$x^\mu$, $(W^\mu)$,

$x_\mu = g_{\mu\nu} x^\nu$

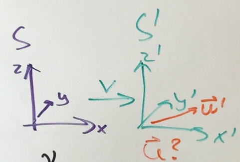
$g_{\mu\nu}$

$\Lambda^\nu{}_\mu \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \to \begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix}$

$(ds^2) = dx^\mu dx_\mu = g_{\mu\nu} dx^\mu dx^\nu$

$\gamma_v = \dfrac{1}{\sqrt{1 - \vec{v}^2/c^2}}$

$\vec{\beta}_v = \vec{v}/c$




$W'^0 = \gamma_v W^0 - \gamma_v \beta_v W^1$

$W'^1 = -\gamma_v \beta_v W^0 + \gamma_v W^1$

$W'^2 = W^2$

$W'^3 = W^3$

$\Lambda_{\text{boost}}{}^\nu{}_\mu(\vec{v}) \to \Lambda^{-1}_{\text{boost}}{}^\nu{}_\mu(\vec{v}) = \Lambda_{\text{boost}}{}^\nu{}_\mu(-\vec{v})$

If $(ds^2) > 0 \Rightarrow d\tau = \frac{1}{c}\sqrt{(ds^2)} = \frac{1}{\gamma_u} dt$

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$\vec{u} = \dfrac{d\vec{r}}{dt} \to U^\mu = \dfrac{dx^\mu}{d\tau} = \left(\gamma_u c, \gamma_u \underbrace{\dfrac{d\vec{r}}{dt}}_{\vec{u}}\right)$, $u^\mu u_\mu = \gamma_u^2 (c^2 - \vec{u}^2) = c^2$

$u'^\mu = (\gamma_u c, \gamma_u u'_x, \gamma_u u'_y, 0) \to u^0 = \gamma_v \gamma_u c + \gamma_v \beta_v \gamma_u u'_x = \gamma_{u+v} c$

$u^1 = \gamma_v \beta_v \gamma_u c + \gamma_v \gamma_u u'_x$

$u^2 = u'^2 = \gamma_u u'_y$

$u_x = \dfrac{U^1}{\gamma_{u+v}}$

$$\boxed{\frac{\partial}{\partial x^\mu} = \partial_\mu \ ; \ \partial^\mu = \frac{\partial}{\partial x_\mu} = g^{\mu\nu} \partial_\nu}$$

$$u_x = \frac{\gamma_v \gamma_u (\beta_v c + u'_x)}{\gamma_v \gamma_u (1 + \beta_v u'_x/c)} = \frac{v + u'_x}{1 + \frac{v}{c}\frac{u'_x}{c}} \ ; \ u_y = \frac{\gamma_u u'_y}{\gamma_u \gamma_v (1 + \beta_v \frac{u'_x}{c})} = \frac{u'_y}{\gamma_v (1 + \frac{v}{c}\frac{u'_x}{c})}$$

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n rel: $\mathcal{L} = \frac{m}{2} \vec{v}^2 + V \ldots$ $\xRightarrow{\text{S.R.}}$ **Free particle**

$\delta \int_{t_1}^{t_2} \underbrace{\mathcal{L}}_{\text{scalar}} \, dt = 0$ $\qquad$ $\delta \int_{\tau_1}^{\tau_2} \mathcal{L} \, d\tau = 0$

What if $\mathcal{L} = \sqrt{U^\mu U_\mu}\, mc$

$U^\mu = \dfrac{dx^\mu}{d\tau}$

$\frac{1}{mc} \mathcal{L} \, d\tau = \sqrt{dct^2 - d\vec{r}^2}$

$\mathcal{L} = mc \sqrt{g_{\mu\nu} U^\mu U^\nu}$

$\Rightarrow$ E.L.F: $\dfrac{d}{d\tau} \dfrac{\partial \mathcal{L}}{\partial U^\mu} = \dfrac{\partial \mathcal{L}}{\partial x^\mu} = 0$

$\mathcal{L} = \dfrac{\partial \mathcal{L}}{\partial U^\mu} = mc \dfrac{1}{2} \dfrac{1}{\sqrt{U^\nu U_\nu}} (g_{\mu\nu} U^\nu + g_{\nu\mu} U^\nu)$

$= m U_\mu = p_\mu$ $\qquad = 2 U_\mu$

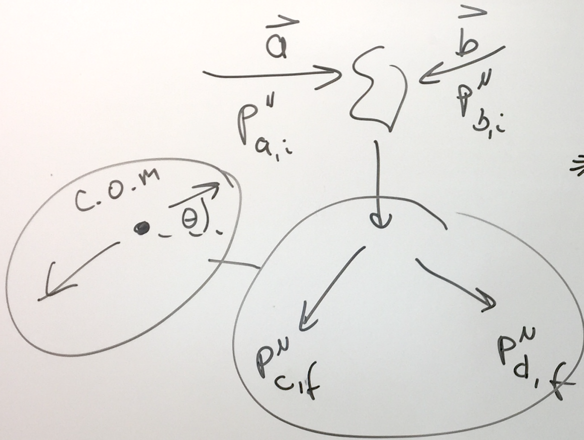
$\Rightarrow \dfrac{dp_\mu}{d\tau} = 0$

$\Rightarrow p^\mu = m U^\mu = (\gamma_u mc, \gamma_u m \vec{u})$

$p^\mu p_\mu = m^2 c^2 = \dfrac{E^2}{c^2}\Big|_{\text{rest frame}}$

$p^0 = E/c \Rightarrow E = mc^2 + \frac{m}{2} \vec{u}^2 + \ldots$

Photon: $p^\mu p_\mu = 0$




$\rightarrow p^\nu_{tot,i} \longrightarrow p^\nu_{tot,i}\, p_{tot,\nu} = M_{inv}^2 c^2$

$\Rightarrow$ c.o.m: $p'^\nu_{tot,i} = (M_{inv}^2 c^2, 0, 0, 0)$

$p^0_{tot,i} = \underline{\underline{\Gamma_v}}(M_{inv} c)$

$\vec{p}_{tot,i} = \underline{\underline{\Gamma_v \vec{\beta}_v}}(M_{inv} c)$

$p^\nu_{tot} = \sum_i p^\nu_i$ conserved (indep. of time)
~~invariant~~

$p^\nu_{tot}\, p_{\nu\, tot} =: M_{inv}^2 c^2$ invariant (indep. of coordinate system)
$=: \frac{E_{inv}^2}{c^2}$ center-of-mass frame