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Galilean Relativity

(1) Simulaneity is independent of reference frame

To define event, we need to know

4 Coordinates ct, x, y, z

(2) Length scale is the same in 2 reference frames

i.e. $\sqrt{(\Delta x)^2+(\Delta y)^2+(\Delta z)^2} = \sqrt{(\Delta x')^2+(\Delta y')^2+(\Delta z')^2}$

(3) Time scale is the same in 2 reference frames

i.e. $\Delta ct = \Delta ct'$

(4) All laws of nature are independent of inertial refrence frames

Assume that S' moving with $\vec{V}$ relative to S (at $t=t'=0$, $\bar{r}_i = \bar{r}_i'$)

so $\bar{r}_i(t)$, $\bar{r}_i' = \bar{r}_i - \vec{V}t$

So the motion of a particle i in the prime coord.

$$\vec{u}_i' = \vec{u}_i - \vec{V}$$

and the momenta in the prime coord. $\bar{P}_i' = \bar{P}_i - m\vec{V}_0$

Assume we have a hamiltonian

$$H = \sum \frac{\bar{P}_i^2}{2m_i} + \sum_{i<j} V(\vec{r}_j - \vec{r}_i)$$

pot. energy due to the pair of particles

It is clear that the transformation from $\bar{r}_i$ to $\bar{r}_i'$ is Canonical and the generating function that does this transformation has the form

$$F_2 = \sum \vec{r}_i \cdot \vec{P}_i' + \vec{V}_0 \cdot \left(\sum m_i \vec{r}_i - \sum \vec{P}_i' t\right)$$

$\boxed{2}$ Since we have

$$\frac{\partial F_2}{\partial \vec{r}_i} = \vec{P}_i = \vec{P}_i' + \vec{V} m_i \quad \checkmark$$

$$\frac{\partial F_2}{\partial \vec{P}_i'} = \bar{r}_i' = \bar{r}_i - \vec{V}t \quad \checkmark$$

Now, what is $K(\bar{r}_i', \bar{P}_i')$?? (New Hamiltonian (Kamiltonian

$$K(\bar{r}_i', \bar{P}_i') = \sum_i \frac{(\vec{P}_i' - m_i\vec{V})^2}{2m_i} + V(\bar{r}_J' - \bar{r}_i') + \underbrace{\vec{V} \cdot \sum_i \vec{P}_i'}_{\frac{\partial F_2}{\partial t}}$$

$$= \sum_i \frac{P_i'^2}{2m_i} + \sum_i \frac{m_i}{2} \vec{V}^2 + V(\bar{r}_J' - \bar{r}_i')$$

$$K(\bar{r}_i', \bar{P}_i') = \sum_i \frac{P_i'^2}{2m_i} + \frac{M}{2} \vec{V}^2 + V(\bar{r}_J' - \bar{r}_i')$$

M is the total mass of the particles which is const. so $\frac{M}{2}\vec{V}^2$ is cons

HEn : $\dot{\bar{r}}_i' = \frac{\partial K}{\partial \bar{P}_i'} = \frac{\bar{P}_i'}{m}$ same as in unprimed system

$\dot{\bar{P}}_i' = -\frac{\partial K}{\partial \bar{r}_i'} = \bar{F}_i' = \bar{F}_i$ (forces are same)

With ICT we can write F_2 as

$$F_2 = \sum_i \bar{r}_i \bar{P}_i' + \delta\vec{V} \cdot \bar{G}$$

where $\bar{G}$ is the generator

$$\bar{G} = \underbrace{\sum m_i \vec{r}_i}_{\text{from definition } M\bar{R}} - \underbrace{\sum_i \vec{P}_i}_{\text{Total momenta } P} t \quad (i)$$

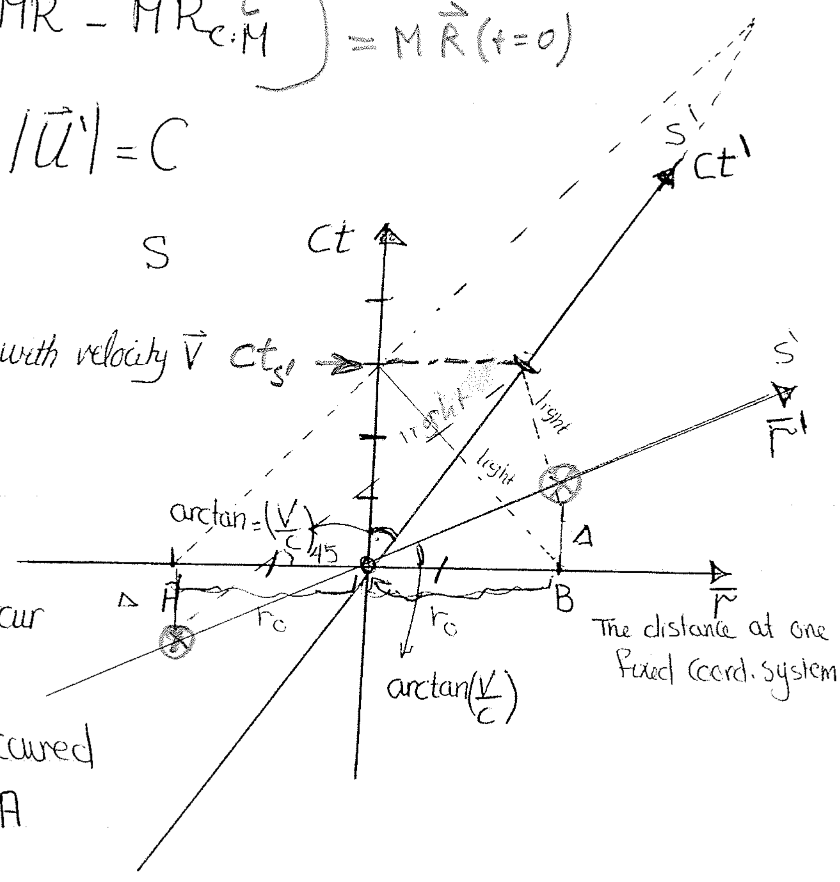
[3] $\dot{\vec{G}} = [G, H] + \dfrac{\partial \vec{G}}{\partial t} =$

$\delta H = [H, \vec{G}] \cdot \delta \vec{V} = (-\sum_i \vec{P}_i) \cdot \delta \vec{V}$

So that $\dot{\vec{G}} = \sum \vec{P}_i - \vec{P}_i = 0$

$$\left.\begin{aligned} \text{Equ. (i)} \quad \vec{G} &= M\vec{R} - M\vec{V}_{CM}\, t \\ &= M\vec{R} - M\dot{\vec{R}}_{C.M}\, t \end{aligned}\right\} \text{is conserved} = M\vec{R}(t=0)$$

Einstein's postulate ⑤ $|\vec{u}| = c \longrightarrow |\vec{u}'| = c$



Since the S' moves with velocity $\vec{V}$ w.r.t. S

In S, events A, B occur Simultanously

In S', event B is occured earlier than event A

For Δ, S' will see the two events simultanous while S will see the two events (one occur earlier than the other)

$(ct_{s'} + \Delta) = r_0 + \frac{V}{c} ct_{s'} \Rightarrow \Delta = r_0 - (1 - \frac{V}{c}) ct_{s'}$

$(ct_{s'} - \Delta) = r_0 - \frac{V}{c} ct_{s'} \Rightarrow \Delta = -r_0 + (1 - \frac{V}{c}) ct_{s'}$

$\boxed{4}$

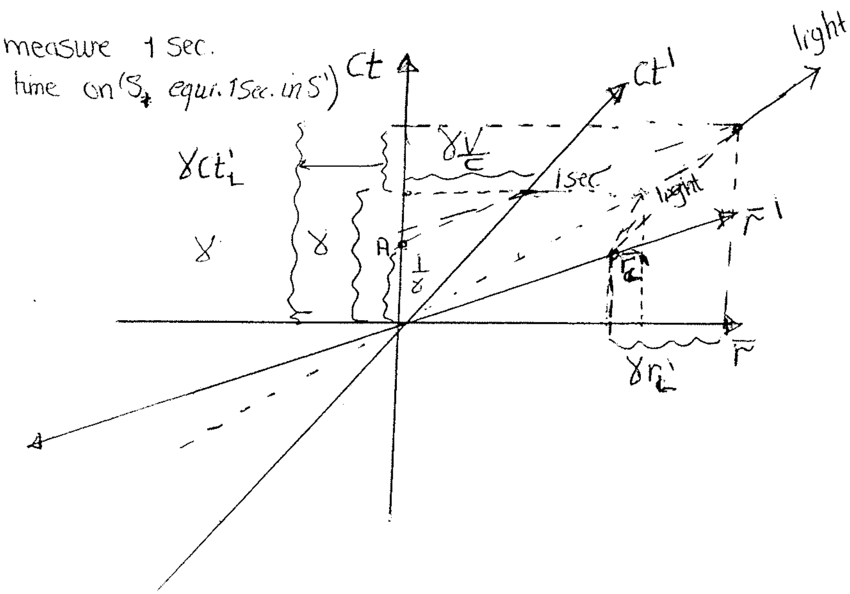
Subst.

$$2r_0 - 2ct_{S'} = 0$$

$$\boxed{ct_{S'} = r_0} \Longrightarrow \boxed{\Delta = +\frac{V}{c} r_0}$$

when a clock on S' measure 1 sec.
we need what is this time on (S, equi. 1sec. in S') actually

~~To measure~~



$$\left(\gamma \frac{V}{c}\right) \cdot \frac{V}{c} = \gamma - \frac{1}{\gamma}$$

$$\gamma^2 \left(\frac{V}{c}\right)^2 = \gamma^2 - 1$$

$$1 = \gamma^2 \left(1 - \frac{V^2}{c^2}\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}} \quad , \quad \frac{V}{c} = \beta$$

clocks in S' run slow according to S:
time elapsed in S after 1 s in S' is $\gamma \cdot 1s$

Clocks in S run slow according to S':
time elapsed in S, after 1s in S' is $\frac{1}{\gamma} \cdot 1s$

From symmetry: Scale in S' is shortened according to S by a factor $\frac{1}{\gamma}$