Electromagnetic Induction

Chapter 21

Part 1

Faraday's Law

Faraday's observation

Electric currents produce magnetic fields.
19th century puzzle: Can magnetic fields produce currents?
A **static** magnet will produce **no current in a stationary coil.**

Faraday: If the magnetic field **changes**, or if the magnet and coil are in **relative motion**, there **will** be an induced emf (and therefore current) in the coil.

Magnetic Flux

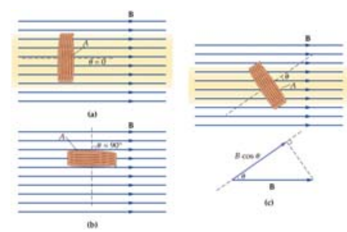
For a "loop" of wire (not necessarily circular) with area *A,* in an external magnetic field *B,* the **magnetic flux** through the loop is:

$$\Phi = BA\cos\vartheta$$

A = area of loop

θ = angle between B and the normal to the loop

SI units of Magnetic Flux: 1 T·m^2 = 1 weber = 1 Wb



Induced emf (Voltage) from changing Magnetic Flux

Faraday: If the magnetic field **changes**, or if the magnet and coil are in **relative motion**, there **will** be an induced emf (and therefore current) in the coil.

Key Concept: The **magnetic flux** through the coil must **change**, this will induce an emf e in the coil, which produces a current I = emf/R in the coil.

Such a current is said to be **induced** by the **time varying** magnet flux that "links" the coil.

Problem

A magnetic field is oriented at an angle of 32° to the normal of a rectangular area 5.5 cm by 7.2 cm. If the magnetic flux through this surface has a magnitude of 4.8 × 10^{-5} T·m^2, what is the strength of the magnetic field?

$$\Phi = BA\cos\theta \quad \text{or} \quad B = \frac{\Phi}{A\cos\theta}$$

$$B = \frac{\Phi}{A\cos\theta} = \frac{4.8\times10^{-5}}{0.055\times0.072\times\cos 32} = \boxed{14.3\text{ mT}}$$

Faraday's Law of Induction

Faraday's Law: The instantaneous **EMF** (voltage) induced in a circuit (w/ *N* loops) equals the **rate of change of magnetic flux through the circuit**:

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Phi_f - \Phi_i}{t_f - t_i}$$

The **minus** sign just indicates the **direction** of the induced emf. To calculate the **magnitude**, we will use:

$$|\varepsilon| = N\left|\frac{\Delta\Phi}{\Delta t}\right| = N\left|\frac{\Phi_f - \Phi_i}{t_f - t_i}\right|$$

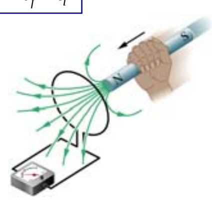
Faraday's Law of Induction

$$\varepsilon = -\frac{\Phi_f - \Phi_i}{t_f - t_i} = -\frac{B_f A_f \cos\theta_f - B_i A_i \cos\theta_i}{t_f - t_i}$$

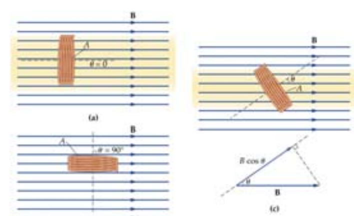
Induction by relative motion

$$\varepsilon = -\frac{\Phi_f - \Phi_i}{t_f - t_i} = -\frac{B_f A_f \cos\theta_f - B_i A_i \cos\theta_i}{t_f - t_i}$$

$$\varepsilon = -A\cos\theta\frac{B_f - B_i}{t_f - t_i}$$



Induction by Rotational Motion



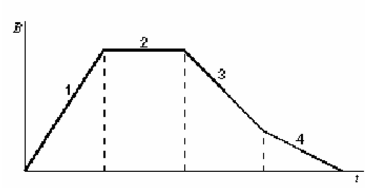
As a coil rotates in a constant magnetic field (uniform or not) the flux through the loop changes, inducing an emf in the coil.

$$\varepsilon = -\frac{\Phi_f - \Phi_i}{t_f - t_i} = -\frac{B_f A_f \cos\theta_f - B_i A_i \cos\theta_i}{t_f - t_i}$$

$$\varepsilon = -BA\frac{\cos\theta_f - \cos\theta_i}{t_f - t_i}$$

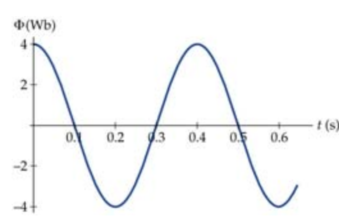
Good to discuss

The graph shows the magnitude B of a uniform magnetic field that is perpendicular to the plane of a conducting loop. Rank the five regions indicated on the graph according to the magnitude of the emf induced in the loop, from least to greatest.



Problem

This is a plot of the magnetic flux through a coil as a function of time. At what times shown in this plot does (a) the **magnetic flux** and (b) the **induced emf** have the greatest **magnitude**?



Problem

A 0.25 T magnetic field is perpendicular to a circular loop of wire with 50 turns and a radius 15 cm. The magnetic field is reduced to zero in 0.12 s. What is the magnitude of the induced emf?

$$emf = -N\frac{\Delta\Phi}{\Delta t} \quad \text{and} \quad |emf| = \left|N\frac{\Delta\Phi}{\Delta t}\right|$$

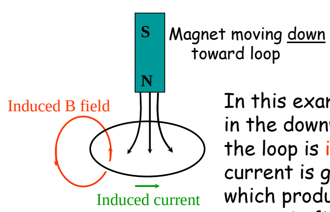
$$emf = \left|50\frac{\Phi_f - \Phi_i}{0.12}\right| = \left|50\frac{\left(0 - 0.25\times\pi\times(0.15)^2\right)}{0.12}\right| = \boxed{7.36 \text{ volts}}$$

Part 2

Lenz's Law

Lenz's Law

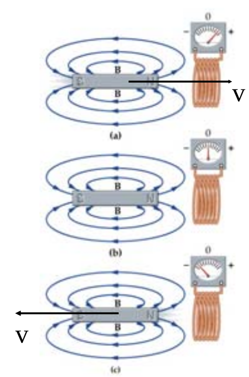
Lenz's Law: An **induced current** always flows in a direction that **opposes** the **change** that caused it.



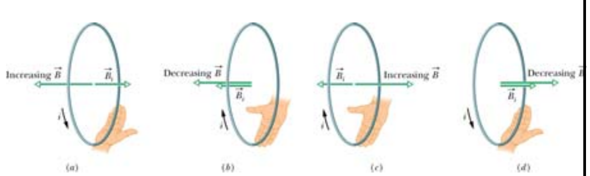
In this example the magnetic field in the downward direction through the loop is increasing. So a current is generated in the loop which produces an upward magnetic field inside the loop to oppose the change.

Induction by Relative Motion

- When a permanent magnet moves relative to a coil, the magnetic flux through the coil **changes** (WHY?), inducing an emf in the coil.
- In a) the magnitude of the flux is increasing
- In c) the flux is decreasing in magnitude.
- In a) and c) the induced current is in the **opposite** direction (Lenz's law).



Lenz's Law



Good to discuss

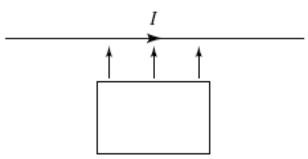
A square loop of wire lies in the plane of the page. A decreasing magnetic field is directed into the page. The induced current in the loop is:

A) counterclockwise
B) clockwise
C) zero
D) depends upon whether or not B is decreasing at a constant rate
E) clockwise in two of the loop sides and counterclockwise in the other two

Good to discuss

A long, straight wire carries a steady current I. A rectangular conducting loop lies in the same plane as the wire, with two sides parallel to the wire and two sides perpendicular. Suppose the loop is pushed toward the wire as shown. Given the direction of I, the induced current in the loop is

1. clockwise.
2. counterclockwise.
3. need more information

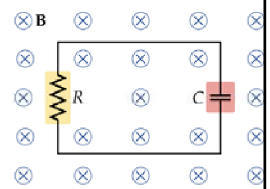


Problem

The figure shows a circuit containing a resistor and an uncharged capacitor. Pointing into the plane of the circuit is a uniform magnetic field B. If the magnetic field increases in magnitude with time, which plate of the capacitor (top or bottom) becomes positively charged?

The magnetic flux through the circuit is changing so that there are more magnetic field lines into the paper "linking" the circuit.

According to Lenz's law the induced cirrent opposes the change. Thus the induced current is counter clock wise. Positive charge builds up on the bottom plate.

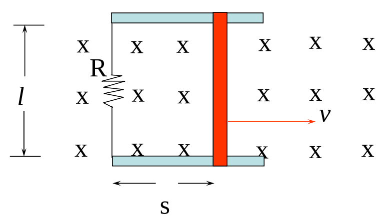


Part 3

Motional EMF

If the moving conductor is part of a circuit, then the magnetic flux through the circuit will change with time and a current will be induced (Area of loop = *ls*):

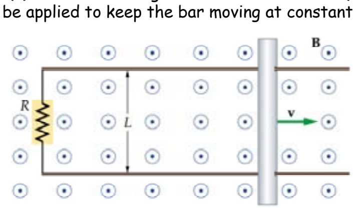
$$\varepsilon = N\left|\frac{\Delta\Phi}{\Delta t}\right| = (1)B\frac{l\Delta s}{\Delta t} = Blv$$



Problem

The figure shows a zero-resistance rod sliding to the right on two zero-resistance rails separated by the distance L = 0.45 m. The rails are connected by a 12.5 Ω resistor, and the entire system is in a uniform magnetic field with a magnitude of 0.75 T.

(a) If the velocity of the bar is 5.0 m/s to the right, what is the current in the circuit? (b) What is the direction of the current in the circuit? (c) What is the magnetic force on the bar? (d) What force must be applied to keep the bar moving at constant velocity?



a) $emf = Blv = IR$ or $I = \frac{Blv}{R} = \frac{0.75\times0.45\times5}{12.5} = \boxed{0.14\text{ A}}$

b) clockwise

c) $\vec{F} = BlI$ $\boxed{\text{to the left}}$.

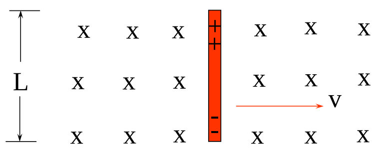
$|\vec{F}| = 0.75\times0.45\times0.14 = \boxed{0.046\text{ N}}$

d) A force must be applied to the right equal to 0.046 N.

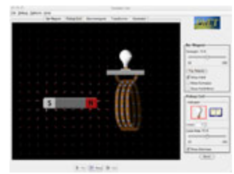
Motional emf

An emf will also be produced if a conductor moves through a magnetic field. The emf comes from the motion of charges, which are free to move in the conductor.

In this example, why do **positive** charges collect at the **top** of the rod?



Computer Simulation



Generators

A **generator** is a device that converts mechanical energy to electrical energy. Consider a current loop which **rotates** in a constant magnetic field:

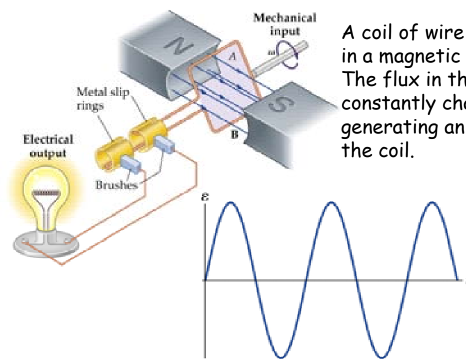
The magnetic flux through the loop **changes**, so an **emf is induced**.

If a loop of area *A* with *N* turns rotates with angular speed ω (period of rotation T = 2π/ω) in a constant *B* field, then the instantaneous induced emf is:

$$\varepsilon = NBA\omega \sin \omega t$$

If this loop is part of a circuit, this emf will induce an **Alternating Current** (AC) in the circuit.

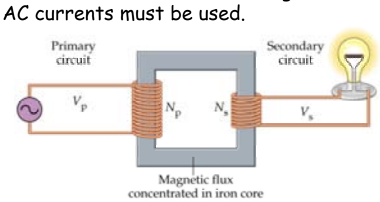
Generator



A coil of wire turns in a magnetic field. The flux in the coil is constantly changing, generating an emf in the coil.

Transformers

A transformer is a device used to change the voltage in a circuit. AC currents must be used.



$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

p = primary

s = secondary

Step-down transformers

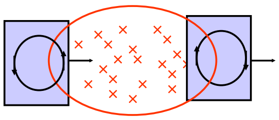
240,000 V in the power lines

2,400 V local substations

120 V in houses

Eddy Currents

When a conductor is **moved** in a magnetic field, there is a force on the **electrons** (remember they are free to move in a conductor), which then move in the metal. This movement is called an **eddy current**.



The **induced** currents produce magnetic fields which tend to **oppose** the motion of the metal.

Part 4

Self-inductance

Solenoids

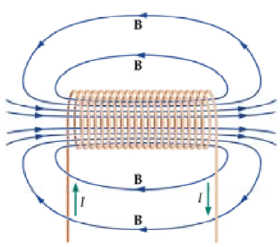
If we stack several current loops together we end up with a **solenoid**.

In the limit of a very long solenoid, the magnetic field inside is very uniform:

$$B=\mu_0 nI$$

n = number of windings per unit length,

I = current in windings



Self-Inductance

If you try to change the current **instantaneously** in a circuit containing a **solenoid**, the response will instead be **gradual**.

This is because the **changing magnetic flux** through the solenoid produces a **self-induced emf** to initially oppose any changes as prescribed by Lenz's Law.

This effect is known as **self-induction**.

Actually **all** circuit elements have some self-inductance. However, only coils with many turns of wire, such as in a **solenoid**, have substantial self-inductance.

Inductance

The self-induced emf is given by: $\varepsilon = N\left|\frac{\Delta\Phi}{\Delta t}\right| = L\left|\frac{\Delta I}{\Delta t}\right|$

where L is defined as the **inductance** of the circuit, usually residing in the "inductor.

For any geometry of loop, the magnetic flux through the loop, produced by current in the loop is proportional to the current. The inductance L is the constant of proportionality.

$$N\Phi = LI \;\Rightarrow\; L = N\left|\frac{\Delta\Phi}{\Delta I}\right|$$

The unit of inductance is the Henry

1 H = 1 T·m²/A = 1 (T·m²/s) (s/A) = 1 V·s/A

Note that inductance, like capacitance, is ***purely geometrical***.

Inductance of a Solenoid

A solenoid has **inductance** given by

$$L = \mu_0\left(\frac{N^2}{\ell}\right)A = \mu_0 n^2 A\ell$$

L = inductance of the solenoid
N = number of turns in solenoid
l = length of solenoid
A = cross sectional area of solenoid
n = number turns per length

Problem

The inductance of a solenoid with 450 turns and a length of 24 cm is 7.3 mH. (a) What is the cross-sectional area of the solenoid? (b) What is the induced emf in the solenoid if its current drops from 3.2 A to 0 in 55 ms?

a) $L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi\times10^{-7}\times(450)^2\times A}{0.24} = 7.3\times10^{-3}$

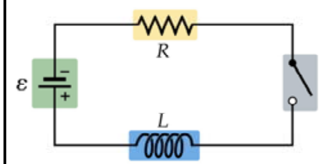
$A = 7.3\times10^{-3}\times\frac{0.24}{4\pi\times10^{-7}\times(450)^2} = \boxed{6.88\times10^{-3}\ \text{m}^2}$

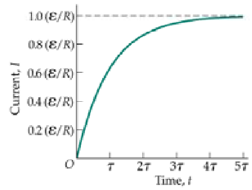
b) $emf = -N\frac{\Delta\Phi}{\Delta t}$ but $L = N\frac{\Delta\Phi}{\Delta I}$ or $N\Delta\Phi = L\Delta I$

so $emf = -L\frac{\Delta I}{\Delta t} = -7.3\times10^{-3}\times\frac{(0-3.2)}{55\times10^{-3}} = \boxed{0.425\ \text{volts}}$

RL Circuits

We can construct a circuit out of inductors and resistors. The circuit will behave similar to an RC circuit, with a time constant given by: $\tau = L/R$





$$I = \frac{\varepsilon}{R}(1 - e^{-t/\tau}) = \frac{\varepsilon}{R}(1 - e^{-tR/L})$$