Minkowski Space:

4-Vector: V^μ $\mu = 0,1,2,3$ (V^k to indicate "spatial part" $\mu = 1,2,3$)

Ex.: $x^\mu = (ct, \vec{r})$; $p^\mu = (E/c, \vec{p}) = m_0 c(\gamma, \gamma\vec{\beta})$

Must transform properly under Lorentz-Transformation (boost $S \to S'$ $\vec{v}$; Rotations)

Scalar Product: $g(V^\mu, U^\nu) = s$ Scalar $s \to s$ Ex.: m_0

Ex.: $g(x^\mu, y^\nu) = ct_x ct_y - \vec{r}_x \cdot \vec{r}_y = x^0 y^0 - x^1 y^1 - x^2 y^2 - x^3 y^3$

$$g(p^\mu, p^\nu) = E^2/c^2 - \vec{p}^2 = m_0^2 c^2$$

$$g(dx^\mu, dx^\nu) = ds^2 = dct^2 - d\vec{r}^2$$

1) $ds^2 = 0$ light-like 2) $ds^2 < 0$ space-like 3) $ds^2 > 0$ time-like $d\tau = \sqrt{ds^2}/c$ Eigen time

4-Form: $U(\;) \xrightarrow{V^\mu}$ scalar U_ν : $U(V^\mu) = u_\nu V^\nu$

Tensors: $T^{\mu\nu\ldots}_{\alpha\beta\ldots}$ 1st: $\Lambda^\mu{}_\nu$ S': $x^{\mu'} = \Lambda^\mu{}_\nu x^\nu$

ex. boost along z: $\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$ $\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$

$$g(\;,\;) \to g_{\mu\nu}$$

$$g(x^\mu, y^\nu) \to g_{\mu\nu} x^\mu y^\nu \qquad g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$g_{\mu\nu} x^\nu = (ct, -\vec{r}) = x_\mu$$

CM: (non-rel) $\vec{r} \cdot \vec{r} = r^2$

To Do: Express "everything" in the form of scalars, vectors, forms, tensors, ...
→ use to re-express laws of Physics: $\vec{F} = m\vec{a}$

$$x^\mu = (ct, \vec{r}) \qquad \vec{u} = \frac{d\vec{r}}{dt} \longrightarrow \qquad \frac{dx^\mu}{d\tau} = \gamma \frac{dx^\mu}{dt} = u^\mu$$

$$d\tau = \sqrt{dct^2 - d\vec{r}^2} / c = \sqrt{1 - \vec{v}^2/c^2}\, dt = \gamma^{-1}\, dt \qquad = (\gamma c, \gamma \vec{u})$$

$$\underline{m_0 u^\mu = (\gamma m_0 c, \gamma m_0 \vec{v}) = \left(\frac{E}{c}, \vec{p}\right) = p^\mu}$$

$$\frac{du^\mu}{d\tau} = \alpha^\mu \longrightarrow \qquad K^\mu = m_0 \alpha^\mu$$

$$\hookrightarrow \qquad K^\mu = \frac{dp^\mu}{d\tau} = \gamma \frac{dp^\mu}{dt} \; ; \; \underline{F^\mu = \frac{K^\mu}{\gamma} = \frac{dp^\mu}{dt}}$$

$$\frac{d\vec{p}}{dt} = \vec{F} \qquad F^0 = \frac{dp^0}{dt} = \frac{dE/c}{dt}$$

$$K^\mu u_\mu = \gamma \frac{dE/c}{dt} \gamma c - \gamma \vec{F} \cdot \gamma \vec{u} = \gamma^2 (\text{Power} - \vec{F} \cdot \vec{u})$$

$$/\gamma \quad \frac{dp^\mu}{dt} u_\mu = \gamma (\text{Power} - \vec{F} \cdot \vec{u}) = \frac{m}{2} \frac{d}{dt} \cdot u^\mu u_\mu = \frac{m_0}{2} \frac{d}{dt} c^2 = 0$$

$$K^\mu = \frac{dp^\mu}{d\tau} \qquad \text{ex.: e.m.} \quad K^\mu = q u_\nu F^{\mu\nu} / \gamma$$

$$\frac{dp^\mu}{dt} = F^\mu = \frac{1}{\gamma} q u_\nu F^{\mu\nu}$$

$$= q(\vec{u} \cdot \vec{E}, \vec{E} + \vec{u} \times \vec{B})$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\vec{E}/c \\ \vec{E}/c & \vec{B} \end{pmatrix}$$