Physics 603

Lecture Notes 04.18.06 Tuesday

A metric can be represented by either a 2-form acting on 2 vectors or a 2-tensor acting on 2 1-forms

$(ct, \vec{x})$ $\underset{\text{2-tensor}}{g^{\mu\nu}}$; $\underset{\text{2-form}}{g_{\mu\nu}}$ $L^{\nu}_{\ \mu}$

"event" $\longrightarrow$ Minkowski Space

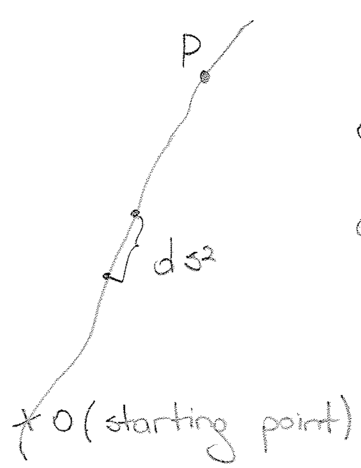
Express all physical quantities as :

i) Lorentz scalors $s \xrightarrow{L} s$

ii) Lorentz vectors $V^{\mu} \xrightarrow{L} V'^{\mu} = L^{\mu}_{\ \nu} V^{\nu}$

iii) n-tensors (n - forms)

"distance" $ds^2 = (dct)^2 - (d\vec{x})^2 \longrightarrow$ i)



physical path

$ds^2 > 0$

$d\tau = \sqrt{ds^2}/c$

$\tau = \int_0^P d\tau$ elapsed rest time

elapsed time according to the clock carried by the particle moving along the path.

Path : $x^\mu(\tau)$

4- Velocity : $u^\mu = \dfrac{dx^\mu}{d\tau} \longrightarrow$ ii)

3 - velocity $\vec{v} = \dfrac{d\vec{x}}{dt}$

$$d\tau = \sqrt{dct^2 - d\vec{x}^2}/c$$

divide by dt $\Rightarrow \dfrac{d\tau}{dt} = \sqrt{\dfrac{dct^2}{dt^2} - \dfrac{d\vec{x}^2}{dt^2}}/c$

$$= \sqrt{c^2 - \vec{v}^2}/c$$

$$= \sqrt{1 - \vec{\beta}^2} = \frac{1}{\gamma}$$

$$u^\mu = \frac{dx^\mu}{dt}\frac{dt}{d\tau} = \gamma(\underset{\uparrow \frac{dx^0}{dt}}{c}, \vec{v})$$

$$u^\nu u_\nu = \gamma^2 c^2 - \gamma^2 \vec{v}^2 = \gamma^2 c^2\left(1 - \frac{\vec{v}^2}{c^2}\right) = c^2$$

this product is a Lorentz scalar and is invariant

Momentum $\vec{p} = m\vec{v} \longrightarrow p^\nu = \underset{\text{rest mass}}{m} u^\nu$

m : mass of the particle measured in the coordinate system in which the particle is at rest.

$$p^\nu = m u^\nu \qquad \vec{p} = \gamma m \vec{v} \qquad p^0 = \gamma m c$$

(always same unless collision)
(in all IS's)

$$\frac{1}{\sqrt{1-\beta^2}} = 1 + \left.\frac{d\gamma}{d\beta^2}\right|_{\beta=0} \beta^2 + \ldots$$

Taylor's Expansion of p^0

$$p^0 = \gamma m c = mc\left(1 - \frac{1}{2}\left.\frac{1}{\sqrt{1-\beta^2}^{\,3}}\right|_{\beta=0} (-\beta^2) + \ldots\right)$$

$$= mc + \frac{1}{2} mc\beta^2 + \ldots$$

$$= mc + \frac{1}{2} m\vec{v}^2/c + \ldots$$

$$= \Big(\underbrace{mc^2}_{\text{rest mass energy}} + \underbrace{\frac{1}{2} m\vec{v}^2}_{T_{kin}}\Big)\Big/c$$

$$p^0 = E/c \qquad E = \underbrace{\gamma m}_{\text{"relativistic mass"}} c^2 \qquad T_{kin} = E - mc^2$$

$$p^\mu p_\mu = m^2c^2 = p^{0\,2} - \vec{p}^{\,2} = (E/c)^2 - \vec{p}^{\,2}$$

$$E^2 = (\vec{p}c)^2 + (mc^2)^2$$

Energy - momentum conservation

$$P^{\mu}_{tot,in} = \sum_i P^{\mu}_{i,in} \qquad P^{\mu}_{tot,fin} = \sum_i P^{\mu}_{i,fin}$$

momentum conservation $\quad P^{\mu}_{tot,in} = P^{\mu}_{tot,fin}$

* Momentum and Energy are always conserved.

i) Inelastic Scattering

$$P^{\mu}_{tot,in} = \left(\gamma m c + M c \,,\, \gamma m \vec{v}\right)$$

$$P^{\mu}_{tot,fin} = \left(\gamma_f M_f c \,,\, \gamma_f M_f \vec{v}_f\right)$$

$$M_f^2 c^2 = \left(\gamma_f M_f c\right)^2 - \left(\gamma_f M_f \vec{v}_f\right)^2 = P^{\nu}_{tot,in} P_{\nu\, tot,in}$$

$$= \gamma^2 m^2 c^2 + 2\gamma m M c^2 + M^2 c^2 - \gamma^2 m^2 \vec{v}^2$$

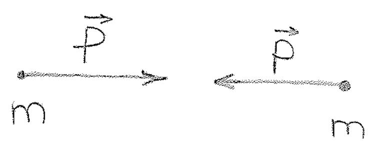
$$= m^2 c^2 + M^2 c^2 + 2\gamma m M c^2$$

$$= (m+M)^2 c^2 + \underbrace{2(\gamma - 1) m M c^2}_{2 T_{kin} M}$$

$$M_f = \sqrt{(m+M)^2 + 2\frac{T_{kin}}{c^2} M}$$

This shows a way to produce particles with higher mass as in high-energy physics. But it is not very effective way.

So,



$$\vec{P}_{tot} = \left(2\sqrt{m^2c^2+\vec{p}^{\,2}}\ ,\ \vec{0}\right)$$

$$M_f c = 2\sqrt{m^2c^2+\vec{p}^{\,2}}$$

c.o.m frame : $P^{\mu}_{tot} \rightarrow M_{inv}\, c = \sqrt{P^{\mu} P_{\mu\, tot}}$

(we want $P^{\nu}_{tot}(\mathrm{com}) = (M_{inv} c, \vec{0})$)

L.T : $\Gamma = \dfrac{P^{0}_{tot}}{M_{inv} c}$ $\qquad \Gamma\vec{\beta} = \dfrac{\vec{P}_{tot}}{M_{inv}\, c}$

how much energy is used to produce an antiproton ?

$$p + p \longrightarrow p + p + \bar{p} + p \qquad M^{min}_{inv} = 4\, m_p$$

using $M_f = \sqrt{(m+M)^2 + 2\dfrac{T_{kin}}{c^2} M}$

$$(4\, m_p)^2 = (2\, m_p)^2 + 2\frac{T_{kin}}{c^2} m_p \qquad \frac{T_{kin}}{c^2} = 6\, m_p$$

using $M_f c = 2\sqrt{m^2c^2 + \vec{p}^{\,2}}$

$$4\, m_p^2 c^2 = m_p^2 c^2 + \vec{p}^{\,2} \Rightarrow |\vec{p}| = \sqrt{3}\, m_p c \qquad (T_{kin} = m_p)$$