Properties of Ellipses

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1) Governing equation :

```
r[phi_] := 1 / (c * (1 + e * Cos[phi]))
```

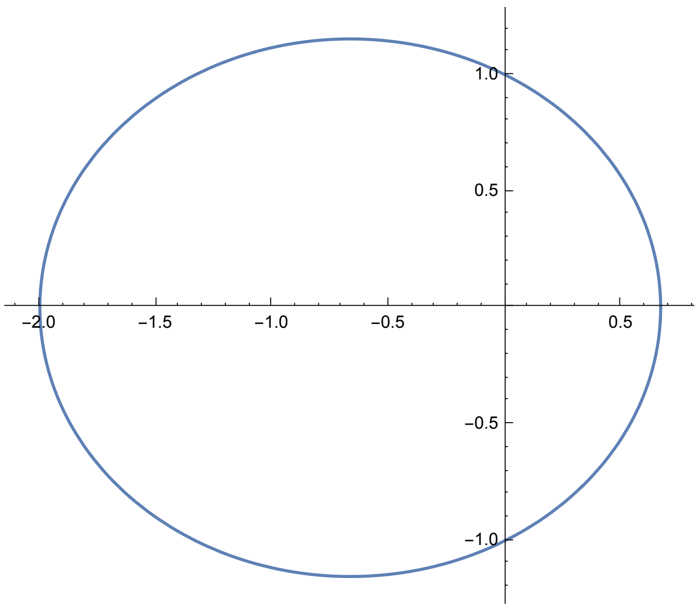
```
c = 1
```

1

```
e = 0.5
```

0.5

```
ParametricPlot[{r[ph] * Cos[ph], r[ph] * Sin[ph]}, {ph, 0, 2 * Pi}]
```




```
Clear[c, e]
```

```
rmin = r[0]
```

$$\frac{1}{c\,(1+e)}$$

```
rmax = r[Pi]
```

$$\frac{1}{c\,(1-e)}$$

```
a = FullSimplify[1 / 2 * (rmin + rmax)]
```

$$\frac{1}{c-c\,e^2}$$

```
offs = Simplify[a - rmin]
```

$$\frac{e}{c - c\, e^2}$$

a*eps which is the distance between the center in the middle of rmin and rmax (= O') and O (origin of coordinate systemin which r and phi are defined).

```
Clear[a]
```

$$cc = \text{Solve}\left[\frac{1}{c - c\, e^2} - a == 0, c\right]$$

$$\left\{\left\{c \to -\frac{1}{a\,(-1 + e^2)}\right\}\right\}$$

$$c = \frac{1}{a\,(1 - e^2)}$$

$$\frac{1}{a\,(1 - e^2)}$$

```
r[xx]
```

$$\frac{a\,(1 - e^2)}{1 + e\,\text{Cos}[xx]}$$

```
Simplify[offs]
```

a e

```
phib = Solve[r[ph] * Cos[ph] == -a * e, ph, Reals]
```

{{ph → ConditionalExpression[-ArcCos[-e] + 2 π C[1], C[1] ∈ Integers && -1 < e < 1]},
{ph → ConditionalExpression[ArcCos[-e] + 2 π C[1], C[1] ∈ Integers && -1 < e < 1]}}

```
rb = r[ArcCos[-e]]
```

a

```
b = Sqrt[rb^2 - (a * e)^2]
```

$$\sqrt{a^2 - a^2\, e^2}$$

```
b = a * Sqrt[1 - e^2]
```

$$a\,\sqrt{1 - e^2}$$

```
c = 1
```

1

```
e = 0.5
```

0.5

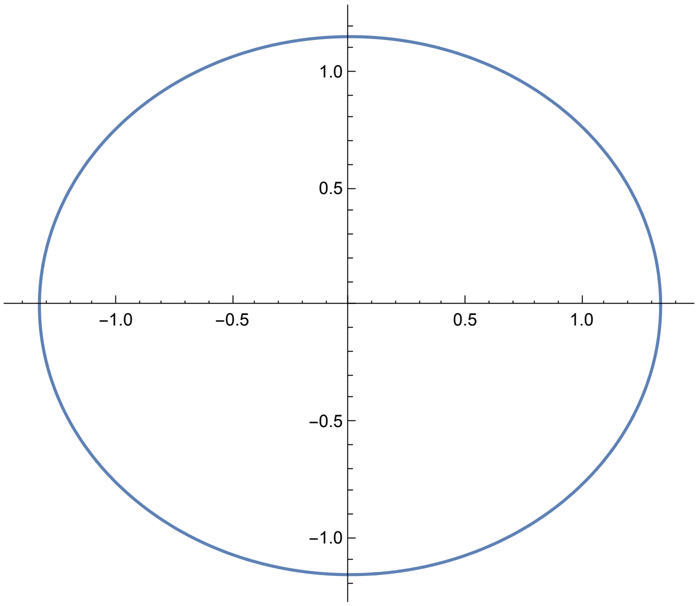
```
a = 1/(c - c e^2)
```

1.33333

```
a * e
```

0.666667

```
ParametricPlot[{a * Cos[ph], b * Sin[ph]}, {ph, 0, 2 * Pi}]
```




```
Clear[a, e, c]
```

```
c = 1/(a (1 - e^2))
```

$$\frac{1}{a\left(1-e^2\right)}$$

```
x[phi_] := r[phi] * Cos[phi]

y[phi_] := r[phi] * Sin[phi]

xp[phi_] := x[phi] + a * e

yp[phi_] := y[phi]

Simplify[xp[ph] ^ 2 / a ^ 2 + yp[ph] ^ 2 / b ^ 2]
```

1

This is the standard equation for an ellipse with major half axis a, minor half axis b, and centered on the origin of the (xp, yp) coordinate system. Finally, we introduce the vector rp from the mirror point -ae to the point on the circumference

```
rp[phi_] := Sqrt[(xp[phi] + a * e) ^ 2 + (yp[phi]) ^ 2]
```

```
Simplify[rp[ph]]
```

$$\sqrt{\frac{a^2 \left(1+e^2+2\, e \operatorname{Cos}[ph]\right)^2}{(1+e \operatorname{Cos}[ph])^2}}$$

```
FullSimplify[rp[ph], {a > 0, 1 + e Cos[ph] > 0, 1 + e^2 + 2 e Cos[ph] > 0}]
```

$$\frac{a \left(1+e^2+2\, e \operatorname{Cos}[ph]\right)}{1+e \operatorname{Cos}[ph]}$$

```
r[ph]
```

$$\frac{a \left(1-e^2\right)}{1+e \operatorname{Cos}[ph]}$$

```
Simplify[r[ph] + rp[ph], {a > 0, 1 + e Cos[ph] > 0, 1 + e^2 + 2 e Cos[ph] > 0}]
```

$$2\, a$$

This is the third generating relationship for an ellipse.

```
cospsi[phi_] := xp[phi] / a
```

```
Simplify[cospsi[ph]]
```

$$\frac{e+\operatorname{Cos}[ph]}{1+e \operatorname{Cos}[ph]}$$