Interference and Diffraction
(wave nature of light)

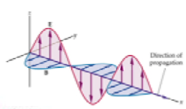
Chapter 26

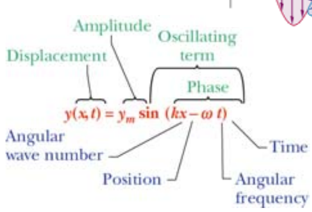
Part 0
Interference of two sinusoidal waves

Waves

EM waves – transverse waves (the E and B fields are perpendicular to the direction of travel)

Transverse waves





wavelength

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

period frequency

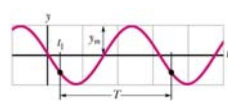
Snapshots of a traveling wave

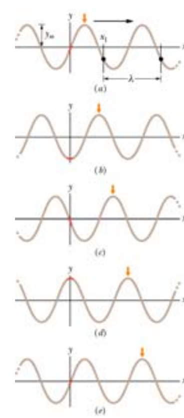
Wave traveling in a positive x direction at different moments of time

$$y(x,t) = y_m \sin(kx - \omega t)$$

$$y(x,t) = y_m \sin(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t)$$

Displacement as a function of time at x=0





interference of waves

The term interference refers to a situation when two or more waves overlap in space.

When this occur, the total displacement at any point at any instant of time is governed by the principle of superposition.

The principle of superposition

When two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at any point by the individual waves if each were presented alone

$$y_{net}(x,t) = \sum_n y_n \sin(k_n x - \omega_n t + \phi_n)$$

Interference of two sinusoidal waves

Special case: two sinusoidal waves of the same wavelength and amplitude.

$$y_1(x,t) = y_m \sin(kx - \omega t)$$

$$y_2(x,t) = y_m \sin(kx - \omega t + \phi)$$

From the principle of superposition

$$y_{net}(x,t) = y_1(x,t) + y_1(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$$

Using

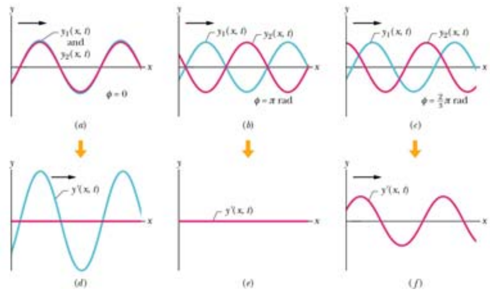
$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

The resultant wave

$$y_{net}(x,t) = \left[2 y_m \cos\frac{\phi}{2}\right]\sin(kx - \omega t + \phi/2)$$

Examples

$$y_{net}(x,t) = \left[2 y_m \cos\frac{\phi}{2}\right]\sin(kx - \omega t + \phi/2)$$



For a clear interference pattern: wave are coherent (constant relative phase ϕ)

Part 1

Interference of light

Monochromatic Light

interference of sinusoidal waves with the same frequency and wavelength

Since for light $c = f\lambda$

$$y(x,t) = y_m \sin\left(\frac{2\pi}{\lambda}(x - ct)\right)$$

in optics we need monochromatic light

(light of a single color)

Two waves

for two monochromatic waves

$$y_1(x,t) = y_m \sin\left(\frac{2\pi}{\lambda}(x - ct)\right)$$

$$y_2(x,t) = y_m \sin\left(\frac{2\pi}{\lambda}(x - ct + \delta)\right)$$

$$y_{net}(x,t) = 2 y_m \sin\left(\frac{2\pi}{\lambda}(x - ct + \frac{\delta}{2})\right)\cos\left(\frac{\pi}{\lambda}\delta\right)$$

for $\delta = m\lambda \qquad \cos\left(\frac{\pi}{\lambda}\delta\right) = \cos(m\pi) = \pm 1$

$\delta = (m + \frac{1}{2})\lambda \qquad \cos\left(\frac{\pi}{\lambda}\delta\right) = \cos\left(m\pi + \frac{\pi}{2}\right) = 0$

Two waves coming from two points

for two monochromatic waves from points r_1 and r_2

$$y_1(x,t) = y_m \sin\left(\frac{2\pi}{\lambda}(r_1 + x - ct)\right)$$

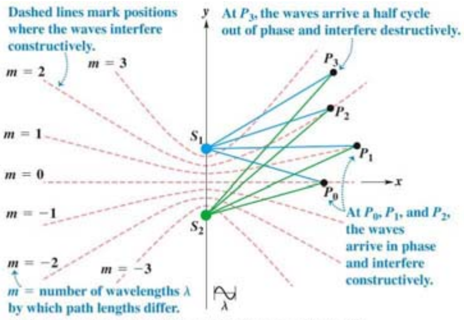
$$y_2(x,t) = y_m \sin\left(\frac{2\pi}{\lambda}(r_2 + x - ct)\right)$$

$$y_{net}(x,t) = 2 y_m \sin\left(\frac{2\pi}{\lambda}(x - ct + \frac{r_2 + r_1}{2})\right)\cos\left(\frac{\pi}{\lambda}(r_2 - r_1)\right)$$

for $r_2 - r_1 = m\lambda \qquad \cos\left(\frac{\pi}{\lambda}(r_2 - r_1)\right) = \cos(m\pi) = \pm 1$

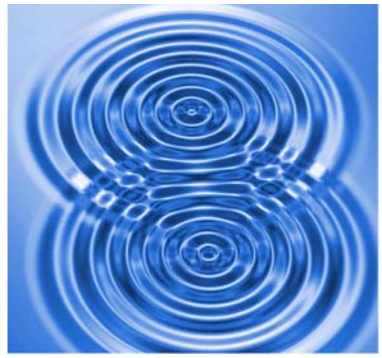
$r_2 - r_1 = (m + \frac{1}{2})\lambda \qquad \cos\left(\frac{\pi}{\lambda}(r_2 - r_1)\right) = \cos\left(m\pi + \frac{\pi}{2}\right) = 0$

Two waves coming from two points



(a) Interference of waves from two monochromatic sources S_1 and S_2 that are in phase and equidistant from the origin

Interference of two waves on a water surface



Constrictive and Destructive interference

Constrictive interference of two waves arriving at a point occurs when the path difference from the two sources is an integer number of wavelengths:

$$r_2 - r_1 = m\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$

Destructive interference of two waves arriving at a point occurs when the path difference from the two sources is a half-integer number of wavelengths:

$$r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$

monochromatic light (cont.)

Most common sources of light do not emit monochromatic light (rather a continuous distribution of wavelength)

However, lasers, some discharge lams, filters emit light in a very narrow band of wavelengths

Coherence

For equations above to hold, the two sources must always be coherent (constant relative phase δ)

$$y_{net}(x,t) = 2y_m \sin\left(\frac{2\pi}{\lambda}(x - ct + \frac{\delta}{2})\right)\cos\left(\frac{\pi}{\lambda}\delta\right)$$

(real interference includes many waves)

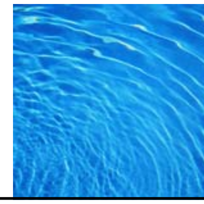
Usually beams of light emitted from two sources have no definite phase relation to each other.

The distinguished feature of light from lasers is that the emission of light from many atoms is synchronized in frequency and phase.

For observing constructive and destructive interference

we need monochromatic and coherent light

otherwise we observe a chaotic interference without clear pattern



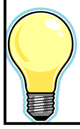
Producing Coherent Sources

Old method

Light from a monochromatic source is allowed to pass through a narrow slit

The light from the single slit is allowed to fall on a screen containing two narrow slits

The first slit is needed to insure the light comes from a tiny region of the source which is coherent



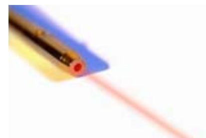
Producing Coherent Sources, cont

New method

Currently, it is much more common to use a laser as a coherent source

The laser produces an intense, coherent, monochromatic beam over a width of several millimeters

The laser light can be used to illuminate multiple slits directly

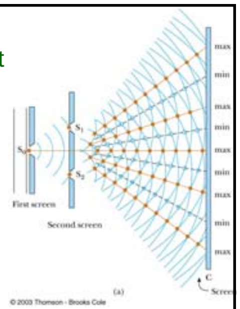


Young's Double Slit Experiment

Thomas Young - interference in light waves from two sources (1801)

Light is incident on a screen with a narrow slit.

The waves emerging from the two next slits originate from the same wave front and therefore *are always in phase*

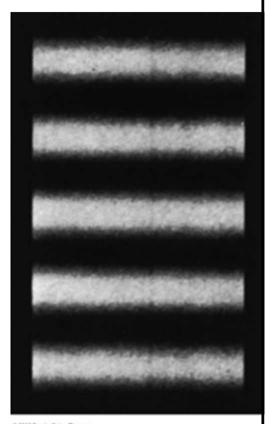


Fringe Pattern

The fringe pattern formed from a Young's Double Slit Experiment

The bright areas represent constructive interference

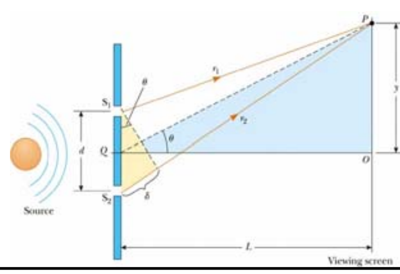
The dark areas represent destructive interference



Interference Equations

The path difference, δ, is found from the triangle

$\delta = r_2 - r_1 = d \sin\theta$



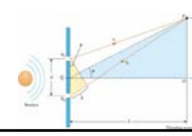
Constructive and destructive interference: two slits

Constructive interference occur at angles for which

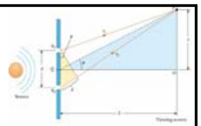
$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$

Destructive interference (cancellation) occurs, forming dark regions, when the path difference is

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$



Young's experimnts



Constructive interference on a screen

$$y_m = R\frac{m\lambda}{d} \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$

Dark fringes

$$y_m = R\frac{\left(m + \frac{1}{2}\right)\lambda}{d} \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$

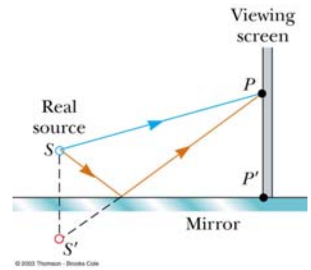
where R is the distance to the screen and d is the distance between slits

Lloyd's Mirror

An arrangement for producing an interference pattern with a single light source

Wave reach point P either by a direct path or by reflection

The reflected ray can be treated as a ray from the source S' behind the mirror

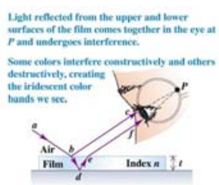


Interference in Thin Films

Interference effects are commonly observed in thin films

Examples are soap bubbles and oil on water

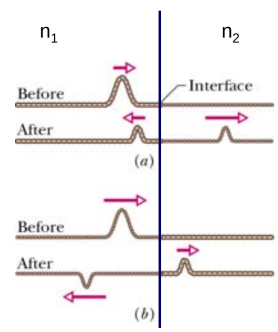
The interference is due to the interaction of the waves reflected from both surfaces of the film



Comment: Phase Changes Due To Reflection

There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction

An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling



Interference in Thin Films, 2

Facts to remember

- An electromagnetic wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change on reflection when $n_2 > n_1$
  - There is no phase change in the reflected wave if $n_2 < n_1$
- The wavelength of light λ_n in a medium with index of refraction n is $\lambda_n = \lambda/n$ where λ is the wavelength of light in vacuum

Interference in Thin Films, 3

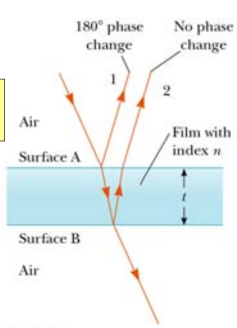
equations

constructive interference

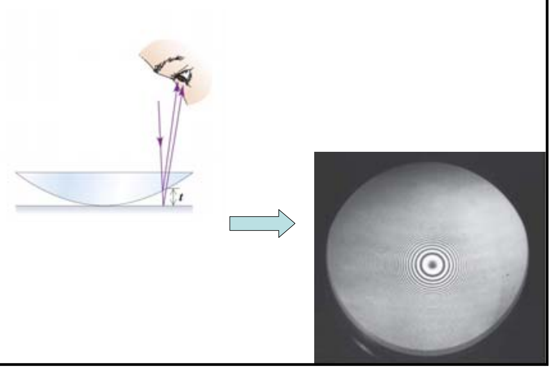
$$2l = (m + \frac{1}{2})\frac{\lambda}{n_2} \quad (m = 0, \pm 1, \pm 2, \ldots)$$

destructive interference

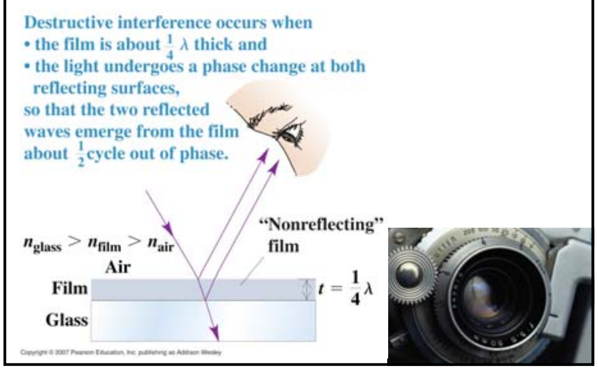
$$2l = m\frac{\lambda}{n_2} \quad (m = 0, \pm 1, \pm 2, \ldots)$$



Newton's Rings



Nonreflective coating

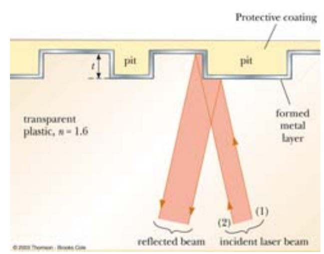


CD's and DVD's

A series of ones and zeros read by laser light reflected from the disk

Reading a CD

The pit depth is made equal to one-quarter of the wavelength of the light



Reading a CD, cont

When the laser beam hits a rising or falling bump edge, part of the beam reflects from the top of the bump and part from the lower adjacent area

The bump edges are read as ones

The flat bump tops and intervening flat plains are read as zeros

DVD's

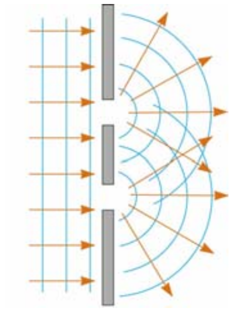
DVD's use shorter wavelength lasers

- The track separation, pit depth and minimum pit length are all smaller
- Therefore, the DVD can store about 30 times more information than a CD
- Blue ray DVD – more GB

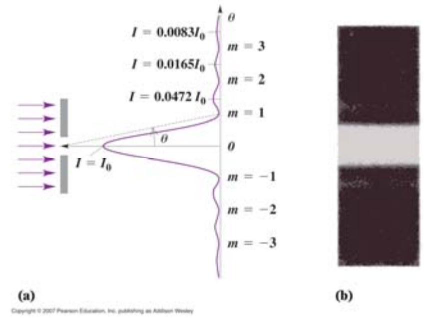
Diffraction

Huygen's principle requires that the waves spread out after they pass through slits

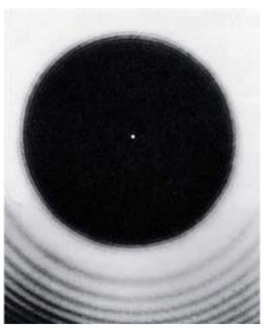
This spreading out of light from its initial line of travel is called diffraction



Diffraction from a single slit



Diffraction from a disk (example – steel ball)

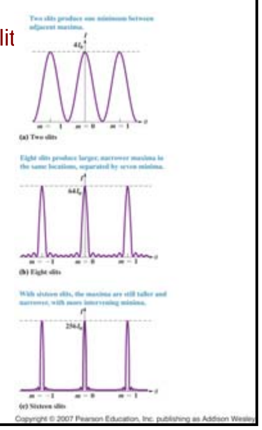


Diffraction Grating (multiple slit

The diffracting grating consists of many equally spaced parallel slits

- A typical grating contains several thousand lines per centimeter

The intensity of the pattern on the screen is the result of the combined effects of interference and diffraction

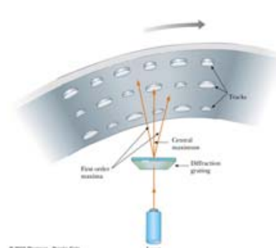


Diffraction Grating in CD Tracking

A diffraction grating can be used in a three-beam method to keep the beam on a CD on track

The central maximum of the diffraction pattern is used to read the information on the CD

The two first-order maxima are used for steering



Polarization of Light Waves

Each atom produces a wave with its own orientation of E

All directions of the electric field vector are equally possible and lie in a plane perpendicular to the direction of propagation

This is an unpolarized wave