$i = 1 \dots k$
general. coord. which fulfill all constraints

$\mathcal{L}(q_i, \dot q_i, t) \rightarrow$ ELEM: $\dot p_i = \frac{\partial \mathcal{L}}{\partial q_i}$ $\quad p_i = \frac{\partial \mathcal{L}}{\partial \dot q_i}$

2nd order PDE for $q_i(t)$

$$h(q_i, \dot q_i, t) = \sum_{i=1}^{k} p_i \dot q_i - \mathcal{L}$$

$$dh(q_i, \dot q_i, t) = \sum_{i=1}^{k} \left(dp_i\, \dot q_i + \cancel{p_i\, d\dot q_i} \right) - \sum_i \left(\cancel{\underbrace{\frac{\partial \mathcal{L}}{\partial \dot q_i}}_{p_i} d\dot q_i} + \frac{\partial \mathcal{L}}{\partial q_i} dq_i \right) - \frac{\partial \mathcal{L}}{\partial t} dt$$

canonical ~~gen.~~ mom. / ~~gen.~~ coord.

$$\underline{\underline{H(p_i, q_i, t)}} = h(q_i, \dot q_i(p_j, q_\sigma, t), t)$$

Hamiltonian

$$\frac{\partial H}{\partial p_i} = \dot q_i \qquad \frac{\partial H}{\partial q_i} = -\frac{\partial \mathcal{L}}{\partial q_i} = -\dot p_i$$

Hamilton's Eqns of Motion

Reminder: $\frac{d}{dt} H = \frac{\partial H}{\partial t}$

---

m, y $\quad k=1 \quad T = \frac{1}{2} m \dot y^2 \quad V = mgy \rightarrow \mathcal{L} = \frac{1}{2} m \dot y^2 - mgy$

ELEM: $p_y = \frac{\partial \mathcal{L}}{\partial \dot y} = m \dot y \Rightarrow \dot p_y = m \ddot y = \frac{\partial \mathcal{L}}{\partial y} = -mg$

$$h = p_y \dot y - \mathcal{L} = m \dot y^2 - \frac{1}{2} m \dot y^2 + mgy = \frac{1}{2} m \dot y^2 + mgy$$

value of $h = E$, conserved

$\rightarrow H$? $\quad \dot y \rightarrow \dot y = \frac{p_y}{m} \Rightarrow H(y, p_y) = \frac{p_y^2}{2m} + mgy$

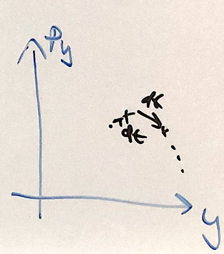
HEM: $\underline{\frac{\partial H}{\partial p_y} = \frac{p_y}{m} \overset{!}{=} \dot y} \qquad \underline{-\frac{\partial H}{\partial y} = \dot p_y = -mg}$

Solve? $p_y(t), y(t)$ $\rightarrow$ Substitution $\Rightarrow p_y = m \dot y \Rightarrow \ddot y = -g$

$$\rightarrow \left[\begin{aligned} y(t) &= y_0 + v_{0y} t - \tfrac{1}{2} g t^2 \\ p_y(t) &= m(v_{0y} - gt) \end{aligned} \right.$$

$$\begin{aligned} &\mathbb{R}^k \{q_i\} \rightarrow \mathcal{L}(q_i, \dot q_i, t) \\ &\downarrow \tfrac{d}{dt} \\ &\mathbb{R}^k \{\dot q_i\} \rightarrow \mathcal{L} \\ &\mathbb{R} \{t\} \rightarrow \mathcal{L} \end{aligned}$$

Phase space



Large class of cases

$$\mathcal{L} = \frac{1}{2}\dot{\vec{q}}^T \underbrace{\mathbb{T}_{(\vec{q})}}_{\text{symmetric}} \dot{\vec{q}} + \dot{\vec{q}}^T \cdot \vec{a}_{(\vec{q})} + \mathcal{L}_0(q_i, t)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = (\mathbb{T}\dot{\vec{q}})_i + (\vec{a})_i = p_i \Rightarrow \dot{\vec{q}} = \mathbb{T}^{-1}(\vec{p} - \vec{a})$$

$$\sum_i p_i \dot{q}_i = \dot{\vec{q}}^T \mathbb{T} \dot{\vec{q}} + \dot{\vec{q}}^T \cdot \vec{a}$$

$$h = \frac{1}{2}\dot{\vec{q}}^T \mathbb{T} \dot{\vec{q}} - \mathcal{L}_0(q_i, t) \overset{?}{=} E$$

a) $\mathcal{L}_0 = V(\vec{q})$
b) $\vec{a} = 0$

$$H = \frac{1}{2}(\vec{p} - \vec{a})^T \mathbb{T}^{-1} (\vec{p} - \vec{a}) - \mathcal{L}_0(\vec{q}, t)$$

Ex.: $\mathcal{L} = \frac{1}{2} m \dot{\vec{r}}^2 - q\Phi + q\vec{A}\cdot\dot{\vec{r}} \Rightarrow \mathbb{T} = \begin{pmatrix} m & & 0 \\ & m & \\ 0 & & m \end{pmatrix}, \vec{a} = q\vec{A}, \mathcal{L}_0 = -q\Phi$

$\Rightarrow$
$\vec{p} = m\dot{\vec{r}} + q\vec{A}$
$H = \frac{1}{2}\frac{(\vec{p} - q\vec{A})^2}{m} + q\Phi = E!$

HEП:
$\frac{\partial H}{\partial p_i} = \dot{r}_i$
$-\frac{\partial H}{\partial q_i} = \dot{p}_i$

$$\Rightarrow \dot{\vec{p}} = \underbrace{\frac{\vec{p} - q\vec{A}}{m}}_{\vec{v}} \cdot q\vec{\nabla}\vec{A} - q\vec{\nabla}\Phi$$

$p_1 = y$
$q_1 = p_y$

$$H = \frac{q_1^2}{2m} + mg\,p_1$$

$$\frac{\partial H}{\partial p_1} = mg = \dot{q}_1$$

$$-\frac{\partial H}{\partial q_1} = -\frac{q_1}{m} = \dot{p}_1$$

$$\ddot{y} = \ddot{p}_1 = -\frac{\dot{q}_1}{m} = -g$$