Using affine models of the term structure to estimate risk premia

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Abstract

This paper uses affine models of the term structure to provide historical estimates of risk premia. The foreign exchange and inflation risk premia can be modelled in the same way since the price level can be thought of as an exchange rate that transforms real prices to nominal prices. Affine models with three latent factors of the Cox, Ingersoll and Ross (1985) type are used, with a common factor between the two pricing kernels (state price vectors) to account for interdependence. In the case of foreign exchange risk premium two factors are used to model the domestic pricing kernel and two factors to model the foreign pricing kernel with a common factor between them. This specification can account for the forward premium anomaly, the tendency for high interest rate currencies to appreciate, which contradicts uncovered interest rate parity. In the case of inflation risk premium two factors are used to model the real pricing kernel and two factors to model the nominal pricing kernel with a common factor between them. The model distinguishes between expected and realised variables and therefore allows the estimation of expectational errors. The model also allows for time-varying market prices of risk and time-varying correlations between the two pricing kernels or between each of the pricing kernels and the foreign exchange rate or the price level. Another contribution, which has been ignored in the previous literature, is that the model is estimated using both bond yields and realised price level or foreign exchange rate changes. Fitting the later is necessary for the model to produce realistic patterns for the price level or foreign exchange rate changes. The results show that the foreign exchange risk premium fell substantially after 96, which is consistent with the large appreciation of sterling. Expectational errors were very large for the whole of the period studied, that is, from 93 to 99. Inflation risk premium was about 100 basis points for most of the period 87 to 97, but fell substantially since Bank independence in March 97, which may be the result of a higher credibility to the new UK monetary policy institutional framework. Inflation expectational errors also became smaller after the adoption of inflation targeting in UK in January 93.
1. Introduction

In this paper, I describe a no-arbitrage approach for modeling and measuring risk premia and investigating their historical patterns. In it, I use a Cox, Ingersoll and Ross (1985) model, CIR hereafter, with three factors to examine both the foreign exchange and the inflation risk premium. These two different risk premia can be looked at in an analogous way, since the price level can be considered as an exchange rate that transforms nominal prices to real prices using the same no-arbitrage framework. This allows us to using the same theoretical framework for modeling and understanding them using market prices of risk for the sources of uncertainty that drive the foreign exchange rate or the price level.

Risk premia drive a wedge between market participants’ expected change in the foreign exchange rate or price level and that implied by interest rate differentials. In a risk-neutral world the difference between the nominal and real interest rate is equal to the expected rise in the price level. Similarly the difference between the dollar and sterling nominal interest rates is equal to the expected change in the dollar-sterling exchange rate. However, because of uncertainty about the future price level or future exchange rate, investors require a risk premium. Knowing the magnitude of the inflation or foreign exchange risk premia can therefore be useful when we are adjusting interest rate differentials to extract market expectations about the price level or the exchange rate respectively. They are also useful in assessing people’s attitudes towards risk, since they depend on their risk preferences.

There is a large literature on applying arbitrage-free affine models of the stochastic discount factor (pricing kernel) to the inflation risk premium, but few studies have applied these models to foreign exchange risk premium. Remolona, Wickens and Gong (1998) and Campbell and Viceira (1998) are two recent studies on applying no-arbitrage affine models of the term structure to inflation risk premium. Remolona, Wickens and Gong (1998) used a generalization of the discrete-time version of a CIR affine-yield model with two factors. They associated one factor with inflation and the other factor with the real rate process. Although their specification allowed for time varying risk premia, their model failed to capture any dependence between the two factors. They used nominal and real yields to estimate their model. Campbell and Viceira (1998) used a discrete-time version of the Vasicek (1977) model with one-factor associated with the real interest rate and the other with inflation. They allowed for non-zero correlation between innovations in the one-period real rate and expected inflation. However, their model failed to capture time variation in risk premia. They used nominal yields and actual inflation data to estimate their model.

In the foreign exchange risk premium case, Backus, Foresi and Telmer (1998) used no-arbitrage affine yield models to account for the forward premium anomaly: the tendency for high interest rate currencies to appreciate, which contradicts uncovered interest rate parity. The anomaly imposes further conditions on affine models: either interest rates must be negative with positive probability or the effects of one or more factors on pricing kernels must differ across currencies. The second class of models seems to account better for the properties of currency prices.

All of the above specifications use unobservable (latent) factors. Wickens and Smith (2000), on the other hand, used observable factors to model the stochastic discount factor. In particular, they applied a vector GARCH-in-mean specification with the exchange rate and the two interest rates as their benchmark. They imposed restrictions on the joint conditional distribution of these variables to avoid no-arbitrage possibilities. They augmented their model with additional macroeconomic factors, such as retail sales growth, CPI, and base money. They found that money growth is the most important source of exchange risk. However, the use of observable factors can have some
drawbacks. Observable factors may not fit the data as well as latent factors. Also the low
frequency and reliability of macroeconomic data may pose further problems in the estimation of
these models.

This paper uses an arbitrage-free affine yield model with three latent factors. Market completeness
is also assumed. In particular, each pricing kernel is described by a two-factor affine model with
the two factors following discrete-time CIR processes. There is a common factor between the two
pricing kernels; that is, there are three factors in total. The shocks to the three factors are
uncorrelated. However, the introduction of a common factor allows for non-zero correlation
between the two pricing kernels and the foreign exchange rate or inflation processes. The model, in
linear state-space form, is estimated using non-linear least squares. In the estimation process I use
term structure data of different maturities: domestic and foreign nominal yields in the case of
foreign exchange risk premium and real and nominal yields in the case of inflation risk premium. I
also used realized inflation data (as in Campbell and Viceira (1998)) in the case of the inflation risk
premium and realized foreign exchange rate changes in the case of the foreign exchange risk
premium.

The main contribution of this paper is that it uses a model that allows for non-zero time-varying
correlations between the two pricing kernels and the inflation or foreign exchange rate processes,
and at the same time allows for time-varying risk premia. Previous literature on estimating the
inflation risk premium using affine models of the term structure assumed either time-varying risk
premia and zero correlations (Remolona, Wickens and Gong (1998)), or constant risk premia and
non-zero constant correlations (Campbell and Viceira (1998)). This paper allows for time-varying
risk premia and non-zero time varying correlations. It also uses information from both the term
structure and actual realized changes in the foreign exchange rate or price level. The same model is
estimated for both inflation and foreign exchange risk premia, since the price level can be thought
of as an exchange rate that transforms nominal assets to real assets.

Section 2 describes how different types of risk premia (i.e., foreign exchange, inflation) can be
modeled in exactly the same way using no-arbitrage theory. This is called the ‘foreign exchange
analogy’. Section 3 describes a specific model with three latent factors, which can be estimated
using bond yields. Section 4 applies the method to estimate foreign exchange, inflation, and swap
risk premia. Section 5 concludes.

2. The foreign exchange analogy

I will first describe the implications of no-arbitrage theory for the foreign exchange rate process.
No-arbitrage theory, under the assumption of market completeness, implies that the foreign
exchange rate is given by the ratio of the pricing kernels of the two economies. Therefore, as long
as the domestic and foreign pricing kernel processes are specified (i.e., using the properties of
domestic and foreign asset returns respectively), the foreign exchange rate process is specified as
well. All three cannot be are independently specified. We consider the following general
specifications for the domestic $m_r$ and foreign $m_{f}$ pricing kernels$^1$:

\[ \text{The domestic and foreign pricing kernels follow the processes given below:} \]

\[ \text{\begin{align*} \end{align*}} \]

$^1$The results of no-arbitrage theory and the specifications used in this and the following section are explained in
Appendix 1.
\[
\frac{\Delta m_{t+1}}{m_t} = -r_t - \lambda_t \epsilon_{t+1} \Rightarrow \log \frac{m_{t+1}}{m_t} = -r_t - \frac{1}{2} \|\lambda_t\|^2 - \lambda_t \epsilon_{t+1}
\]  
(1)

\[
\frac{\Delta m_{t+1}^f}{m_t^f} = -r_t^f - \lambda_t^f \epsilon_{t+1} \Rightarrow \log \frac{m_{t+1}^f}{m_t^f} = -r_t^f - \frac{1}{2} \|\lambda_t^f\|^2 - \lambda_t^f \epsilon_{t+1}
\]  
(2)

where, \(r_t\) is the domestic one-period (short) rate, \(r_t^f\) is the foreign one-period (short) rate, \(\lambda_t\) is the domestic market price of risk (the market price of risk of a particular source of uncertainty is equal to the expected excess return of an asset that is perfectly correlated with that source of uncertainty and has unit volatility\(^2\)), \(\lambda_t^f\) is the foreign market price of risk, and \(\epsilon_t\) is the vector of the sources of uncertainty that may affect the domestic and foreign pricing kernels and thus the exchange rate.

Then, the process for the log exchange rate \(Y_t\) (the price of domestic currency in terms of foreign currency) will be given by:

\[
\Delta \log Y_{t+1} = \Delta \log m_{t+1} - \Delta \log m_{t+1}^f = r_t^f - r_t + \frac{1}{2} (\|\lambda_t^f\|^2 - \|\lambda_t\|^2) + (\lambda_t^f - \lambda_t)^\top \epsilon_{t+1}
\]  
(3)

This is, essentially, an Uncovered Interest Parity (UIP) condition adjusted for the foreign exchange risk premium. If sterling is the domestic currency and euro the foreign currency, it says that sterling rates are higher than euro rates to compensate investors for the expected depreciation of sterling relative to euro. This ensures that there are no-arbitrage opportunities when investors are risk neutral.

When investors are risk averse they require a risk premium because of the uncertainty about future exchange rate. The risk premium is given by:

\[
\frac{1}{2} (\|\lambda_t^f\|^2 - \|\lambda_t\|^2)
\]  
(4)

That is, the risk premium is equal to the difference in magnitude between the market prices of risk in the two economies. This risk premium ensures that there are no-arbitrage opportunities when investors are risk averse.

Consider an investor who can hold either a UK asset or a German asset of the same volatility (quantity of risk) in a world with only one source of uncertainty. Suppose that the market price of risk in the United Kingdom is higher than in Germany, i.e., UK assets are more ‘risky’ than German assets of the same volatility. This can be either because UK investors are more risk averse or because their consumption growth is more volatile than German investors’ consumption growth. In this case, the excess return over the UK risk free rate of an asset held in the United Kingdom will be higher than the excess return over the German risk free rate of an asset of the same quantity of risk held in Germany. Suppose also that the exchange rate is expected to remain the same. Consider, now, the return on the risky asset. Clearly, it will have the same return irrespective of where it is held. But, since its excess return is higher in the United Kingdom than in Germany, the German risk-free rate must be higher than the risk-free rate in the United Kingdom. Given no expected change in the sterling-Deutschmark exchange rate (in Deutschmarks per unit of sterling), the foreign exchange risk premium from the point of view of a German investor must be negative.

\(^2\) It is to show that in a standard representative agent framework, the market price of risk is positively related to investor’s risk preferences (risk aversion) and the volatility of his consumption growth.
This is the key result: if the ‘riskiness’ of assets of a given volatility in the United Kingdom (UK market price of risk) rises, the foreign exchange risk premium will fall.

Consider now the price level $P_t$. The real price of an asset is equal to its nominal price divided by the price level. The price level is therefore an “exchange rate” that transforms nominal prices to real prices. In particular $P_t$ is the price of “real currency” (a unit of consumption good) in terms of nominal currency. An equation similar to (3) holds for the price level,

$$\Delta \log P_{t+1} = \Delta \log m_{t+1}' - \Delta \log m_{t+1} = r_t - r_t' + \frac{1}{2} (\lambda_t^2 - \lambda_t'^2) + (\lambda_t - \lambda_t')^T \epsilon_{t+1}$$

(5)

where superscript $r$ denotes the parameters of the real pricing kernel.

Equation (5) is like a UIP equation for the price level. Nominal rates are higher than real rates to compensate investors for the expected depreciation of nominal currency relative to consumption goods, that is, the expected increase in the price level (inflation). However, because of the uncertainty about future inflation there is also a risk premium, which is given by $\frac{1}{2} (\|\lambda(t)\|^2 - \|\lambda'(t)\|^2)$. Uncertainty about future inflation makes real investors require higher excess returns per unit of volatility than nominal investors, that is, the real market price of risk is higher than nominal market price of risk. The term $\frac{1}{2} (\|\lambda(t)\|^2 - \|\lambda'(t)\|^2)$ then becomes negative. As explained in the next section, this causes the difference between nominal and real yields to be higher than expected inflation, that is, investors require a positive premium on nominal yields to compensate for inflation uncertainty. This positive premium is usually called in the literature inflation risk premium.

3. The empirical model

The model used for estimation purposes is a three-factor model, with the factors following discrete-time versions of the CIR model. There is one factor specific to each of the two pricing kernels, and a common factor that captures the correlation between the two pricing kernels. In fact, the introduction of the common factor allows for non-zero correlations between the two pricing kernels and the foreign exchange rate. The reason for using three factors is that each pricing kernel, which is estimated using bond yields, needs at least two factors to capture the shape of the yield curve, the patterns of yields autocorrelation and volatility with maturity, and the departures from the expectations hypothesis. The discrete time model is described in more details below.

No-arbitrage theory with market completeness implies a relation between the domestic pricing kernel, the foreign pricing kernel, and the foreign exchange rate. The process of one can be inferred from the processes of the other two. I first specify two-factor processes for the foreign and domestic pricing kernel with a common factor between them. The foreign exchange rate will then be given by the ratio between the two pricing kernels.

The domestic and foreign pricing kernels are following the following processes:

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3 In the case of inflation risk premium the “foreign exchange rate” is the price level.

4 The need for two factors has been documented extensively in the literature. For example, see Backus, Foresi, Mozumdar and Wu (2000).
\[-\log m_{t+1}^m = x_0 (\gamma + \frac{1}{2} \lambda_0^2) + \lambda_{01} x_0^2 \epsilon_{0,t+1} + x_{1t} (1 + \frac{1}{2} \lambda_1^2) + \lambda_{12} x_{1t}^2 \epsilon_{1,t+1} \]  

(6)

\[-\log m_{t+1}^m = x_0 (1 + \frac{1}{2} \lambda_0^2) + \lambda_{02} x_0^2 \epsilon_{0,t+1} + x_{2t} (1 + \frac{1}{2} \lambda_2^2) + \lambda_{22} x_{2t}^2 \epsilon_{2,t+1} \]  

(7)

where \( x_n, i = 0, 1, 2 \) follow first order autoregressive processes with rates of mean reversion \( \phi_i \) and long run means \( \mu_i \).

\[ x_{i,t+1} = (1 - \phi_i) \mu_i + \phi_i x_{i,t} + x_n^2 \sigma_i \epsilon_{i,t+1} \]  

(8)

The error terms \( \epsilon_{i,t+1} \) are iid with zero mean and unit variance processes. The factor \( x_0 \) is the common factor between the two pricing kernels. The common factor can affect the two pricing kernels in a different way when \( \lambda_{01} \neq \lambda_{02} \). The coefficient \( \gamma \) is needed to capture the forward premium anomaly, as explained in Backus, Foresi and Telmer (1998).

The domestic log bond prices, which drive the domestic bond yields, are affine (linear) functions of the factors \( x_0 \) and \( x_1 \):

\[ -p_m = A_n + B_{n0}^0 x_0 + B_{n1}^1 x_1 \]  

(9)

where \( p_m \) is the time \( t \) logarithm of a domestic bond price that matures in \( n \) periods. Assuming joint lognormality between the domestic bond prices and the domestic pricing kernel we obtain:

\[ A_n = A_{n-1} + B_{n0}^0 (1 - \phi_0) \mu_0 + B_{n1}^1 (1 - \phi_1) \mu_1 \]  

(10)

\[ B_{n0}^0 = \gamma + B_{n0}^0 \phi_0 - \frac{1}{2} \sigma_0^2 (B_{n0}^0)^2 - \lambda_{00} \sigma_0 B_{n1}^0 \]  

(11)

\[ B_{n1}^1 = 1 + B_{n0}^0 \phi_1 - \frac{1}{2} \sigma_1^2 (B_{n1}^1)^2 - \lambda_{10} \sigma_1 B_{n1}^1 \]  

(12)

We require that \( p_0 = 0, \forall t \Rightarrow A_0 = B_{00}^0 = B_{01}^1 = 0 \).

The domestic short-rate or one-period yield is given by:

\[ r_{1t} = -p_{1t} = A_1 + B_{10}^0 x_0 + B_{11}^1 x_1 = \gamma x_0 + x_1 \]  

(13)

The risk premium on one period holding returns for an \( n \)-period domestic bond is equal to:

\[ E_t[p_{n-1,t+1} - p_{nt} - r_{1t}] = -\frac{1}{2} \sigma_0^2 (B_{n0}^0)^2 x_0 - \lambda_{00} \sigma_0 B_{n0}^0 x_0 - \frac{1}{2} \sigma_1^2 (B_{n1}^0)^2 x_0 - \lambda_{10} \sigma_1 B_{10}^0 x_1 + \omega_{n-1,t} \]  

(14)

If we average the above equation from time \( t \) to time \( t + n \) we get the following expression for the \( n \)-period yield:

\[ p_n = E_t[\log m_{t+1} + p_{n-1,t+1}] + \frac{1}{2} Var_t[\log m_{t+1} + p_{n-1,t+1}] \].

Joint lognormality between bond prices and the pricing kernel means that
\[ r_{nt} = -\frac{1}{n} p_{mt} = -\frac{1}{n} \gamma \sum_{k=1}^{n} E_t x_{0,t+k-1} + \frac{1}{n} \sum_{k=1}^{n} E_t x_{1,t+k-1} + \frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k,t+k-1} \]  \hspace{1cm} (15)

So the \( n \)-period domestic yield is not simply given by the average of expected future short rates, but there also is a term premium, which is given by \( \frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k,t+k-1} \) (including Jensen’s inequality terms).

The vector of the domestic market price of risk, which is defined as the expected excess return on an \( n \)-period domestic bond \( E_t[p_{n-1,t+1}] - p_{nt} - r_t \) (excluding Jensen’s inequality terms) divided by the conditional volatility vector of that excess return \( Stdev_t[p_{n-1,t+1}] \) is simply the vector:

\[
\lambda_{domestic} = \begin{bmatrix}
- \lambda_{01} x_{0t}^\frac{1}{2}
- \lambda_{01} x_{0t}^\frac{1}{2}
- \lambda x_{lt}^\frac{1}{2}
0
\end{bmatrix}
\hspace{1cm} (16)
\]

The element in the third row is zero since only the first and the second factor are priced in the domestic economy. As suggested by no-arbitrage theory, the market price of risk is independent of the bond maturity \( n \).

Similarly, the foreign log bond prices, which drive the foreign bond yields, will be affine (linear) functions of the factors \( x_{ot} \) and \( x_{zt} \):

\[- p_{ot}' = A_n' + B_{o2} x_{ot} + B_n'^2 x_{zt} \]  \hspace{1cm} (17)

where \( p_{ot}' \) is the time \( t \) logarithm of a foreign bond price that matures in \( n \) periods.

Assuming joint lognormality between the foreign bond prices and the foreign pricing kernel we get

\[ A_n' = A_{n-1}' + B_{o2} (1 - \phi_0) \mu_0 + B_{n-1}' (1 - \phi_2) \mu_2 \]  \hspace{1cm} (18)

\[ B_{o2} = 1 + B_{n-1}' \phi_0 - \frac{1}{2} \sigma_0^2 (B_{n-1}')^2 - \lambda_{o2} \sigma_0 B_{n-1}' \]  \hspace{1cm} (19)

\[ B_n'^2 = 1 + B_{n-1}' \phi_2 - \frac{1}{2} \sigma_2^2 (B_{n-1}')^2 - \lambda_2 \sigma_2 B_{n-1}' \]  \hspace{1cm} (20)

We, again, require that \( p_{ot}' = 0, \forall t \Rightarrow A_0' = B_{o2}' = B_0'^2 = 0 \).

The foreign short-rate or one-period yield is given by

\[ r_{ot}' = -p_{ot}' = A_1' + B_{1} x_{ot} + B_1' x_{zt} = x_{ot} + x_{zt} \]  \hspace{1cm} (21)

The risk premium on one period holding returns for an \( n \)-period foreign bond is equal to:

\[ E_t[p_{n-1,t+1}] - p_{nt} - r_t' = \frac{1}{2} \sigma_0^2 (B_{n-1})^2 x_{ot} - \lambda_{o2} \sigma_0 B_{n-1}' x_{0t} - \frac{1}{2} \sigma_2^2 (B_{n-1})^2 x_{zt} - \lambda_2 \sigma_2 B_{n-1}' x_{zt} = \omega_{o1,t} \]  \hspace{1cm} (22)
If we average the above equation from time $t$ to time $t + n$ we get the following expression for the $n$-period yield:

$$r_{nt}^f = -\frac{1}{n} p_{nt}^f = \frac{1}{n} \sum_{k=1}^{n} E_t x_{0,t+k-1} + \frac{1}{n} \sum_{k=1}^{n} E_t x_{2,t+k-1} + \frac{1}{n} \sum_{k=1}^{n} E_t \omega_{t+k}^f$$  \hspace{1cm} (23)

The vector of the foreign market price of risk, which is defined as the expected excess return on an $n$-period foreign bond $E_t[p_{n-1,t+1}^f] - p_{nt}^f - r_t^f$ (excluding Jensen’s inequality terms) divided by the conditional volatility vector of that excess return $Stdev_t[p_{n-1,t+1}^f]$ is simply the vector:

$$\lambda_{\text{foreign}} = \begin{bmatrix} -\lambda_{02} x_{0t}^2 \\ 0 \\ -\lambda_{2} x_{2t}^2 \end{bmatrix}$$  \hspace{1cm} (24)

The element in the second row is zero since only the first and the third factor are priced in the foreign economy.

The foreign exchange rate (the price of domestic currency in terms of foreign currency is given by the no-arbitrage equation

$$y_{t+1} - y_t = \log \frac{Y_{t+1}}{Y_t} = -\log \frac{m_{t+1}}{m_t} + \log \frac{m_{t+1}}{m_t} = x_{0t} (1 + \frac{1}{2} \lambda_{02}^2) + \lambda_{02} x_{0t}^2 e_{0,t+1} + x_{2t} (1 + \frac{1}{2} \lambda_{2}^2) + \lambda_{2} x_{2t}^2 e_{2,t+1}$$

$$-x_{0t} (\gamma + \frac{1}{2} \lambda_{01}^2) - \lambda_{01} x_{0t}^2 e_{0,t+1} - x_{2t} (1 + \frac{1}{2} \lambda_{2}^2) - \lambda_{2} x_{2t}^2 e_{2,t+1}$$

$$= x_{0t} (1 - \gamma) + x_{2t} - x_{0t} + \frac{1}{2} x_{0t} (\lambda_{02}^2 - \lambda_{01}^2) + \frac{1}{2} x_{2t} \lambda_{2}^2 - \frac{1}{2} x_{0t} \lambda_{01}^2$$

$$+ (\lambda_{02} - \lambda_{01}) x_{0t}^2 e_{0,t+1} + \lambda_{2} x_{2t}^2 e_{2,t+1} - \lambda_{01} x_{0t}^2 e_{1,t+1}$$

Therefore:

$$E_t (y_{t+1} - y_t) = x_{0t} (1 - \gamma) + x_{2t} - x_{0t} + \frac{1}{2} x_{0t} (\lambda_{02}^2 - \lambda_{01}^2) + \frac{1}{2} x_{2t} \lambda_{2}^2 - \frac{1}{2} x_{0t} \lambda_{01}^2 = x_{2t} - x_{0t} + \phi_t$$  \hspace{1cm} (25)

where $x_{0t} (1 - \gamma) + x_{2t} - x_{0t}$ is the one-period interest rate differential or forward premium\(^6\) (the difference between the foreign and the domestic short rate) and

$$\phi_t = \frac{1}{2} x_{0t} (\lambda_{02}^2 - \lambda_{01}^2) + \frac{1}{2} x_{2t} \lambda_{2}^2 - \frac{1}{2} x_{0t} \lambda_{01}^2$$ is the one-period foreign exchange risk premium (including

\(^6\) The model is consistent with the forward premium anomaly, which requires that the coefficient of regressing expected depreciation $E_t (y_{t+1} - y_t)$ on forward premium $x_{0t} (1 - \gamma) + x_{2t} - x_{0t}$ is negative. This coefficient is equal in our model to $1 + \frac{(1 - \gamma) \frac{1}{2} (\lambda_{02}^2 - \lambda_{01}^2) Var x_{0t} + \frac{1}{2} \lambda_{2}^2 Var x_{2t} + \frac{1}{2} \lambda_{01}^2 Var x_{1t}}{(1 - \gamma) Var x_{0t} + Var x_{1t} + Var x_{2t}}$, which can be negative for sufficient values of coefficients $\gamma, \lambda_{01}, \lambda_{02}$.}
Jensen’s inequality terms\(^7\)). If we average the above equation from time \(t\) to time \(t + n\) we get the following expression for the \(n\)-period expected change of the logarithm of the foreign exchange rate

\[
\frac{1}{n} E_t(y_{t+n} - y_t) = \frac{1}{n} (1 - \gamma) \sum_{k=1}^{n} E_t x_{0,t+k-1} + \frac{1}{n} \sum_{k=1}^{n} E_t x_{1,t+k-1} - \frac{1}{n} \sum_{k=1}^{n} E_t x_{2,t+k-1} + \frac{1}{n} \sum_{k=1}^{n} E_t \phi_{t+k-1} \tag{26}
\]

The expected change of the logarithm of the foreign exchange rate over \(n\) periods is not simply given by the difference of the averages of expected one-period domestic and foreign rates, but there is also a risk premium given by \(\frac{1}{n} \sum_{k=1}^{n} E_t \phi_{t+k-1}\). This foreign exchange risk premium depends on expected one-period interest rates (which are unobservable) and doesn’t include term premia.

If we combine equations (15), (23) and (26) we get

\[
\frac{1}{n} E_t(y_{t+n} - y_t) = r_{nt}^f - r_{nt} - \frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k,t+k-1}^f + \frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k,t+k-1}^d + \frac{1}{n} \sum_{k=1}^{n} E_t \phi_{t+k-1} \tag{27}
\]

The expected change of the logarithm of the foreign exchange rate over \(n\) periods is not simply given by the difference of the \(n\)-period domestic and foreign yields, but there is also a risk premium given by \(-\frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k,t+k-1}^f + \frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k,t+k-1}^d + \frac{1}{n} \sum_{k=1}^{n} E_t \phi_{t+k-1}\). This foreign exchange risk premium depends on the term premia of the domestic and foreign nominal yields, that is, it is given by the quantity \(\frac{1}{n} \sum_{k=1}^{n} E_t \phi_{t+k-1}\) plus the difference between the domestic and foreign term premia

\[
-\frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k,t+k-1}^f + \frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k,t+k-1}^d .
\]

The model is estimated using domestic and foreign nominal zero-coupon yields. The same analysis applies to the inflation risk premium. The only difference is that the coefficient \(\gamma\) is restricted to be equal to 1, since there is no need to account for the forward premium anomaly in the case of inflation risk premium. In the case of the inflation risk premium the domestic pricing kernel is replaced by the real, and the foreign pricing kernel is replaced by the nominal. The model is estimated using nominal and index-linked zero-coupon yields.

### 4. Estimation

The discrete time model described above system of equations can be estimated using the Kalman filtering methodology, similar to that described in Remolona, Wickens and Gong (1998). To estimate the 3-factor model analyzed in the previous section, I used monthly data on 2, 3.5, 5, 7.5 and 10-year yields of domestic and foreign bonds (the time period used in the estimation of the discrete model is one month). The time period studied is from January 1993 to July 1999 (that is, 79 monthly observations) for the foreign exchange case and from February 1987 to July 1999 (that is, 149 monthly observations) for the inflation case. The yields were produced with the variable roughness penalty method of fitting the yield curve as described in Anderson and Sleath (1999).

---

\(^7\) It is easy to show that

\[
E_t \frac{Y_{t+1}}{Y_t} = x_{0t} (1 - \gamma) + x_{2t} - x_{1t} + x_{2t} \lambda_2^2 + x_{0t} (\lambda_{02}^2 - \lambda_{02}^2 \lambda_{01}) .
\]

So the one-period risk premium excluding Jensen’s inequality terms is given by

\[
x_{2t} \lambda_2^2 + x_{0t} (\lambda_{02}^2 - \lambda_{02}^2 \lambda_{01}) .
\]
the estimation process I also used one-year-ahead actual foreign exchange or price level changes or that were fitted to the model’s implied foreign exchange rate or price level changes respectively. The Kalman filtering methodology allows for measurement errors in bond yields, which can arise because of model mis-specification, errors in fitting the yield curve or price distortions because of low liquidity. It also allows for errors in fitting actual foreign exchange rate changes because of expectational errors or model mis-specification. The model in linear state-space form is described in Appendix 2. It was estimated using maximum likelihood. There are 14 parameters to be estimated the values of which are shown in Appendix 3 along with their standard errors.

I applied the model to UK and German zero-coupon yields and actual 1-year GBPDEM exchange rate changes to model the foreign exchange risk premium.

The results are shown in Diagram 1:

The foreign exchange risk premium of the GBPDEM exchange rate in the above diagram is the ex-ante 2-year foreign exchange risk premium given by \( \frac{1}{n} \sum_{k=1}^{n} E_{t+k-1} \) in equation (26) corrected for Jensen’s inequality terms. The expected 2-year foreign exchange rate change is given by equation (26). The realized foreign exchange rate change is the actual 2-year change of the GBPDEM exchange rate. The difference between realized and expected is the result of expectational errors or model mis-specification. This difference is quite large during the large appreciation of sterling in 96 and 99. We can see the large fall in the foreign exchange risk premium since mid 96, which is consistent with the large appreciation of sterling relative to Deutschmark. The pattern of the foreign exchange risk premium is similar to that derived form Consensus Survey. However, the derived risk premium fails to explain the large appreciation of sterling since 99, since the risk premium has risen since February 99. This could be because of changes in the medium-term equilibrium exchange rate. A change in the spot exchange rate can be the result of either a change in the risk premium or the equilibrium exchange rate. Therefore, the risk premium alone cannot always explain spot rate movements. The appreciation of sterling since February 99 can be the result of an upward revision of the medium equilibrium exchange rate and not because of the change in the risk premium.

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8 Since index-linked gilts are linked to RPI, the model was fitted to RPI changes.
9 GBPDEM denotes the price of sterling in terms of Deutschmarks.
The same methodology was applied to UK real and nominal zero-coupon yields to model the inflation risk premium. As explained in the previous section, the coefficient $\gamma$ is restricted to be equal to 1, since there is no need to account for the forward premium anomaly in the case of inflation risk premium. Therefore, there is one parameter less to be estimated in the case of inflation risk premium. The results are shown in Diagram 2:

![Diagram 2](image-url)

The inflation risk premium in the above diagram is, again, the *ex-ante* 2-year inflation risk premium given by the term \[ \frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k+1} - \frac{1}{n} \sum_{k=1}^{n} E_t \omega_{n-k+1} - \frac{1}{n} \sum_{k=1}^{n} E_t \phi_{n-k+1} \] in equation (27) corrected for Jensen’s inequality terms. It is the difference between 2-year nominal and real yield differential minus 2-year expected inflation\(^{10}\). The expected inflation is given by equation (26). The realized inflation is the actual 2-year inflation rate. The difference between realized and expected inflation was quite large before 1993. After the introduction of inflation targeting in UK this difference became small: less than 50 basis points. This difference is the result of expectational errors or model mispecification. The inflation risk premium was about 100 basis points for most of the period before 97 and fell rapidly after Bank independence (May 97) to about 0 in mid 2000, probably because investors considered the new institutional framework for UK monetary policy more effective for ensuring price level stability.

5. Conclusions

The paper applied a 3-factor CIR model to model both inflation and foreign exchange risk premia. The same model can be estimated for both inflation and foreign exchange risk premia, since the price level can be thought of as an exchange rate that transforms nominal assets to real assets. The model allowed for a common factor between the pricing kernels, that is, the real and nominal pricing kernels in the case of the inflation risk premium and the UK nominal and German nominal pricing kernels in the case of the foreign exchange risk premium. An additional coefficient is used in the foreign exchange rate case to account for the *forward premium anomaly*, that is, the tendency for high interest rate currencies to appreciate, which contradicts uncovered interest rate parity.

\(^{10}\) It is mainly driven by the quantity \[-\frac{1}{n} \sum_{k=1}^{n} E_t \phi_{n-k+1} \], that is, the negative of the risk premium defined in the foreign exchange case.
Previous literature on estimating risk premia using affine models of the term structure assumed either time-varying risk premia and zero correlations or constant risk premia and non-zero constant correlations. Conversely, this paper allowed for time-varying risk premia and non-zero time-varying correlations. It also used information from both the term structure and actual realized changes in the foreign exchange rate or price level. The 3-factor model was estimated using the Kalman filtering algorithm and non-linear least squares.

The results show a large fall in the sterling-Deutschmark risk premium since 1996 that is consistent with the large appreciation of sterling. The pattern is similar to the Consensus survey-based risk premium measure. However, the derived risk premium failed to capture the large appreciation of sterling since 1999. This could be because of changes in the medium-term equilibrium exchange rate. Expectational errors were generally very large. The magnitude of the inflation risk premium was about 100 basis points for most of the period before 96 and declined rapidly after Bank of England independence. This could be the result of higher credibility to UK monetary policy. Also, inflation expectational errors were reduced after January 93, when the Bank adopted inflation targeting.

The methodology used in this paper focuses only on measuring risk premia. It ignores the issue of explaining risk premia. This is because latent (non-observable) factors are used. An extension of this work would be to relate the latent factors to some observable factors, such as the money supply or productivity. The same framework could also be applied to credit risk, that is, to the nominal government bond term structure and either the swap or corporate bond term structure. This will help us to decompose swap or corporate spreads into two parts: the part of spread that is related to the probability of default (the potential loss in value because of default) and the pure risk premium arising from the uncertainty about this loss.
Appendix 1:

The main result of no-arbitrage theory is the existence of the pricing kernel, or stochastic discount factor, which we denote as $m_t$. If we further assume that the markets are complete (that is, for each state one can combine available assets to get a non-zero payoff in that state and zero payoffs in all other states), the pricing kernel is unique. The pricing kernel is a stochastic process that governs the prices of the different states of the world, that is, the prices of state-contingent claims, which pay £1 if that state of the world occurs and 0 elsewhere. The price $P_t$ at time $t$ of any claim, with payoff $d_{t+1}$ at time $t+1$ is given by

$$P_t = E_t \left( \frac{m_{t+1}}{m_t} d_{t+1} \right) = E_t \left( \frac{m_{t+1}}{m_t} R_{t+1} \right)$$  \hspace{1cm} (A1)

where $E_t(.)$ denotes the expectation conditional on information available at time $t$, and $R_{t+1} = \frac{d_{t+1}}{P_t}$ denotes the asset’s gross nominal return.

The above valuation equation says simply that the price of any asset is equal to the conditional at time $t$ expectation of its payoff at different states of the world multiplied by the price of each state. The price of each state is equal to the pricing kernel multiplied by the probability of the state. In other words, the pricing kernel is the state price vector divided by the probability of each particular state. Equation (A1) involves both a discounting through the ratio $\frac{m_{t+1}}{m_t}$, and an averaging through the conditional expectation.

Consider a one-period risk-free bond with return $R_{0,t+1} = 1 + r_t$ (it is known at time $t$). Since the payoff is certain and equal to £1 in all states of the world, according to equation (A1) it will satisfy

$$\frac{1}{R_{0,t+1}} = \frac{1}{1 + r_t} = E_t \left( \frac{m_{t+1}}{m_t} \right)$$

If we assume $r$ is small, we can write this as

$$1 - r_t = E_t \left( \frac{m_{t+1}}{m_t} \right) \Rightarrow E_t \left( \frac{\Delta m_{t+1}}{m_t} \right) = -r_t$$ \hspace{1cm} (A2)

that is, the proportional change of the pricing kernel has an expected value equal to the negative of the short rate. The stochastic process of the pricing kernel can then be written as

$$\frac{\Delta m_{t+1}}{m_t} = -r_t \Delta t - \lambda_t^T \Delta W_{t+1}$$ \hspace{1cm} (A3)

where $W_{t+1}$ is the source of uncertainty vector (negatively related to the pricing kernel) and the quantity $\lambda_t$ is the volatility of the pricing kernel, commonly referred to as the market price of risk.

The market price of risk is the excess return per unit of volatility (or unit of quantity of “risk”) of an asset that is perfectly correlated with the source of uncertainty $W_t$ (that is, perfectly negatively correlated with the pricing kernel).

To see this we rewrite equation (A1) as
Consider an asset that is perfectly negatively correlated with the pricing kernel. The above equation then implies

\[ E_t(R_{t+1}) = R_{0,t+1} (1 - \text{Cov}(\frac{m_{t+1}}{m_t}, R_{t+1})) \Rightarrow E_t(R_{t+1} - R_{0,t+1}) = -R_{0,t+1} \text{Cov}(\frac{\Delta m_{t+1}}{m_t}, R_{t+1}) \]

that is, the standard deviation of the proportional change of the pricing kernel (the volatility parameter \( \lambda_t \)) is equal to the excess return per unit of volatility of an asset perfectly negatively correlated with the pricing kernel. Since the pricing kernel is related to marginal utility of consumption, which is then negatively related to consumption itself, the asset will be positively correlated to consumption growth. This asset has a positive risk premium since it delivers wealth when consumption if high (that is, when it is less valuable). So the market price of risk is a positive.

I now describe the results of no-arbitrage theory for the foreign exchange rate process.

Suppose that we have an asset with gross nominal return \( R_{t+1} = \frac{d_{t+1}}{P_t} \) in the domestic currency, then it must satisfy the standard pricing kernel equation (A1).

The gross nominal return of the same asset in the foreign currency is \( R_{t+1}^f = \frac{Y_{t+1}}{Y_t} R_{t+1} \) where \( Y_t \) is the exchange rate at time \( t \) of one unit of domestic currency in terms of foreign currency. \( R_{t+1}^f \) must satisfy the foreign economy’s analogue to equation (A1), that is

\[ 1 = E_t\left( \frac{m_{t+1}}{m_t} R_{t+1}^f \right) \Rightarrow 1 = E_t\left( \frac{m_{t+1}^f}{m_t^f} \frac{Y_{t+1}}{Y_t} \frac{R_{t+1}^f}{R_{t+1}} \right) \]

If we compare equations (A1) and (A6), and the asset is traded in both currencies, we can easily see that

\[ \frac{m_{t+1}^f}{m_t^f} \frac{Y_{t+1}}{Y_t} \Rightarrow \log\left( \frac{Y_{t+1}}{Y_t} \right) = \log\left( \frac{m_{t+1}}{m_t} \right) - \log\left( \frac{m_{t+1}^f}{m_t^f} \right) \]

must hold, that is the foreign exchange depreciation rate is given by the ratio of the pricing kernels of the two economies. In other words, the two pricing kernels are identical when measured in the same currency. It is, perhaps, easiest to think of a ‘global’ pricing kernel driven by global uncertainty that prices all state contingent claims in terms of a single good that can be consumed in both countries. The exchange rate will adjust to bring the domestic and foreign pricing kernels into line with the global pricing kernel. Now, the ‘global’ source of uncertainty affects the two pricing kernels in a different way. The smaller the volatility of the exchange rate, the more the two pricing kernels are moving together.
The above relation, which is a result of no-arbitrage and market completeness, means that, the three random variables, $\frac{m_{r+1}}{m_i}, \frac{m_{r+1}^f}{m_r^f}, \frac{Y_{r+1}}{Y_r}$, cannot all be independent. Therefore, as long as we specify the processes for the two pricing kernels, equation (A7) will then define the exchange rate process.
Appendix 2:

The Kalman filtering algorithm is described below following Harvey (1990) and Roncalli (1995).

The three factors follow first order autoregressive processes. The transition equations for the three factors are:

$$x_t = \Phi_t x_{t-1} + C_t + \Sigma \epsilon_t, \quad t = 1, \ldots, \tau \quad \text{(B1)}$$

where $x_t = \begin{bmatrix} x_{0,t} \\ x_{1,t} \\ x_{2,t} \end{bmatrix}$, $C_t = \begin{bmatrix} (1-\phi_0)\mu_0 \\ (1-\phi_1)\mu_1 \\ (1-\phi_2)\mu_2 \end{bmatrix}$, $\Phi_t = \begin{bmatrix} \phi_0 & 0 & 0 \\ 0 & \phi_1 & 0 \\ 0 & 0 & \phi_2 \end{bmatrix}$, $\Sigma_t = \begin{bmatrix} x_{0,t-1}^{1/2} \sigma_0 & 0 & 0 \\ 0 & x_{1,t-1}^{1/2} \sigma_1 & 0 \\ 0 & 0 & x_{2,t-1}^{1/2} \sigma_2 \end{bmatrix}$,

$$\epsilon_t = \begin{bmatrix} \epsilon_{0,t} \\ \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix}$$

t = 1, \ldots, \tau and the error terms $\epsilon_{i,t+1}$ are iid $(0,1)$, that is, $E[\epsilon_t] = 0$, $V[\epsilon_t] = I_3$, where $I_3$ is the $3 \times 3$ identity matrix.

The are 11 measurement equations, for the actual 1-year foreign exchange rate change and the domestic and foreign yields used in the estimation process. The measurement equations are:

$$r_t = A_t + B_t x_t + u_t \quad \text{(B2)}$$

where $r_t = \begin{bmatrix} f x_{12,t} \\ r_{24,t} \\ r_{42,t} \\ r_{60,t} \\ r_{90,t} \end{bmatrix}$, $A_t = \begin{bmatrix} a_{12}^{fx} \\ a_{24}^{fx} \\ a_{42}^{fx} \\ a_{60}^{fx} \\ a_{90}^{fx} \end{bmatrix}$, $B_t = \begin{bmatrix} b_{0,12}^{fx} & b_{1,12}^{fx} & b_{2,12}^{fx} \\ b_{0,24}^{fx} & b_{1,24}^{fx} & 0 \\ b_{0,42}^{fx} & b_{1,42}^{fx} & 0 \\ b_{0,60}^{fx} & b_{1,60}^{fx} & 0 \\ b_{0,90}^{fx} & b_{1,90}^{fx} & 0 \end{bmatrix}$, $u_t = \begin{bmatrix} u_{12,t}^{fx} \\ u_{24,t}^{fx} \\ u_{42,t}^{fx} \\ u_{60,t}^{fx} \\ u_{90,t}^{fx} \end{bmatrix}$.

$f x_{12,t}$ is the 12-month foreign exchange rate change, $r_{n,t}$ ($n = 24, 42, 60, 90, 120$ months) is the zero-coupon yield on a domestic bond for maturity $n$ months, and $r_{n,f}$ is the zero-coupon yield on a foreign bond for maturity $n$ months. The quantities $u_{12,t}^{fx}$, $u_{n,t}^{fx}, u_{n,f}$ are measurement errors which are iid $(0, \nu)$, that is, $E[u_t] = 0$, $V[u_t] = H_t = \nu I_{10}$. It is therefore assumed that the measurement errors of all bond yields have the same standard deviation irrespective of their maturity or bond type. This assumption can reduce substantially the number of parameters that need to be estimated, which can improve the convergence of the optimization process. Also it can reduce the possibility of overfitting some yields at the expense of the others. The coefficients in matrices $A_t$ and $B_t$ above are related to the parameters of the model by the following equations:
\[ a_n = \frac{A_n}{n}, \quad a^{f}_n = \frac{A^{f}_n}{n}, \quad b_{0,n} = \frac{B^{01}_n}{n}, \quad b^{f}_{0,n} = \frac{B^{02}_n}{n}, \quad b_{1,n} = \frac{B^{1}_n}{n}, \quad b_{2,n} = \frac{B^{2}_n}{n}, \]

where \( A_n, A^{f}_n, B^{01}_n, B^{02}_n, B^{1}_n, B^{2}_n \) follow the recursive equations (10), (11), (12), (18), (19), (20),

\[
a^{f}_{12} = M \cdot (11 + 10 \cdot \Phi + \ldots + \Phi^{10} ) \cdot C, \quad b^{f}_{12} = \begin{bmatrix} b^{f}_{0,12} & b^{f}_{1,12} & b^{f}_{2,12} \end{bmatrix} = M \cdot (1 + \Phi + \Phi^2 + \ldots + \Phi^{11} )
\]

where \( M = \frac{1}{12} \left[ 1 - \gamma + 0.5 \lambda^{2}_{02} - 0.5 \lambda^{2}_{01} - 1 - 0.5 \lambda^{2}_{1} + 1 + 0.5 \lambda^{2}_{2} \right] \).

The parameters to be estimated are 14: the degrees of persistence of the 3 factors \( \phi_0, \phi_1, \phi_2 \), their long-run means \( \mu_1, \mu_2, \mu_3 \), their volatility parameters \( \sigma_1, \sigma_2, \sigma_3 \), the market prices of risk \( \lambda_{01}, \lambda_{02}, \lambda_1, \lambda_2 \), and the standard deviation of the measurement equation errors \( v \).

Let \( E_t[x_t] = \chi_t \) and \( E_t[(\chi_t - x_t)(\chi_t - x_t)^T] = P_t \). Based on given initial values \( \chi_0, P_0 \), the Kalman algorithm recursive equations are:

\[
\chi_{t}\mid_{t-1} = \Phi_t \chi_{t-1} + C_t \quad \text{(B3)}
\]

\[
P_{t}\mid_{t-1} = \Phi_t P_{t-1} \Phi^T_t + \Sigma_t I^T \Sigma^T_t \quad \text{(B4)}
\]

\[
r_{t}\mid_{t-1} = A_t + B_t \chi_{t}\mid_{t-1} \quad \text{(B5)}
\]

\[
s_t = r_t - r_{t-1} \quad \text{(B6)}
\]

\[
F_t = B_t P_{t-1} B^T_t + H_t \quad \text{(B7)}
\]

\[
\chi_{t} = \chi_{t}\mid_{t-1} + P_{t}\mid_{t-1} B^T_t F_t^{-1} s_t \quad \text{(B8)}
\]

\[
P_t = (I_t - P_{t}\mid_{t-1} B_t^{-1} B_t F_t^{-1} B_t) P_t\mid_{t-1} \quad \text{(B9)}
\]

The notation \( t \mid t-1 \) denotes the time \( t \) expectation conditional on information known at time \( t - 1 \). \( s_t \) is the innovations’ process and \( F_t \) is their variance-covariance matrix. The logarithm of the likelihood of the observations at time \( t \) based on the assumption of normally distributed innovations is given by

\[
l_t = -\frac{N}{2} \log 2\pi - \frac{1}{2} \log |F_t| - \frac{1}{2} s_t^T F_t^{-1} s_t \quad \text{(B10)}
\]

The likelihood function is then the sum \( L = \sum_{t=1}^{T} l_t \).

The linear state-space model described above was estimated using non-linear least squares and not maximum likelihood, because maximum likelihood had poor convergence properties. The persistence coefficients \( \phi_0, \phi_1, \phi_2 \) were restricted to have values less than or equal to 1, to avoid explosive behavior for the 3 factors.
Appendix 3:

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<th>(p-values in parenthesis)</th>
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