Graduate Quantum Mechanics - Problem Set 1 - Solutions

Problem 1)

$$E = T_{kin} + V(x) = H(p,x) = \frac{p^2}{2m} + mgx \,.$$

$$\frac{\partial H}{\partial p} = \frac{p}{m} = v = \dot{x}$$

$$-\frac{\partial H}{\partial x} = -mg = F = \dot{p}$$

Problem 2)

$$\vec{A}(r_\perp,\varphi,z) = \frac{r}{2} b\,\hat{\varphi} \Rightarrow \quad A_\varphi = \frac{r}{2}b,\, A_r = 0,\, A_Z = 0 \Rightarrow$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = -\frac{\partial A_\varphi}{\partial z}\hat{r} + \frac{1}{r}\frac{\partial r A_\varphi}{\partial r}\hat{z} = \frac{1}{r} r b \hat{z} = b\,\hat{z}$$

where we took only the non-zero terms from the curl in cylindrical coordinates in the formula sheet. This leads to motion in a circle of radius

Problem 3)

$$\vec{F} = q\vec{v}\times\vec{B} = -e\left(0.1c\right)\left(0.1T\right)(-\hat{y}) = 3\cdot 10^7 e \text{ N/C } \hat{y} = 4.8\cdot 10^{-13} \text{ N } \hat{y}$$

The magnitude of this force is constant and it is always perpendicular to the direction of motion. Therefore, the electron moves on a circle of radius $R = \frac{mv}{eB} = 0.0017$ m centered at $x = 0$, $y = R$. Its angular velocity is $\omega = \frac{v}{R} = 1.76\cdot 10^{10}$ rad/s which corresponds to a full orbit every 0.36 ns.

Problem 4)

For each of the following statements about Quantum Mechanics, indicate whether you believe them to be correct or wrong. Give a 1-2 sentence explanation for each of your responses:

a. If all possible information on a system is given, Quantum Mechanics can predict the outcome of any future measurement on the system accurately. WRONG: in general, only probabilities can be predicted
b. Quantum Mechanics cannot predict anything precisely. WRONG – then it wouldn't be physics!

c. Quantum Mechanics cannot predict with certainty the result of any particular measurement on a single particle. DEPENDS – if a particle is in an eigenstate of an observable, I can predict the outcome of a measurement of that observable precisely.
d. The Heisenberg Uncertainty principle means that nothing can be measured precisely. WRONG – see above
e. The x- and y- components of any angular momentum cannot simultaneously be measured with arbitrary precision. CORRECT
f. The time evolution of a quantum mechanical wave function is described by a unitary operator. CORRECT

Problem 5)

The most general solution is
$y(x) = A\exp(mx) + B\exp(-mx)$
which can be shown by plugging it in (as a 2nd order differential equation, there must be two integration constants, A and B).
Since $y(0)=A+B$ and $y'(0)=mA-mB$, we can solve for A and B in terms of the initial conditions at $x=0$.

Problem 6)

$$z = \exp(c) = \exp(\mathrm{Re}(c) + i\,\mathrm{Im}(c)) = \exp(\mathrm{Re}(c))\big(\cos(\mathrm{Im}(c)) + i\sin(\mathrm{Im}(c))\big)$$
$$z^* = \exp(\mathrm{Re}(c))\big(\cos(\mathrm{Im}(c)) - i\sin(\mathrm{Im}(c))\big) = \exp(\mathrm{Re}(c))\exp(-i\,\mathrm{Im}(c)) = \exp(c^*)$$

Problem 7)

See next recitation