Geometrical optics

Chapter 24

Nature of Light

Ancient times & Middle Ages: light consist of stream of particles *corpuscles*

19th century: light is a wave (EM wave)

20th century: several effects associated with the emission and absorption of light revel that it also has a particle aspect and that the energy carried by light waves is packaged in discrete bundles called *photons*.

Quantum electrodynamics – a comprehensive theory that includes both wave and particle properties.

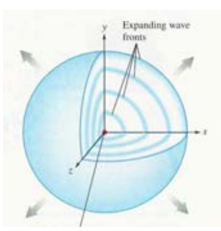
The *propagation* of light is best described by a wave model

The *emission and absorption* by atoms and nuclei requires a particle approach

Wave Fronts

Wave Front – a convenient concept to describe wave propagation

A wave front is *the locus of all adjacent points at which the phase of vibration of the wave is the same*



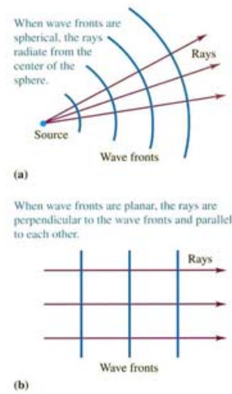
Rays

It is very convenient to represent a light wave by rays rather than by wave fronts

A ray is an imaginary line along the direction of travel of the wave

The branch of optics for which the ray description is adequate is called geometric optics

The branch dealing specifically with wave behavior is called physical optics



spherical and planar wave fronts and rays

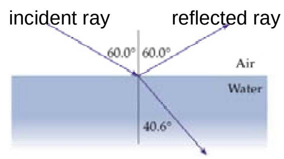
Part 1

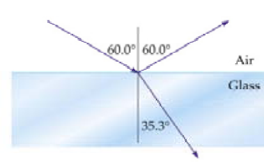
The reflection of light

Reflection and Refraction

When a light ray travels from one medium to another, part of the incident light is reflected and part of the light is transmitted at the boundary between the two media.

The transmitted part is said to be refracted in the second medium.

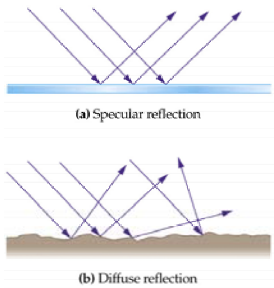




Types of Reflection

If the surface from which the light is reflected is smooth, then the light undergoes specular reflection (parallel rays will all be reflected in the same directions).

If, on the other hand, the surface is rough, then the light will undergo diffuse reflection (parallel rays will be reflected in a variety of directions)



Eduard Manet – A Bar at the Foleis-Bergère (1882)

Types of Images for Mirrors and Lenses

A real image is one in which light actually passes through the image point

Real images can be displayed on screens

A virtual image is one in which the light does not pass through the image point

The light appears to diverge from that point

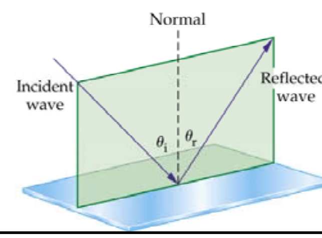
Virtual images cannot be displayed on screens

The Law of Reflection

For specular reflection the incident angle θ_i equals the reflected angle θ_r:

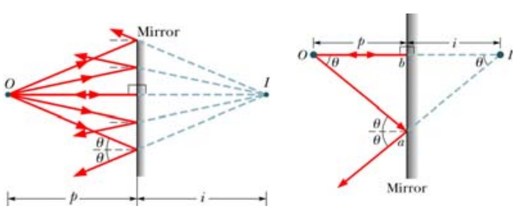
$$\theta_i = \theta_r$$

The angles are measured relative to the normal as shown.



Plane Mirrors

A mirror is a surface that can reflect a beam of light in one direction instead of either scattering it widely in many directions or absorbing it.



Forming Images with a Plane Mirror

A plane mirror is simply a flat mirror.

Consider an object placed at point P in front of a plane mirror. An image will be formed at point P´ behind the mirror.

d_o = distance from object to mirror

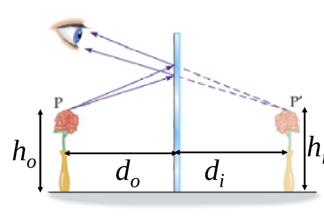
d_i = distance from image to mirror

h_o = height of object

h_i = height of image

For a plane mirror:

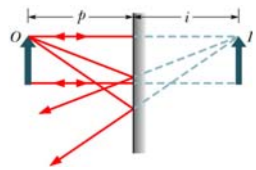
$d_o = d_i$ and $h_o = h_i$



Plane Mirrors

A plane mirror image has the following properties:

- ✓ The image distance equals the object distance
- ✓ The image is unmagnified
- ✓ The image is virtual
- ✓ The image is not inverted

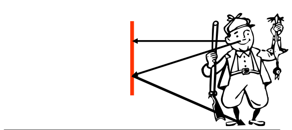


What is wrong with the painting?

Conceptual Checkpoint

To save expenses, you would like to buy the shortest mirror that will allow you to see your entire body. Should the mirror be equal to your height?

Does the answer depend on how far away from the mirror you stand?



Part 2

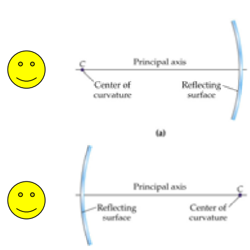
Spherical Mirrors

Spherical Mirrors

A spherical mirror is a mirror whose surface shape is spherical with radius of curvature R. There are two types of spherical mirrors: concave and convex.

We will orient the mirrors so that the reflecting surface is on the left side of the mirror. The object will be to left of the mirror itself.

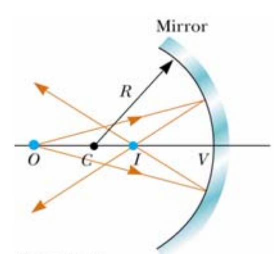
concave



convex

Concave Mirror, Notation

- The mirror has a *radius of curvature* of R
- Its *center of curvature* is the point C
- Point V is the center of the spherical segment
- A line drawn from C to V is called the *principle axis* of the mirror

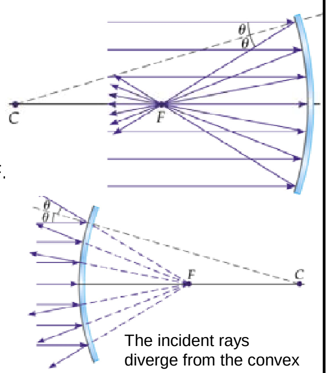


Focal Point

When parallel rays are incident upon a spherical mirror, the reflected rays intersect at the focal point F.

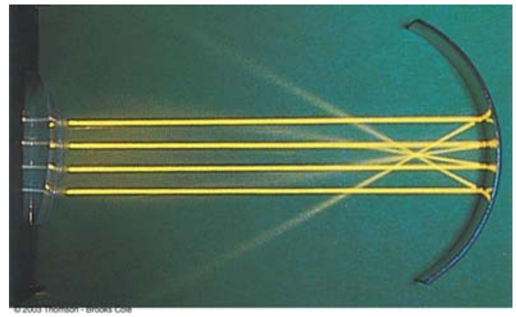
For a concave mirror, the focal point is in front of the mirror.

For a convex mirror, the focal point is behind the mirror.



The incident rays diverge from the convex mirror, but they **trace back to the focal point F.**

Focal Length Shown by Parallel Rays



Focal Length

The focal length f is the distance from the surface of the mirror to the focal point. It can be shown that the focal length is half the radius of curvature of the mirror.

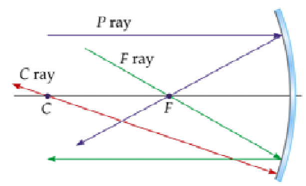
Sign Convention: the focal length is negative if the focal point is behind the mirror.

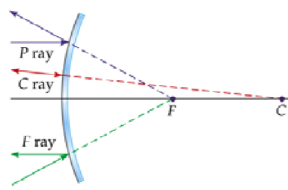
For a concave mirror, $f = \frac{1}{2}R$

For a convex mirror, $f = -\frac{1}{2}R$ (R is always positive)

Ray Tracing

We will use three principal rays to determine where an image will be located.



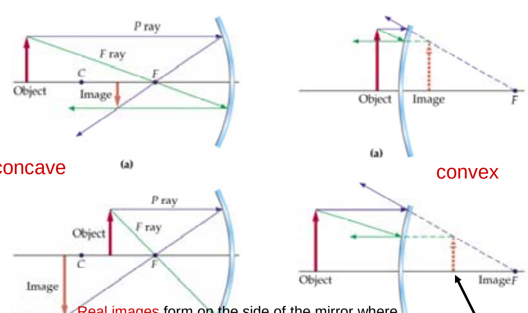


The parallel ray (P ray) reflects through the focal point.

The focal ray (F ray) reflects parallel to the axis,

and the center-of-curvature ray (C ray) reflects back along its incoming path.

Ray Tracing – Examples



concave

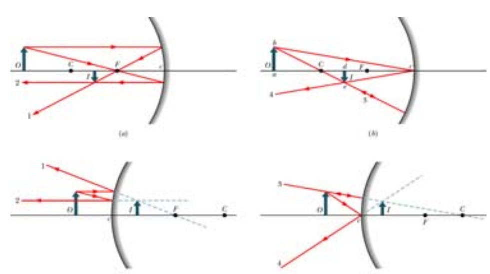
convex

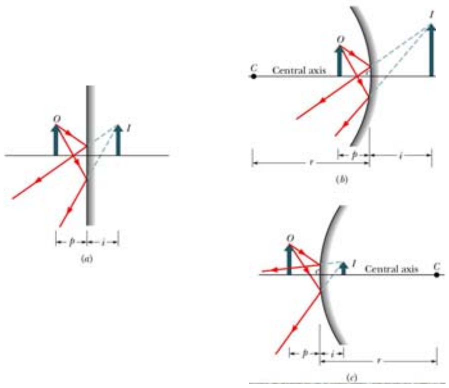
Real images form on the side of the mirror where the object is, and virtual images form on the opposite side

Real image

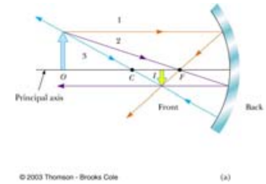
Virtual image

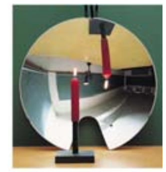
More examples: Images from spherical mirrors





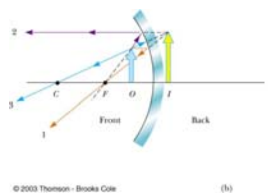
Ray Diagram for Concave Mirror, p > R





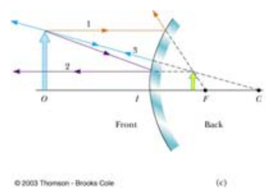
- The image is real
- The image is inverted
- The image is smaller than the object

Ray Diagram for a Concave Mirror, p < f



- The image is virtual
- The image is upright
- The image is larger than the object

Ray Diagram for a Convex Mirror



- The image is virtual
- The image is upright
- The image is smaller than the object

Spherical Aberration

- Mirror is not exactly spherical
- This results in a blurred image
- This effect is called spherical aberration

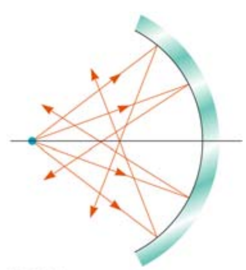
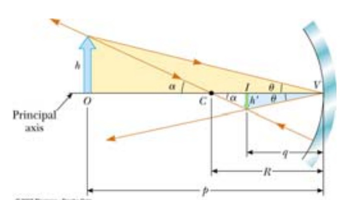


Image Formed by a Concave Mirror

- Geometry shows the relationship between the image and object distances

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$

This is called the **mirror equation**



The Mirror Equation

The ray tracing technique shows qualitatively where the image will be located. The distance from the mirror to the image, d_i, can be found from the mirror equation:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

d_o = distance from object to mirror
d_i = distance from image to mirror
f = focal length

Sign Conventions:
- d_i is positive if the image is in front of the mirror (real image)
- d_i is negative if the image is behind the mirror (virtual image)
- f is positive for concave mirrors
- f is negative for convex mirrors
- m is positive for upright images
- m is negative for inverted images

image magnification. If m is negative, the image is inverted (upside down).

$$m=\frac{\text{image height}}{\text{object height}}=\frac{h_i}{h_o}=-\frac{d_i}{d_o}$$

problem

Example 1

An object is placed 30 cm in front of a concave mirror of radius 10 cm. Where is the image located? Is it real or virtual? Is it upright or inverted? What is the magnification of the image?

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f} \quad \frac{1}{30}+\frac{1}{d_i}=\frac{1}{\left(+\frac{10}{2}\right)}$$

$$\frac{1}{d_i}=\frac{1}{5}-\frac{1}{30} \quad or \quad d_i=\frac{5\times 30}{30-5}=6\text{ cm}$$

$$m=-\frac{d_i}{d_0}=-\frac{6}{30}=-0.2$$

Image is smaller, real and inverted.

problem

Example 2

An object is placed 3 cm in front of a concave mirror of radius 20 cm. Where is the image located? Is it real or virtual? Is it upright or inverted? What is the magnification of the image?

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f} \quad \frac{1}{3}+\frac{1}{d_i}=\frac{1}{\left(\frac{20}{2}\right)}$$

$$\frac{1}{d_i}=\frac{1}{10}-\frac{1}{3} \quad or \quad d_i=-\frac{3\times 10}{7}=-4.3\text{ cm}$$

$$m=-\frac{d_i}{d_0}=\frac{4.3}{3}=1.43$$

Image is larger, virtual, and upright.

problem

Example 3

An object is placed 5 cm in front of a convex mirror of focal length 10 cm. Where is the image located? Is it real or virtual? Is it upright or inverted? What is the magnification of the image?

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f} \quad \frac{1}{5}+\frac{1}{d_i}=-\frac{1}{10}$$

$$\frac{1}{d_i}=-\frac{1}{10}-\frac{1}{5} \quad or$$

$$d_i=-\frac{5\times 10}{15}=-3.33\text{ cm}$$

$$m=-\frac{d_i}{d_0}=\frac{3.33}{5}=0.67$$

Image is smaller, virtual, and upright.

problem

Problem

A concave mirror produces a virtual image that is three times as tall as the object. (a) If the object is 22 cm in front of the mirror, what is the image distance? (b) What is the focal length of this mirror?

a.) $m=-\frac{d_i}{d_0}$

$3=-\frac{d_i}{22} \quad d_i=-66\text{ cm}$

b.) $\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f} \quad \frac{1}{22}-\frac{1}{66}=\frac{1}{f}$

$f=\frac{66\times 22}{66-22}=33\text{ cm}$

Part 3

The Refraction of Light

The Refraction of Light

The speed of light is different in different materials. We define the index of refraction, *n*, of a material to be the ratio of the speed of light in vacuum to the speed of light in the material:

$$n = \frac{c}{v}$$

When light travels from one medium to another its velocity and wavelength change, but its frequency remains constant.

For a vacuum, n = 1

For other media, n > 1

n is a unitless ratio

Snell's Law

In general, when light enters a new material its **direction** will change. The **angle of refraction** θ_2 is related to the **angle of incidence** θ_1 **by Snell's Law:**

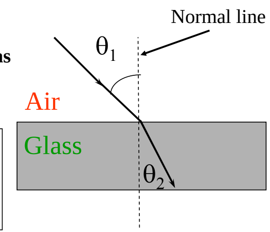
$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

where ***v*** is the velocity of light ***in the medium***.

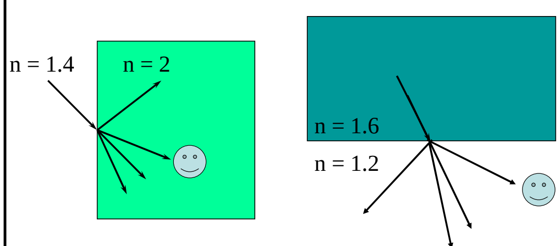
Snell's Law can also be written as

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

The angles θ_1 and θ_2 are measured **relative to the line normal** to the surface between the two materials.



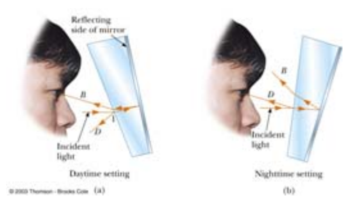
Example: Which way will the rays bend?



Which of these rays can be the refracted ray?

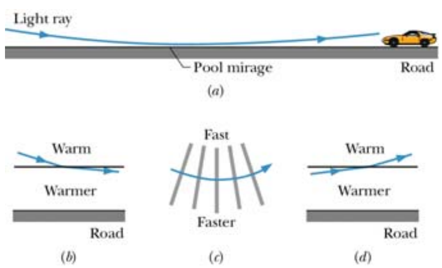
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Application – Day and Night Settings on Auto Mirrors



- With the daytime setting, the bright beam of reflected light is directed into the driver's eyes
- With the nighttime setting, the dim beam of reflected light is directed into the driver's eyes, while the bright beam goes elsewhere

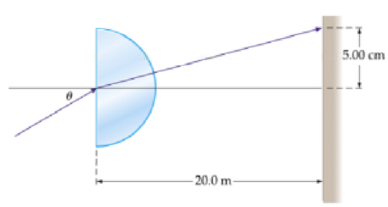
A Common Mirage



Problem

problem

You have a semicircular disk of glass with an index of refraction of *n* = 1.52. Find the incident angle θ for which the beam of light in the figure will hit the indicated point on the screen.



problem

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$1 \times \sin\theta_1 = 1.52 \times \sin\theta_2 \qquad \tan\theta_2 = \frac{5}{20}$$

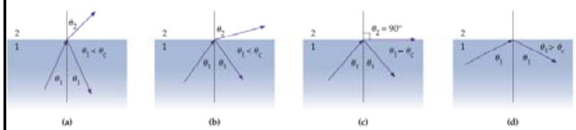
Therefore $\theta_2 = 14^\circ$

$$\sin\theta_1 = 1.52 \times \sin 14^\circ$$

Therefore $\theta_1 = 21.6^\circ$

Total Internal Reflection

When light travels from a medium with $\mathbf{n_1 > n_2}$, there is an angle, called the **critical angle** θ_c, at which all the light is reflected and none is transmitted. This process is known as **total internal reflection.**



The incident ray is both reflected and refracted.

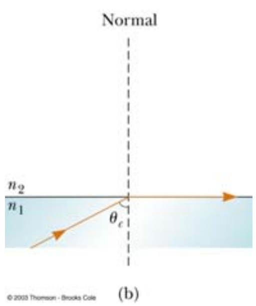
Total Internal Reflection

Critical Angle

A particular angle of incidence will result in an angle of refraction of 90°

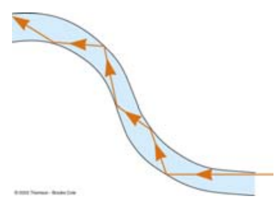
- This angle of incidence is called the *critical angle*

$$\sin\theta_c = \frac{n_2}{n_1} \quad for \quad n_1 > n_2$$



Fiber Optics

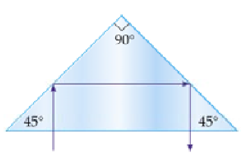
- An application of internal reflection
- Plastic or glass rods are used to "pipe" light from one place to another
- Applications include
  - medical use of fiber optic cables for diagnosis and correction of medical problems
  - Telecommunications



problem

Problem

A ray of light enters the long side of a 45°-90°-45° prism and undergoes two total internal reflections, as indicated in the figure. The result is a reversal in the ray's direction of propagation. Find the minimum value of the prism's index of refraction, *n*, for these internal reflections to be total.



problem

At the first reflection,

$$n_{med} \sin 45^\circ = 1$$

This is the same for the second reflection.

Therefore,

$$\sin 45^\circ \geq \frac{1}{n_{med}} \quad or$$

$$n_{med} \geq \frac{1}{\sin 45^\circ} = \sqrt{2}$$

$$n_{med} = 1.414$$

Question

Sometimes when looking at a window, one sees two reflected images, slightly displaced from each other.
What causes this effect?

?

Question

A student claims that, because of atmospheric refraction, the sun can be seen after it has set and that the day is therefore longer than it would be if the earth had no atmosphere.

What does the student mean by saying the sun can be seen after it has set?

Does the same effect also occur at sunrise?

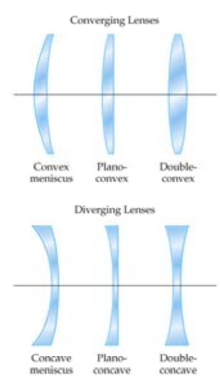
?

Part 4

Lenses

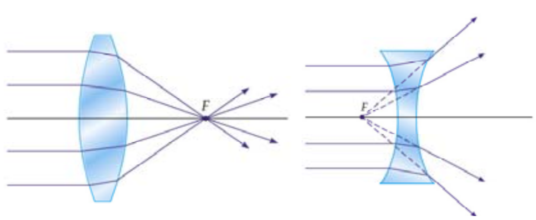
Lenses

Light is reflected from a mirror. Light is refracted through a lens.



Focal Point

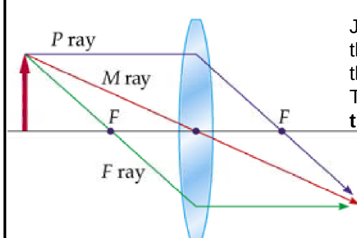
The focal point of a lens is the place where parallel rays incident upon the lens converge.



converging lens

diverging lens

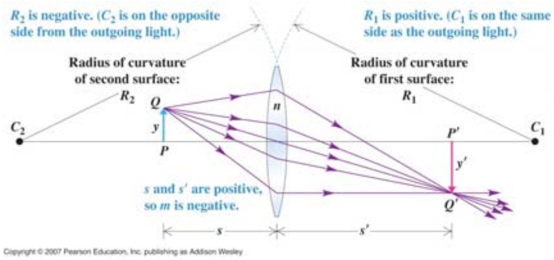
Ray Tracing for Lenses



Just as for mirrors we use three “easy” rays to find the image from a lens. The lens is assumed to be **thin**.

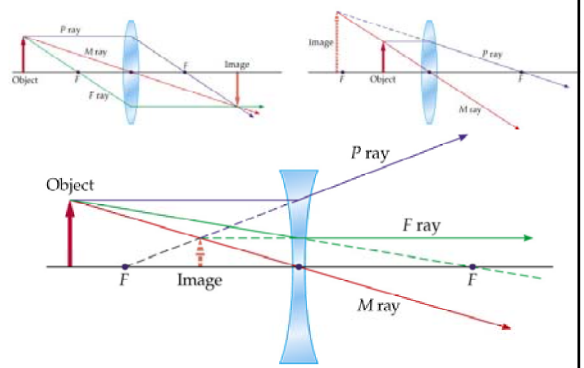
The **P ray propagates parallel** to the principal axis until it encounters the lens, where it is refracted to **pass through the focal point** on the far side of the lens. The **F ray passes through the focal point on the near side of the lens, then leaves the lens parallel to the principal axis**. The **M ray passes through the middle** of the lens with **no** deflection.

The focal length for a thin lens



$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Ray Tracing Examples



The Thin Lens Equation

The ray tracing technique shows qualitatively where the image from a lens will be located. The distance from the lens to the image, d_i, can be found from the thin-lens equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Sign Conventions:

d_i is positive for real images (on the opposite side of the lens from the object)

d_i is negative for virtual images (same side as object)

f is positive for converging (convex) lenses

f is negative for diverging (concave) lenses

m is positive for upright images

m is negative for inverted images

Example 4

problem

An object is placed 20 cm in front of a converging lens of focal length 10 cm. Where is the image? Is it upright or inverted? Real or virtual? What is the magnification of the image?

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f} \qquad \frac{1}{20} + \frac{1}{d_i} = \frac{1}{10} \qquad \frac{1}{d_i} = \frac{1}{10} - \frac{1}{20}$$

$$d_i = \frac{10 \times 20}{20 - 10} = \boxed{20 \text{ cm}}$$

Since d_i is positive, the image is $\boxed{\text{real.}}$

$$m = -\frac{d_i}{d_0} = -1$$ Therefore the image in $\boxed{\text{inverted}}$.

Example 5

problem

An object is placed 5 cm in front of a converging lens of focal length 10 cm. Where is the image? Is it upright or inverted? Real or virtual? What is the magnification of the image?

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f} \qquad \frac{1}{5} + \frac{1}{d_i} = \frac{1}{10} \qquad \frac{1}{d_i} = \frac{1}{10} - \frac{1}{5}$$

$$d_i = \frac{10 \times 5}{5 - 10} = \boxed{-10 \text{ cm}}$$

Since d_i is negative, the image is $\boxed{\text{virtual.}}$

$$m = -\frac{d_i}{d_0} = \frac{10}{5} = 2$$ Therefore the image in $\boxed{\text{upright}}$.

Example 6

problem

An object is placed 8 cm in front of a diverging lens of focal length 4 cm. Where is the image? Is it upright or inverted? Real or virtual? What is the magnification of the image?

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f} \qquad \frac{1}{8} + \frac{1}{d_i} = -\frac{1}{4} \qquad \frac{1}{d_i} = -\frac{1}{4} - \frac{1}{8} = -\left(\frac{1}{4} + \frac{1}{8}\right)$$

$$d_i = -\left(\frac{4 \times 8}{4 + 8}\right) = \boxed{-2.67 \text{ cm}}$$

Since d_i is negative, the image is $\boxed{\text{virtual.}}$

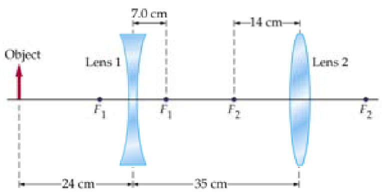
$$m = -\frac{d_i}{d_0} = \frac{2.67}{8} = 0.33$$ Therefore the image in $\boxed{\text{upright and smaller}}$.

problem

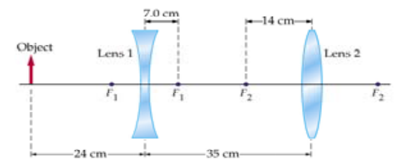
Problem

(a) Determine the distance from lens 1 to the final image for the system shown in the figure. (b) What is the magnification of this image?

When you have two lenses, the image of the first lens is the object for the second lens.



problem



First lens $\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}$ $\quad \frac{1}{24}+\frac{1}{d_i}=-\frac{1}{7}$ $\quad \frac{1}{d_i}=-\frac{1}{7}-\frac{1}{24}=-\left(\frac{1}{7}+\frac{1}{24}\right)$

$d_i=-\left(\frac{7\times 24}{7+24}\right)=\boxed{-5.42\text{ cm}}$

Second lens $\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}$ $\quad \frac{1}{(35+5.42)}+\frac{1}{d_i}=\frac{1}{14}$ $\quad \frac{1}{d_i}=\frac{1}{14}-\frac{1}{40.42}$

$d_i=\left(\frac{14\times 40.42}{40.42-14}\right)=\boxed{21.42\text{ cm}}$

Distance from lens 1 is $21.42+35=56.42$ cm

$m=m_1\times m_2=\left(-\frac{d_i}{d_0}\right)_1\times\left(-\frac{d_i}{d_0}\right)_2=\left(\frac{5.42}{24}\right)\times\left(-\frac{21.42}{40.42}\right)=-0.12$

Part 5

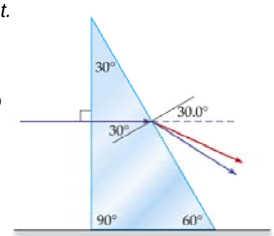
Dispersion

Dispersion

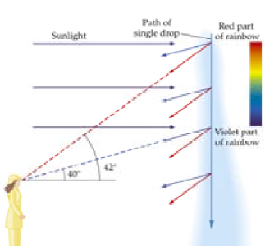
In a material, the **velocity of light** (and therefore the index of refraction) can **depend** on the **wavelength**. This is known as **dispersion**. ***Blue*** *light travels* ***slower*** *in glass and water than does* ***red*** *light.*

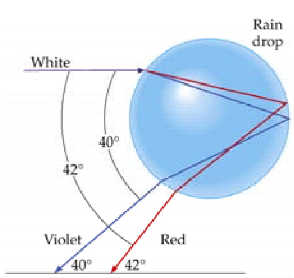
As a result of dispersion, different colors entering a material will be refracted into different angles.

Dispersive materials can be used to separate a light beam into its spectrum (the colors that make up the light beam). Example: prism



The Rainbow





Observing the Rainbow

- If a raindrop high in the sky is observed, the red ray is seen
- A drop lower in the sky would direct violet light to the observer
- The other colors of the spectra lie in between the red and the violet

Using Spectra to Identify Gases

All hot, low pressure gases emit their own characteristic spectra

The particular wavelengths emitted by a gas serve as "fingerprints" of that gas

Some uses of spectral analysis

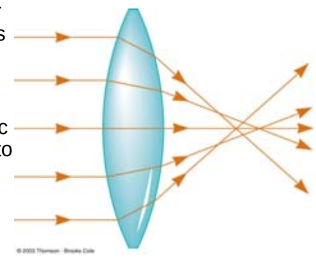
Identification of molecules

Identification of elements in distant stars

Identification of minerals

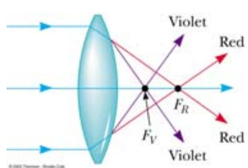
Spherical Aberration

- Results from the focal points of light rays far from the principle axis are different from the focal points of rays passing near the axis
- For a mirror, parabolic shapes can be used to correct for spherical aberration



Chromatic Aberration

- Different wavelengths of light refracted by a lens focus at different points
  - Violet rays are refracted more than red rays
  - The focal length for red light is greater than the focal length for violet light
- Chromatic aberration can be minimized by the use of a combination of converging and diverging lenses



problem

Problem

The index of refraction for red light in a certain liquid is 1.320; the index of refraction for violet light in the same liquid is 1.332. Find the dispersion ($\theta_v - \theta_r$) for red and violet light when both are incident on the flat surface of the liquid at an angle of 45.00° to the normal.

SNELL'S LAW

$$n_{1r} \sin\theta_1 = n_{2r} \sin\theta_{2r}$$

$$\sin\theta_{2r} = \frac{n_{1r}\sin\theta_1}{n_{2r}} = \frac{\sin 45^\circ}{1.32} \quad \theta_{2r} = 32.39^\circ \quad \text{and}$$

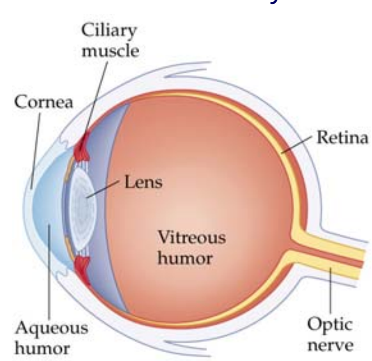
$$\sin\theta_{2b} = \frac{n_{1b}\sin\theta_1}{n_{2b}} = \frac{\sin 45^\circ}{1.332} \quad \theta_{2b} = 32.06^\circ$$

$$\boxed{\theta_{2r} - \theta_{2b} = 32.39^\circ - 32.06^\circ = 0.33^\circ}$$

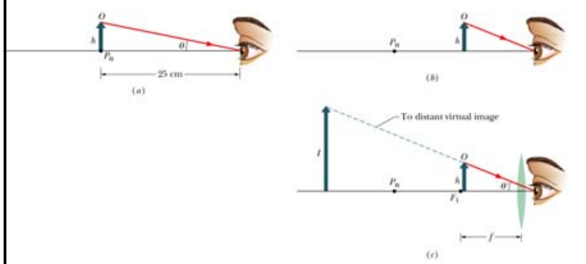
Part 6

Instruments

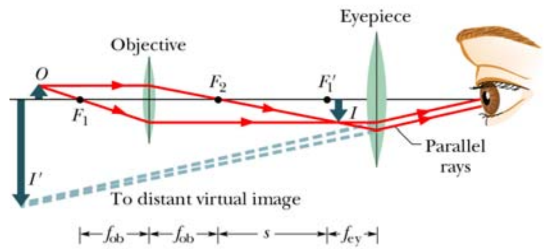
The Human Eye



Simple Magnifying Lens



Compound Microscope



Mirrors

Convex Mirror

Object location	Image orientation	Image size	Image type
Arbitrary	Upright	Reduced	Virtual

Concave Mirror

Object location	Image orientation	Image size	Image type
Beyond C	Inverted	Reduced	Real
C	Inverted	Same as object	Real
Between F & C	Inverted	Enlarged	Real
Just beyond F	Inverted	Approaching Infinity	Real
Just inside F	Upright	Approaching Infinity	Virtual
Between F & mirror	Upright	Enlarged	Virtual

Lenses

Concave Lens

Object location	Image orientation	Image size	Image type
Arbitrary	Upright	Reduced	Virtual

Convex Lens

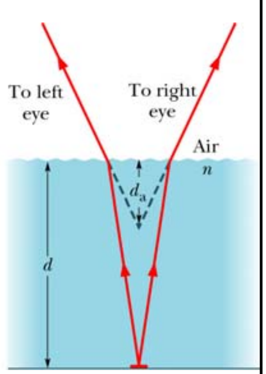
Object location	Image orientation	Image size	Image type
Beyond F	Inverted	Reduced or enlarg.	Real
Just beyond F	Inverted	Approaching Infinity	Real
Just inside F	Upright	Approaching Infinity	Virtual
Between F & lens	Upright	Enlarged	Virtual

You look downward at a coin that lies at the bottom of a pool of liquid with depth d and index of refraction n.

Because you view with two eyes, which intercept different rays of light from the coin, you perceive the coin to be where extensions of the intercepted rays cross, at depth d_a instead of d.

Assuming that the intercepted rays are close to a vertical axis through the coin, show that $d_a = d/n$.

(Hint: Use the small-angle approximation that $\sin\theta = \tan\theta = \theta$.)



?

Question

A concave mirror (sometimes surrounded by light) is often used as an aid for applying cosmetics to the face.

Why is such a mirror always concave rather than convex?

What considerations determine its radius of curvature?

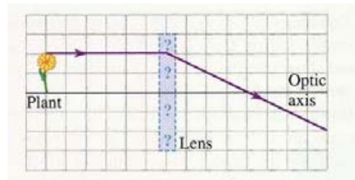
?

Question

A person looks at her reflection in the concave side of a shiny spoon.
Is this image right side up or inverted?
What does she see if she looks in the convex side?

problem

Figure shows a small plant near a thin lens. The ray shown is one of the principal rays for the lens. Each square is 1.0 cm along the horizontal direction, but the vertical direction is not to the same scale.
(a) Using only the ray shown, decide what type of lens this is.
(b) What is the focal length of the lens?
(c) Locate the image by drawing the other two principal rays,
(d) Calculate where the image should be, and compare with solution in (c).



problem

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