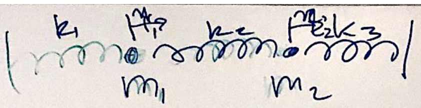

few d.o.f.

$$\mathcal{L} = \frac{1}{2}\dot{\vec{\eta}}\,\mathbb{T}\,\dot{\vec{\eta}} - \frac{1}{2}\vec{\eta}\,\mathbb{V}\,\vec{\eta}$$

$$\mathbb{T} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$|\lambda \mathbb{T} - \mathbb{V}| = 0 \rightarrow \lambda_1 \ldots \lambda_n$$

$$\vec{a}_1 \ldots \vec{a}_n$$

$$\lambda \mathbb{T}\,\vec{a}_i = \mathbb{V}\,\vec{a}_i$$

$$\lambda \underbrace{A^T \mathbb{T} A} = \underbrace{A^T \mathbb{V} \vec{A}} \qquad A = (\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n)$$

new generalized coordinates:

$$\zeta_k(t) = C_k e^{i\omega_k t}$$

$$\vec{\eta}(t) = \mathrm{Re}\left(\sum_k C_k \vec{a}_k \cdot e^{-i\omega_k t}\right) = A(\vec{\zeta})$$

$$\omega_k = \sqrt{\lambda_k}$$

$$T = \frac{1}{2}\dot{\vec{\eta}}^T \mathbb{T} \dot{\vec{\eta}}$$

$$= \frac{1}{2}\dot{\vec{\zeta}}^T \underbrace{A^T \mathbb{T} A}_{\mathbb{T}'} \dot{\vec{\zeta}}$$

$$= \frac{1}{2}\dot{\vec{\zeta}}^T T' \dot{\vec{\zeta}}$$

$t = 0 \rightarrow \vec{\eta}(0) = \mathrm{Re} \sum_k C_k \vec{a}_k = \sum_k (\mathrm{Re}\, C_k) \vec{a}_k$

$\dot{\vec{\eta}}(0) = \mathrm{Re} \sum_k -i\omega C_k \vec{a}_k \cancel{e^{-i\omega t}}$

$= \sum_k (\mathrm{Im}\, C_k)\, \omega \vec{a}_k$

So IF: $\vec{a}_k$ are orthonormal $\Rightarrow \mathrm{Re}\, C_\ell = \vec{a}_\ell^T \cdot \vec{\eta}(0)$

$\mathrm{Im}\, C_\ell = \frac{1}{\omega_\ell} \vec{a}_\ell^T \cdot \dot{\vec{\eta}}(0)$

Case 1: $\eta_1(0) = \eta_2(0) = \Delta x$

Case 2

$$\vec{\eta}(0) = \begin{pmatrix} \Delta x \\ \cancel{\Delta x}=0 \end{pmatrix}$$

$$\vec{C} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta x \end{pmatrix} = \cancel{\begin{pmatrix} 0 \\ \sqrt{2}\,\Delta x \end{pmatrix}} \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta x \\ \frac{1}{\sqrt{2}}\Delta x \end{pmatrix}$$