$$1+e=0,\ e\approx 1$$

$$E=-\frac{GM\mu}{2a}\qquad t_{fall}=\frac{\pi}{\sqrt{GM}}\left(\frac{r}{2}\right)^{3/2}$$

$$e=\sqrt{1+\frac{2Ep_\varphi^2}{G^2M^2\mu^3}}$$

always!

$e=0\ \to\ ?\quad \Rightarrow r(\varphi)=\boxed{\frac{1}{c}=\frac{p_\varphi^2}{GM\mu^2}}=\frac{\mu^2 r^4\dot\varphi^2}{GM}$

$E=-\frac{\mu k^2}{2p_\varphi^2}=\min!\ \to$ solve: $r\dot\varphi^2=\frac{GM}{r^2}$ Circle

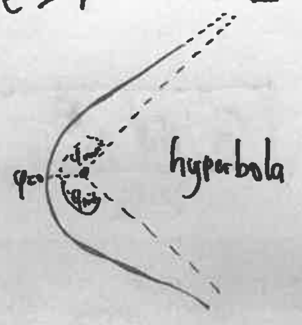
$\dot\varphi=\sqrt{\frac{GM}{r^3}}$

$T=\frac{2\pi}{\dot\varphi}=\frac{2\pi}{\sqrt{GM}}r^{3/2}$

$e=1\ \to\ E=0$

or $p_\varphi=0$

$e>1\ \to\ E>0$



$0<e<1$:

$r_{min}=\frac{1}{c(1+e)}=a(1-e)\qquad r_{max}=\frac{1}{c(1-e)}$

$2a=\frac{1}{c(1-e)}+\frac{1}{c(1+e)}=\frac{\cancel{2}1}{c(1-e^2)}$

$\frac{1}{c}=a(1-e^2)$

$x(\varphi)=\frac{a(1-e^2)}{1+e\cos\varphi}\cos\varphi\qquad \varphi(x=-ae)$

$r(\varphi)$

$y(\varphi)=\bigcirc\ \sin\varphi\ \to b=a\sqrt{1-e^2}$

$\left(\frac{x+ae}{a}\right)^2+\left(\frac{y}{b}\right)^2=1\ \in$ ellipse eqn

$a=\frac{p_\varphi^2}{GM\mu^2}\cdot\frac{1}{\frac{-2Ep_\varphi^2}{G^2M^2\mu^3}}=GM\mu/2|E|$

y_{max}? $\frac{\partial}{\partial\cos\varphi}\ a(1-e^2)\frac{\sqrt{1-\cos^2\varphi}}{1+e\cos\varphi}=\cdot\ \frac{\frac{-\cos\varphi}{\sqrt{1-\cos^2\varphi}}(1+e\cos\varphi)-e\sqrt{1-\cos^2\varphi}}{(1+e\cos\varphi)^2}=0$

$\Rightarrow -\cos\varphi-e\cos^2\varphi-e+e\cos^2\varphi=0\ \Rightarrow\cos\varphi=-e$

$x(y_{max})=\frac{a(1-e^2)}{1-e^2}(-e)=-ae$

$y_{max}=b\quad a$

$0<\quad a\quad e\ ae$

$y_{max}=\frac{a(1-e^2)}{1-e^2}\sqrt{1-e^2}=a\sqrt{1-e^2}=b$

$x'=x+ae\Rightarrow\frac{x'^2}{a^2}=\left(\frac{(1-e^2)\cos\varphi}{1+e\cos\varphi}+e\right)^2\quad\frac{y'^2}{b^2}=\frac{1}{1-e^2}\left(\frac{1-e^2}{1+e\cos\varphi}\sin\varphi\right)^2$

$+\ =\cancel{1+e^2}\ \frac{(1-e^2)^2}{(1+e\cos\varphi)^2}\cos^2\varphi+\frac{1-e^2}{(1+e\cos\varphi)^2}(1-\cos^2\varphi)+2e\frac{(1-e^2)\cos\varphi}{1+e\cos\varphi}+e^2$

$=\frac{1-e^2}{(1+e\cos\varphi)^2}\left(\cancel{\cos^2}-e^2\cos^2\varphi+1-\cos^2\varphi+2e(1+e\cos\varphi)\cos\varphi\right)+e^2$

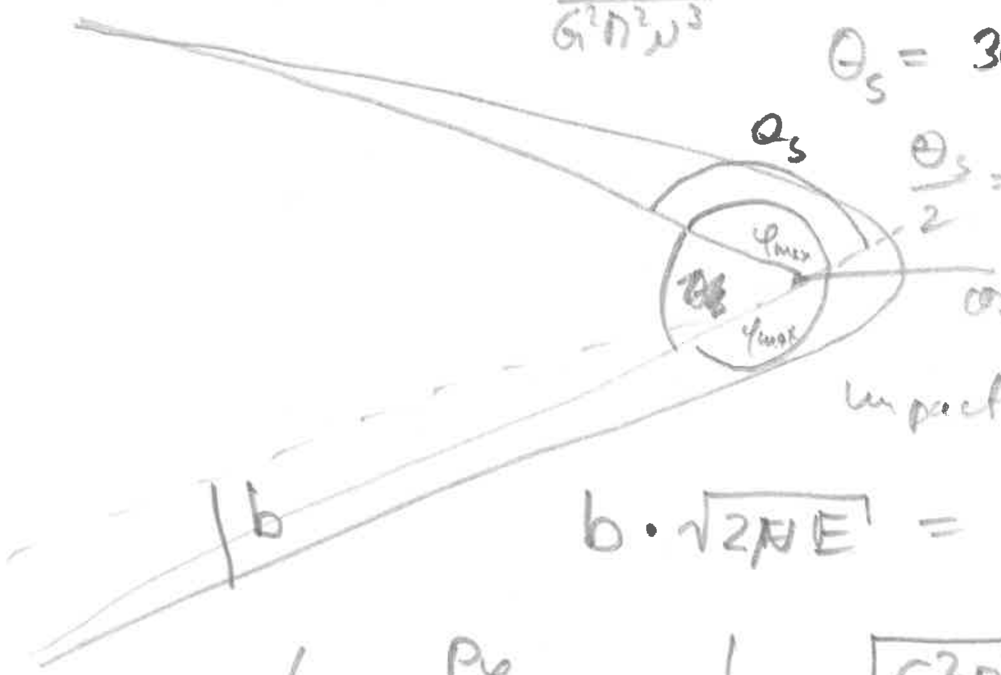
$\cancel{2}\ \cancel{+e^2\cos^2\varphi}+1+2e\cos\varphi\qquad =1-e^2+e^2=1\ \checkmark$

$b=\frac{GM\mu}{2|E|}\sqrt{\frac{2Ep_\varphi^2}{G^2M^2\mu^3}}=\frac{p_\varphi}{\sqrt{2|E|\mu}}$

$\pi ab=\pi a^2\sqrt{\frac{2Ep_\varphi^2}{G^2M^2\mu^3}}=\pi a^2\frac{p_\varphi}{\sqrt{GM}\mu\sqrt{a}}\cdot\frac{1}{\ }$

$=\frac{\pi}{\sqrt{GM}}\cdot\frac{p_\varphi}{\mu}a^{3/2}\qquad\frac{p_\varphi}{2\mu}=2\,dA/dt\Rightarrow T=\frac{2\pi}{\sqrt{GM}}a^{3/2}$

1 remember: $1-e^2 = \frac{-2E p_\varphi^2}{G^2 M^2 \mu^3}$



$\Theta_s = 360° - 2\varphi_{max} - 180°$

$\frac{\Theta_s}{2} = \cancel{180°}\,90° - \varphi_{max}$

$\cos\varphi_{max} = -\frac{1}{e}$

impact parameter

$$b\cdot\sqrt{2\mu E} = p_\varphi$$

$$b = \frac{p_\varphi}{\sqrt{2\mu E}} = \frac{1}{\sqrt{2\mu E}}\sqrt{\frac{G^2M^2\mu^3}{2E}(e^2-1)}$$

$$= \frac{GM\mu}{2E}\sqrt{\frac{1}{\cos^2\varphi_{max}}-1} = \frac{GM\mu}{2E}\,\frac{\sin\varphi_{max}}{\cos\varphi_{max}}$$

$$db = \frac{GM\mu}{2E}\,\frac{1}{\cos^2\varphi_{max}}\,d\varphi_{max}$$

$$2\pi b\,db = 2\pi\frac{G^2M^2\mu^2}{4E^2}\,\frac{1}{\cos^3\varphi_{max}}\,\sin\varphi_{max}\,d\varphi_{max}$$

$$\varphi_{max} = \frac{\pi}{2} - \frac{\Theta_s}{2} \qquad \sin\varphi_{max} = \cos\frac{\Theta_s}{2}$$

$$d\varphi_{max} = \frac{d\Theta_s}{2} \qquad \cos\varphi_{max} = \sin\frac{\Theta_s}{2}$$

$$\Rightarrow d\sigma = 2\pi\frac{G^2M^2\mu^2}{4E^2}\,\frac{\cos\frac{\Theta_s}{2}\sin\frac{\Theta_s}{2}}{\sin^4\frac{\Theta}{2}}\,\frac{d\Theta_s}{2}$$

(whatever multiplies $\frac{1}{r}$)^2 → Rutherford

$$= \frac{G^2M^2\mu^2}{16E^2\sin^4\frac{\Theta}{2}}\underbrace{2\pi\sin\Theta_s\,d\Theta_s}_{d\Omega} = \frac{G^2M^2\mu^2}{4E^2}\,2\pi\underbrace{\frac{d\cos\Theta_s}{(1-\cos\Theta_s)^2}}_{d\frac{1}{1-\cos\theta}}$$