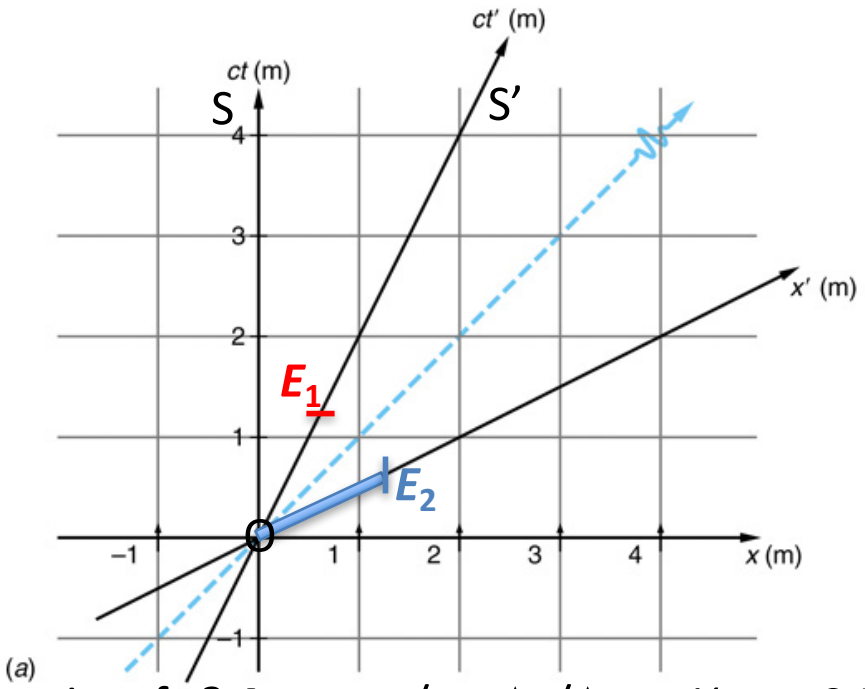
Quiz 2

Assume S' contains a rod 1 m in length, aligned with the *x*' axis. It has a (rest) mass *m* = 1 kg.
The origin of S' is identical with the origin of S. Please answer the following questions.



A. What is the velocity *v* of S' relative to S in units of *c*?
B. At what point (*ct*, *x*) in S does the event E_1 = "clock in S' strikes 1m" occur?
C. What is the invariant interval $(\Delta s)^2$ between the origin and that event in S?
D. What is the invariant interval $(\Delta s)^2$ between the origin and that event in S'?
E. If S measures both ends of the rod simultaneously (at *ct* = 0), what length does it measure?
F. What are the coordinates of the event E_2 = "S' measures the far side end of the rod, simultaneously with its origin (*ct'*, *x'*) = (0,0)"?
G. What is the invariant interval $(\Delta s)^2$ between the origin and **that** event in S?
H. What is the total 4-momentum of the rod in S'? (P^0,P^1,P^2,P^3)
I. What is the total 4-momentum of the rod in S? (P^0,P^1,P^2,P^3)
J. The rod collides with an identical one at rest in S. What is the invariant mass of the combined object after the collision?

Quiz 2

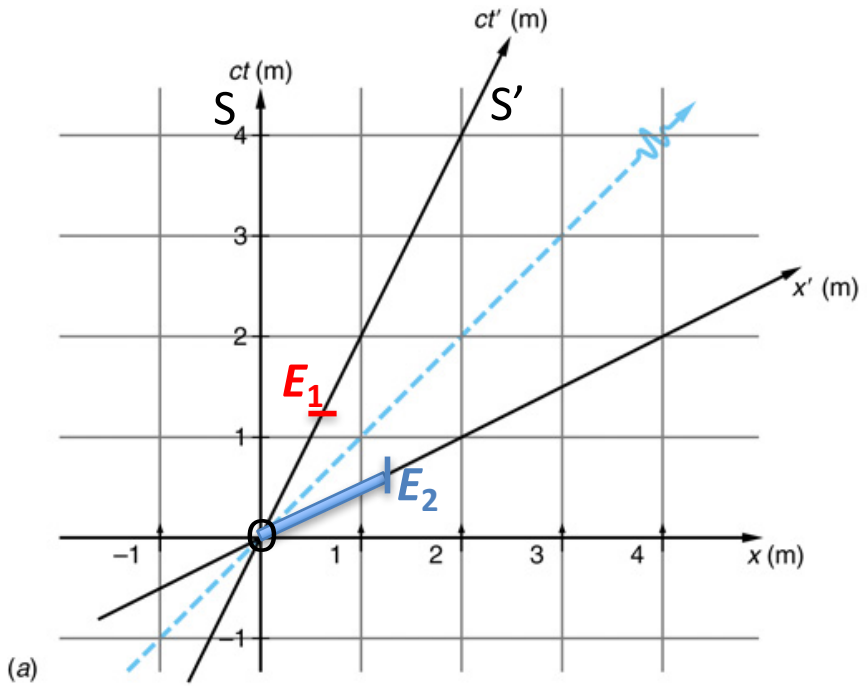


Assume S' contains a rod 1 m in length, aligned with the x' axis. It has a (rest) mass m = 1 kg.
The origin of S' is identical with the origin of S. Please answer the following questions.

A. What is the velocity v of S' relative to S in units of c? **Answ.:** $v/c = \Delta x/\Delta ct = ½$; $v = 0.5\ c$
B. At what point (ct, x) in S does the event E_1 = "clock in S' strikes 1m" occur?
Answ.: $ct = \gamma ct' = (1 + 0.25)^{-1/2} \cdot 1\ m = 1.155$ m (time dilation); $x = v\ ct = 0.577$ m
C. What is the invariant interval $(\Delta s)^2$ between the origin and that event in S?
Answ.: 1 m^2 - either by direct calculation ($ct^2 - x^2$) or by observing that it must be the same as in S' (see below)
D. What is the invariant interval $(\Delta s)^2$ between the origin and that event in S'?
Answ.: 1 m^2 by definition for a time-like interval or by direct calculation
E. If S measures both ends of the rod simultaneously (at $ct = 0$), what length does it measure?
Answ.: $1/\gamma$ m = 0.866 m (length contraction or use Lorentz transformation with $ct = 0$)
F. What are the coordinates of the event E_2 = "S' measures the far side end of the rod, simultaneously with its origin (ct', x') = (0,0)"? **Answ.:** $ct = \gamma ct' + \gamma\ v/c\ x' = 0 + 0.577$m;
$x = \gamma\ v/c\ ct' + \gamma\ x' = 0 + 1.155$ m

Quiz 2

Assume S' contains a rod 1 m in length, aligned with the *x*' axis. It has a (rest) mass *m* = 1 kg.
The origin of S' is identical with the origin of S. Please answer the following questions.



F. What are the coordinates of the event E_2 = "S' measures the far side end of the rod, simultaneously with its origin (ct', x') = (0,0)"? **Answ.:** $ct = \gamma ct' + \gamma\, v/c\, x' = 0 + 0.577$m; $x = \gamma\, v/c\, ct' + \gamma\, x' = 0 + 1.155$ m

G. What is the invariant interval $(\Delta s)^2$ between the origin and **that** event in S?
Answ.: Again, 1 m^2 by definition (space-like interval), direct calculation or invariance

H. What is the total 4-momentum of the rod in S'? (P^0,P^1,P^2,P^3)
Answ.: $P^0 = E/c = mc = 2.9979 \cdot 10^8$ kg m/s; $P^1 = P^2 = P^3 = 0$ ($u = 0$!)

I. What is the total 4-momentum of the rod in S? (P^0,P^1,P^2,P^3)
Answ.: $P^0 = E/c = \Gamma mc = 3.462 \cdot 10^8$ kg m/s; $P^1 = \Gamma mv = 1.731 \cdot 10^8$ kg m/s; rest = 0 ($u = v$!)

J. The rod collides with an identical one at rest in S. What is the invariant mass of the combined object after the collision?
Answ.: $P^0_{tot} = mc + \Gamma mc = 6.46 \cdot 10^8$ kg m/s; $P^1_{tot} = \Gamma mv = 1.731 \cdot 10^8$ kg m/s
$m^2c^2 = (P^0_{tot})^2 - (P^1_{tot})^2 = [(1+\Gamma)^2 - 0.5^2\,\Gamma^2\,](1\text{kg c})^2 \Rightarrow$ m = 2.076 kg