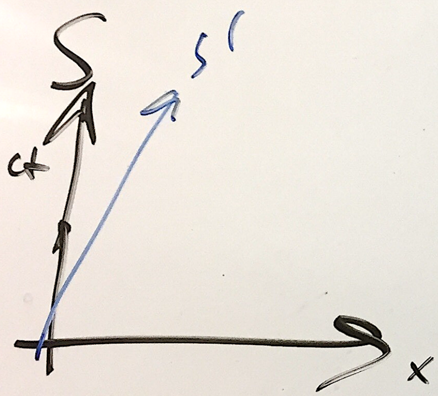
Mass m Force $\vec{F}$ acceleration $\vec{a}$

Momentum $\vec{p} = \Gamma m \vec{u}$ $\qquad \sum^{N} \vec{p}_i$ is conserved

$$\vec{F}_{obj} = \frac{d\vec{p}_{obj}}{dt}$$

mass as measured in rest frame = "rest mass"



S': $m_1 = 1$ kg $\qquad \vec{u}'_1 = (0, u, 0)$

S: $m_2 = 1$ kg $\qquad \vec{u}_2 = (0, -u, 0)$

$$u_{y_{1+2}} = u'_{y_{1+2}} = 0$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$p_{tot_y} = \underbrace{-mu}_{2} + \underbrace{\cancel{\Gamma} m \frac{1}{\gamma} u}_{1}$$

$\vec{u}_1$? $\qquad u_{1y} = \frac{1}{\gamma} u$

$$\Rightarrow \Gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\vec{u}_1 = \left(v, \frac{u}{\gamma}, 0\right)$$

Relativistic (3-)momentum: Most fundamental object, conserved. New: scale factor Gamma multiplies rest mass m.
Example: two equal "putty" masses thrown sideways from platform and train stick together

$$\vec{p} = \Gamma m \vec{u} \qquad \Gamma = \frac{1}{\sqrt{1-\vec{u}^2/c^2}}$$

in S'

m 1 $-u'$ M u' m 2

Assume $u' = v$

$$P_{2x} = \frac{\cancel{1+v^2/c^2}}{\sqrt{(1+v^2/c^2)^2 - 4v^2/c^2}}\, m \frac{2v}{\cancel{1+v^2/c^2}}$$

$$1 + 2\frac{v^2}{c^2} + \frac{v^4}{c^4} - 4\frac{v^2}{c^2}$$

$$= 1 - 2\frac{v^2}{c^2} + \frac{v^4}{c^4}$$

$$= \left(1 - \frac{v^2}{c^2}\right)^2$$

S: $U_{1x} = 0$ $\quad P_{1x} = 0$

apparent mass $= \Gamma m = m$

$$U_{2x} = \frac{2u'}{1+u'^2/c^2}$$

$$P_{2x} = \frac{1}{\sqrt{1-\left(\frac{2u'/c}{1+u'^2/c^2}\right)^2}} \cdot m \cdot \frac{2u'}{1+u'^2/c^2}$$

$$P_{2x} = \frac{2m\cancel{u'}}{1-u'^2/c^2} = \frac{1}{\sqrt{1-u'^2/c^2}}\, M \cancel{u'} \Rightarrow M = \frac{2m}{\sqrt{1-u'^2/c^2}}$$

Mass is NOT conserved. But Gamma*Mass is! Example: exploding firecracker in S', one fragment at rest in S

$$\Gamma \cdot mc^2 = \text{Energy !}$$

$$\Gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1 - \frac{1}{2}\left(-\frac{v^2}{c^2}\right) - \frac{3}{8}\left(-\frac{v^2}{c^2}\right)^2 \ldots$$

$$\Gamma mc^2 = mc^2 + \frac{1}{2}mv^2 + \ldots\ldots$$

$$\text{Energy} = \Gamma mc^2$$

$$\text{Momentum} = \Gamma m \vec{v}$$