$$T = \frac{1}{2}\underbrace{\iiint}_{\text{Body}} \underbrace{d^3\vec{r}}_{dz\,dy\,dx}\; \rho(\vec r)\, (\vec\omega\times\vec r)^2 = \frac{1}{2}(\vec\omega)^T \;\mathbb{I}\; (\vec\omega)$$

$$\mathbb{I} = \iiint d^3\vec r\,\rho(\vec r)\left[\mathbb{1}\cdot(\vec r^{\,2}) - \underbrace{\vec r \circ \vec r}_{(\vec r)(\vec r)^T}\right] = \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix}$$

$\mathbb{I}$ symmetric $\Rightarrow \exists\ \hat n_1, \hat n_2, \hat n_3$ such that $\mathbb{I}\hat n_i = I_i \hat n_i$ (principal axes), $\hat n_i \perp \hat n_j$, $i \neq j$

Tensor : $\mathbb{T} \xrightarrow[\mathbb{R}]{\text{rotation}} \mathbb{R}\,\mathbb{T}\,\mathbb{R}^T$ $\qquad \mathbb{R}_n \mathbb{I} \mathbb{R}_n^T = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}$, $\exists \mathbb{R}_n$

Vector : $\vec v \rightarrow \mathbb{R}\vec v$

Principal axes:
1) rotational symmetry axis $\rightarrow \hat k$; $\hat i, \hat j$ as you like
2) symmetry plane : $\hat k \perp$

$$\vec L = \iiint d^3\vec r\,\rho(\vec r)\ \vec r\times(\vec\omega\times\vec r) = \vec\omega(\vec r^{\,2}) - (\vec r\cdot\vec\omega)\vec r$$

$$= \underbrace{\iiint d^3\vec r\,\rho(\vec r)\left(\mathbb{1}\vec r^{\,2} - \vec r\circ\vec r\right)}_{\mathbb{I}}\ \vec\omega \qquad \vec L = \mathbb{I}\,\vec\omega$$

$$\mathbb{I}_{c.o.m.} \xrightarrow[\text{Theorem}]{\text{Parallel Axes}} \mathbb{I}_{\vec R} = \iiint d^3\vec r_{cm}\,\rho(\vec r)\left(\mathbb{1}(\vec r_{cm}+\vec R)^2 - \underbrace{(\vec r_{cm}+\vec R)\circ(\vec r_{cm}+\vec R)}\right)$$

$\vec R$: from origin to c.o.m.

$(\vec r_{cm}+\vec R)^2 = \vec r_{cm}^{\,2} + \cancel{2\vec r_{cm}}\vec R + \vec R^2$

$(\vec r_{cm}+\vec R)\circ(\vec r_{cm}+\vec R) = \vec r_{cm}\circ\vec r_{cm} + \cancel{\vec r_{cm}\circ\vec R} + \cancel{\vec R\circ\vec r_{cm}} + \vec R\circ\vec R$

$$= \mathbb{I}_{c.o.m.} + M\left(\mathbb{1}\vec R^2 - \vec R\circ\vec R\right)$$

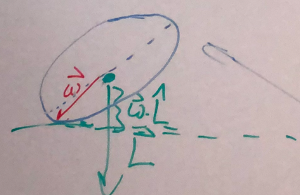
Inertia Ellipsoid : Surface spanned by all vectors $\vec{\varrho}$ such that

$$\vec{\varrho}\, \mathbb{I}\, \vec{\varrho} = 1 = \varrho^2 \cdot \hat{\varrho}\, \mathbb{I}\, \hat{\varrho} \, , \quad \varrho^2 = \frac{1}{\hat{\varrho}\, \mathbb{I}\, \hat{\varrho}}$$

coord = principal axes $\rightarrow$ $\varrho_x^2 \underset{\frac{1}{a^2}}{\underset{\shortparallel}{I_1}} + \varrho_y^2 \underset{\frac{1}{b^2}}{\underset{\shortparallel}{I_2}} + \varrho_z^2 \underset{\frac{1}{c^2}}{\underset{\shortparallel}{I_3}}$

Free motion $\rightarrow$ $\vec{L}$, T are conserved ; $\vec{\omega} \cdot \vec{L} = 2T$ conserved

$\hookrightarrow$ $\vec{\omega}\, \mathbb{I}\, \vec{\omega} = 2T$; $\vec{\omega} \cdot \hat{L} = \frac{2T}{L}$ conserved



= Inertia ellipsoid blown up by $\sqrt{2T}$