Graduate Quantum Mechanics - Problem Set 6

Problem 1)

The normalized wave function $\psi(x,t)$ satisfies the **time-dependent** Schrödinger equation for a **free** particle of mass m, moving along the x-axis (in 1 dimension). Consider a second wave function of the form $\phi(x,t) = \exp(i(ax - bt))\psi(x - vt,t)$. Show that $\phi(x,t)$ obeys the same time-dependent Schrödinger equation provided the constants a, b and v are related by $b = \frac{\hbar a^2}{2m}, \quad v = \frac{\hbar a}{m}$.

Calculate the expectation value of position <X>, momentum <P> and energy <H> for a particle in the state $\phi(x,t)$ in terms of those for a particle in the state $\psi(x,t)$. Show that the uncertainty in momentum is the same in both states.

What physical interpretation can be given to the transformation from the state $\psi(x,t)$ to the state $\phi(x,t)$?

Problem 2)

A particle is in the ground state of a box of length L with infinitely high walls at $-L/2$ and $+L/2$ (see Shankar pp. 157 and our first HW problem set). Suddenly, the box expands (symmetrically) to length $2L$, leaving the wave function momentarily undisturbed. Calculate the probability that measuring the energy of the system afterwards yields as result the ground state energy of the **new** box. (Hint: the answer is $\left(\frac{8}{3\pi}\right)^2$).

Problem 3)

Consider a particle (confined in one dimension along the x-axis) within a potential given by the delta-function at the origin: $V(x) = -aV_0\delta(x)$. (You could consider this the extreme limit of a particle in a box – the box has infinite depth and infinitesimally small width). Surprisingly, there is a **bound** state solution to the stationary (time-independent) Schrödinger equation, and your task is to find it (and the corresponding energy eigenvalue). You can find hints in Shankar on page 163. See also the discussion on page 156 – you can not assume that ψ'(x) is continuous everywhere in this special case. In fact, the second derivative is (obviously) not even finite everywhere; however, integrating it over a very small interval centered around zero gives a finite answer which allows you to solve the problem.