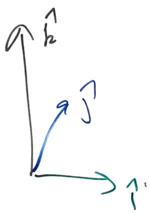
$$\mathrm{Re}\left(C_1 \begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix} e^{i\omega_1 t} + C_2 \begin{pmatrix} a_{21} \\ a_{22} \end{pmatrix} e^{i\omega_2 t} \right) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \omega_1 = \sqrt{\lambda_1}, \quad \omega_2 = \sqrt{\lambda_2}$$

$$|\mathbb{V} - \lambda \mathbb{T}| = 0$$

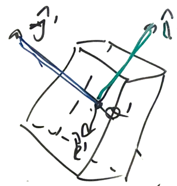
rigid body $\Rightarrow$ 3 position coordinates : $\vec{R}_{cm}$ or $\vec{R}_{\text{fixed point}}$

3 "orientation coordinates"



Space
fixed
Lab

} Coord. Syst.



Body
C. S.

$$(\hat{\imath}')_x = \hat{\imath}' \cdot \hat{\imath} = \cos\theta_{11}$$

$$(\hat{\imath}')_y = \hat{\imath}' \cdot \hat{j} = \cos\theta_{12}$$

$$\rightarrow \hat{\imath}' = \underbrace{(\hat{\imath}' \cdot \hat{\imath})}_{\cos\theta_{11}} \hat{\imath} + \overset{\cos\theta_{12}}{(\hat{\imath}' \cdot \hat{j})} \hat{j} + \overset{\cos\theta_{13}}{(\hat{\imath}' \cdot \hat{k})} \hat{k}$$

$$\Rightarrow \mathbb{R} = \begin{pmatrix} \cos\theta_{11} & \cos\theta_{12} & \cos\theta_{13} \\ \cos\theta_{21} & \cos\theta_{22} & \cos\theta_{23} \\ \cos\theta_{31} & \cos\theta_{32} & \cos\theta_{33} \end{pmatrix}$$

$$\vec{V} \Rightarrow (\vec{V})_x = \vec{V} \cdot \hat{\imath}, \quad (\vec{V})_y = \vec{V} \cdot \hat{j}, \quad (\vec{V})_z = \vec{V} \cdot \hat{k} \rightarrow (\vec{V})$$

$$\hookrightarrow (\vec{V})'_{x,y,z} = \vec{V} \cdot \hat{\imath}' = \vec{V} \cdot \hat{\imath} (\cos\theta_{11}) + \vec{V} \cdot \hat{j} (\cos\theta_{12}) \cdot \vec{V} \cdot \hat{k} (\cos\theta_{13}) \rightarrow (\vec{V})'$$

2..
3..

$$(\vec{V})' = \mathbb{R} (\vec{V})$$ Passive rotation

$\hat{\imath}'$ is $\hat{\imath}$ actively rotated

$$(\hat{\imath}') = \begin{pmatrix} \cos\theta_{11} \\ \cos\theta_{12} \\ \cos\theta_{13} \end{pmatrix} = \mathbb{R}' \begin{pmatrix} \hat{\imath} \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbb{R}^T \mathbb{R} = \mathbb{R} \mathbb{R}^T = \mathbb{1}$$ orthogonal, $(\det(\mathbb{R}))^2 = 1$, $\mathbb{R}' = \mathbb{R}^T$

3 degrees of freedom

$$\vec{U} \cdot \vec{V} = (\vec{U})'^T (\vec{V})' = (\vec{U}) \mathbb{R}^T \mathbb{R} (\vec{V}) = (\vec{U})^T (\vec{V})$$

If $\det = 1$ $\Rightarrow$ Special $\mathbb{R} \in SO(3)$

Ex.:

$$(\mathbb{R}) = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

passive rotation into a coordinate system that is rotated counterclockwise ($\circlearrowleft \uparrow \hat{k}$) around $\hat{k}$

ALSO active rotation of a vector clockwise around $\hat{k}$

rotation around $\hat{\imath}'$: $\mathbb{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$

rotation around $\hat{k}''$ $\mathbb{R} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Euler