

$$\left(\overset{k_1}{m_1} \overset{k_2}{m_2} \overset{k_3}{m_3} \right)$$

few d.o.f.

$$\mathcal{L} = \frac{1}{2} \dot{\vec{\eta}}^T \mathbb{T} \dot{\vec{\eta}} - \frac{1}{2} \vec{\eta}^T \mathbb{V} \vec{\eta}$$

$$\mathbb{T} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$|\lambda \mathbb{T} - \mathbb{V}| = 0 \rightarrow \lambda_1, \dots, \lambda_n$$

$$\vec{a}_1, \dots, \vec{a}_n$$

$$\lambda \mathbb{T} \vec{a}_i = \mathbb{V} \vec{a}_i$$

$$\rightarrow \underbrace{A^T \mathbb{T} A}_{\text{new generalized coordinates}} = \underbrace{A^T \mathbb{V} A}_{\text{new generalized coordinates}}$$

new generalized coordinates:

$$\vec{\eta}(t) = \sum_k C_k e^{i\omega_k t}$$

$$A = (\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n)$$

$$\vec{\eta}(t) = \text{Re} \left(\sum_k C_k \vec{a}_k e^{-i\omega_k t} \right) = A \begin{pmatrix} \vec{c} \\ \vec{d} \end{pmatrix}$$

$$\omega_k = \sqrt{\lambda_k}$$

$$\begin{aligned}
 T &= \frac{1}{2} \dot{\vec{\eta}}^T T \dot{\vec{\eta}} \\
 &= \frac{1}{2} \dot{\vec{S}}^T \underbrace{A^T \Pi A}_{\Pi'} \dot{\vec{S}} \\
 &= \frac{1}{2} \dot{\vec{S}}^T T' \dot{\vec{S}}
 \end{aligned}$$

$$t=0 \rightarrow \vec{\eta}(0) = \text{Re} \sum_k C_k \vec{a}_k = \sum_k (\text{Re } C_k) \vec{a}_k$$

$$\begin{aligned}
 \dot{\vec{\eta}}(0) &= \text{Re} \sum_k -i\omega C_k \vec{a}_k e^{-i\omega t} \\
 &= \sum_k (\text{Im } C_k) \omega \vec{a}_k
 \end{aligned}$$

So IF: \vec{a}_k are orthonormal \Rightarrow

$$\begin{aligned}
 \text{Re } C_e &= \vec{a}_e^T \cdot \vec{\eta}(0) \\
 \text{Im } C_e &= \frac{1}{\omega_e} \vec{a}_e^T \cdot \dot{\vec{\eta}}(0)
 \end{aligned}$$

Case 1:

$$\eta_1 = \eta_2 = \Delta x$$

$$\vec{\eta}(0) = \begin{pmatrix} \Delta x \\ \Delta x = 0 \end{pmatrix}$$

$\vec{C} =$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta x \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2} \Delta x \end{pmatrix} \quad \begin{pmatrix} \frac{1}{\sqrt{2}} \Delta x \\ \frac{1}{\sqrt{2}} \Delta x \end{pmatrix}$$

Case 2