Relativity

Chapter 27

Part 1

Special theory of relativity

Historical Background

By the beginning of 20^{th} century – experimental observation that Newtonian's mechanics was not able to explain (Michelson's and Morley's experiment)

Most important steps:

Hendrik Lorentz (Lorenz transformation)
Jules Henri Poincaré (invariance of Maxwell's equations)
Hermann Minkowski (Minkowski space)
Albert Einstein (Special theory of relativity)

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Special Theory of Relativity

The theory deals with only with inertial reference frames

An inertial reference frame of reference is one in which Newton's Laws are valid.

The frame of reference for a person or object is the coordinate system which moves with the person or with the object.

General Theory of Relativity deals with reference frames which accelerate.

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What is so special (difficult) about the special theory of relativity?

- ✓ The theory can contradict our experience
- ✓ Space and time are entangled (the time between two events depends how far apart they occur) and vice versa
- ✓ The entanglement is different for observers who move relative to each other
- ✓ Time does not pass at a fixed rate!
- ✓ Relative motion can change the rate at which time passes
- ✓ Newtonian mechanics is a special case of the special theory of relativity

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The most difficult

- ✓ Who measures what about an event
- ✓ How that measurement is made

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The Postulates of Relativity

The relativity postulate:
The laws of physics are the same for observers in all inertial reference frames. No frame is preferred

The speed of light postulate:
The speed of light in vacuum has the same value c in all directions and in all inertial reference frames

c = 299 792 458 m/s

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Testing the speed of light postulate

Numerous experiments: $\pi^0 \rightarrow \gamma + \gamma$

CERN (European Center for Nuclear Research) – 1964:
Neutral pion decay
Beam of pions moving at a speed of 0.99975c with respect to the laboratory. Speed of light was the same!

PROVED!

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Part 2

The Relativity of Time and Time Dilation

Measuring an event

An event is something that happens, to which an observer can assign three space coordinates and one time coordinate

A given event may be recorded by any number of observers, each in a different reference frame

In general different observers will assign different space-time coordinates for the same event.

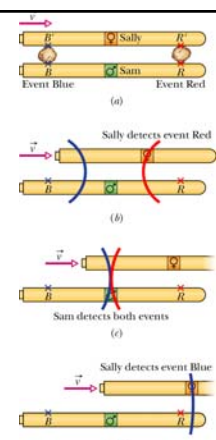
The Relativity of Simultaneity

If two observers are in relative motion, they will not, in general, agree as to whether two events are simultaneous. If one observer finds them to be simultaneous, the other generally will not

Simultaneity is not an absolute concept but a relative one, depending on the motion of the observer

A Closer Look at Simultaneity

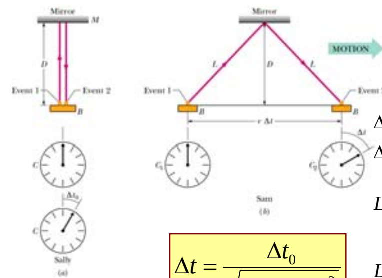
Two space ships and two meteors



The Relativity of Time

The time interval between two events depends on how far apart they occur, in both space an time; that is their spatial and temporal separations are entangled

The Relativity of Time: example



Sally – on the train

Sam – watching from the station

$$\Delta t_0 = 2D/c \quad (Sally)$$

$$\Delta t = 2L/c \quad (Sam)$$

$$L = \sqrt{\left(\frac{1}{2}v\Delta t\right)^2 + D^2}$$

$$L = \sqrt{\left(\frac{1}{2}v\Delta t\right)^2 + \left(\frac{1}{2}c\Delta t_0\right)^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}$$

the key – the speed of light is the same for both observers

The Proper Time

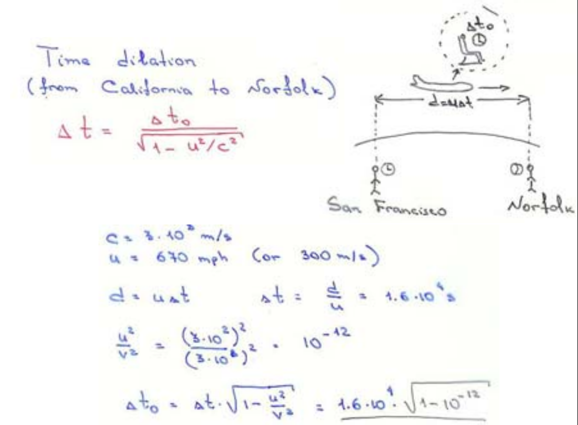
$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}$$

When two events occur at the same location in an inertial reference frame, the time interval between them, measured in that frame, is called the proper time interval or the proper time. Measurements of the same time interval from any other inertial reference frame are always greater.

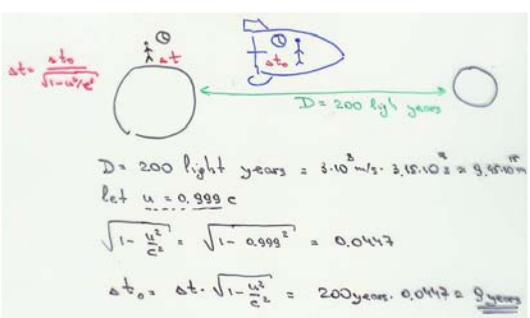
The amount by which a measured time interval is greater than the corresponding proper time interval is called time dilation.

Experiments with macroscopic clocks
a) flying with atomic clocks twice around the world.
b) flying with atomic clocks around Chesapeake bay

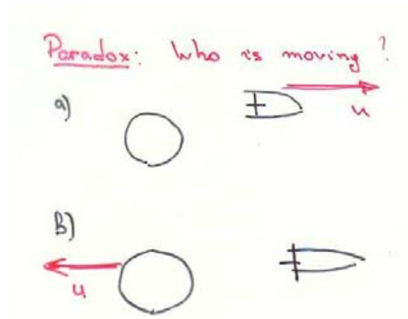
Proved!



Space Travel and the Twin Paradox



Space Travel and the Twin Paradox



Question

The spaceship U.S.S. Enterprise, traveling through the galaxy, sends out a smaller explorer craft that travels to a nearby planet and signals its findings back. The proper time for the trip to the planet is measured by clocks:

A) on board the Enterprise
B) on board the explorer craft
C) on Earth
D) at the center of the galaxy
E) none of the above

Ans. B

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Question

As we watch, a spaceship passes us in time t. The crew of the spaceship measures the passage time and finds it to be t'. Which of the following statements is true?

A) t is the proper time for the passage and it is smaller t'
B) t is the proper time for the passage and it is greater than t'
C) t' is the proper time for the passage and it is smaller than t
D) t' is the proper time for the passage and it is greater than t
E) None of the above statements are true.

Ans. C

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Question

Spaceship A, traveling past us at 0.7c, sends a message capsule to spaceship B, which is in front of A and is traveling in the same direction as A at 0.8c relative to us. The capsule travels at 0.95c relative to us. A clock that measures the proper time between the sending and receiving of the capsule travels:

A) in the same direction as the spaceships at 0.7c relative to us
B) in the opposite direction from the spaceships at 0.7c relative to us
C) in the same direction as the spaceships at 0.8c relative to us
D) in the same direction as the spaceships at 0.95c relative to us
E) in the opposite direction from the spaceships at 0.95c relative to us

Ans. D

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Question

A millionairess was told in 1992 that she had exactly 15 years to live. However, if she travels away from the Earth at 0.8 c and then returns at the same speed, the last New Year's day the doctors expect her to celebrate is:

A) 2001
B) 2003
C) 2007
D) 2010
E) 2017

Ans. E

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Problem

You wish to make a round trip from Earth in a spaceship, traveling at constant speed in a straight line for 6 months and then returning at the same constant speed.
You wish further, on your return, to find Earth as it will be 1000 years in the future.
(a) How fast must you travel?
(b) Does it matter whether you travel in a straight line on your journey? If, for example, you traveled in a circle for 1 year, would it still find 1000 years had elapsed by Earth clock when you returned?

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The round-trip (discounting the time needed to "turn around") should be one year according to the clock you are carrying (this is your proper time interval Δt_0) and 1000 years according to the clocks on Earth which measure Δt.

$$v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = (299792458\,\mathrm{m/s})\sqrt{1-\left(\frac{1\,\mathrm{y}}{1000\,\mathrm{y}}\right)^2} = 299792308\ \mathrm{m/s}$$

$$v = c\sqrt{1-(1000)^{-2}} = 0.999\,999\,50c.$$

The equations do not show a dependence on acceleration (or on the direction of the velocity vector), which suggests that a circular journey (with its constant magnitude centripetal acceleration) would give the same result (if the speed is the same) as the one described in the problem.

it should be admitted that this is a fairly subtle question which has occasionally precipitated debates among professional physicists.

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Part 3

The Relativity of Length and Length Contraction

The Relativity of Length

The length L_0 of an object measured in the rest frame of the object is its proper length or rest length. Measurements of the length from any reference frame that is in relative motion parallel to that length are always less that the proper length

$$L = L_0\sqrt{1-(v/c)^2}$$

Proof: Sam and Sally measure the length of the station platform.

Sam and Sally measure the length of the station platform.

Sally on a train moving through a station
Sam on the station platform
Sam's result (proper length) L_0
for Sam Sally moves through this length in a time $L_0 = v\Delta t$
for Sally the measurement is at the same place but different times $L = v\Delta t_0$

using $$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}$$

we get $$L = L_0\sqrt{1-(v/c)^2}$$

Question

A measurement of the length of an object that is moving relative to the laboratory consists of noting the coordinates of the front and back:

A) at different times according to clocks at rest in the laboratory
B) at the same time according to clocks that move with the object
C) at the same time according to clocks at rest in the laboratory
D) at the same time according to clocks at rest with respect to the fixed stars
E) none of the above

Ans. C

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Question

A certain automobile is 6 m long if at rest. If it is measured to be 4/5 as long, its speed is:

A) 0.1c
B) 0.3c
C) 0.6c
D) 0.8c
E) > 0.95c

Ans. C

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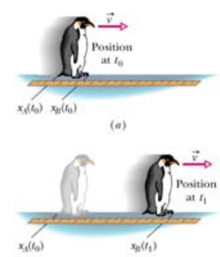
Length contraction

Length contraction occurs only along the direction of relative motion!!!

$$L = L_0\sqrt{1-(v/c)^2}$$



Problem

A cubical box is 0.50 m on a side.

(a) What are the dimensions of the box as measured by an observer moving with a speed of 0.88c parallel to one of the edges of the box?

(b) What is the volume of the box as measured by this observer?

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Only one side of the box is "contracted."

$$\Delta L = \Delta L_P \sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$(a) \quad \Delta L = 0.5\sqrt{1-(0.88)^2} = \boxed{0.24\ \text{m}}$$

The "observed" dimensions are $0.24 \times 0.50 \times 0.50$.

$$(b) \quad \boxed{Vol = 0.24 \times 0.50 \times 0.50 = 0.059\ \text{m}^3}$$

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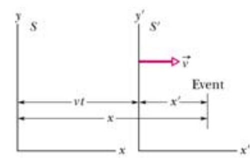
Part 4

The Lorentz Transformation

and some consequences



Galilean Transformation	Lorentz Transformation
$x' = x - vt$	$x' = \dfrac{x - vt}{\sqrt{1 - v^2/c^2}}$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$t' = t$	$t' = \dfrac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$

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The Lorentz transformation equations are valid at all possible physical speeds

The Lorentz transformation equations for pairs of events
Frame S' moves at velocity v relative to frame S

1. $\Delta x = \gamma(\Delta x' + v\Delta t')$ 1'. $\Delta x' = \gamma(\Delta x - v\Delta t)$
2. $\Delta t = \gamma(\Delta t' + v\Delta x'/c^2)$ 2'. $\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

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Some consequences of the Lorentz equations

Time Dilation $\quad \Delta t = \dfrac{\Delta t_0}{\sqrt{1-(v/c)^2}}$

Length Contraction $\quad L = L_0\sqrt{1-(v/c)^2}$

Simultaneity $\quad \Delta t = \dfrac{1}{\sqrt{1-(v/c)^2}}\dfrac{v\Delta x'}{c^2}$

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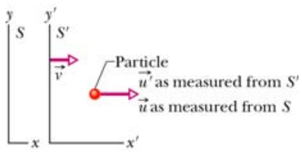
The relativity of velocities

Follows from the Lorentz equations



$$u = \frac{u' + v}{1 + u'v/c^2}$$

$$u' = \frac{u - v}{1 - uv/c^2}$$

if S' moves to the left, then replace v on -v

Problem

Galaxy A is reported to be receding from us with a speed of 0.35c. Galaxy B, located in precisely the opposite direction, is also found to be receding from us at the same speed.

What recessional speed would an observer on Galaxy A find

(a) for our Galaxy

(b) for Galaxy B

One thing Einstein's relativity has in common with the more familiar (Galilean) relativity is the reciprocity of relative velocity. If we see Galaxy A moving away from us at 0.35c then an observer in Galaxy A should see our galaxy move away from him at 0.35c.

We take the positive axis to be in the direction of motion of Galaxy A, as seen by us. The problem indicates v = +0.35c (velocity of Galaxy A relative to Earth) and u = −0.35c (velocity of Galaxy B relative to Earth). We solve for the velocity of B relative to A:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{(-0.35c) - 0.35c}{1 - (-0.35)(0.35)} = -0.62c$$

Problem

An armada of spaceships that is 1.0 yl long (in its rest frame) moves with the speed 0.8c relative to ground station S. A messenger travels from the rear of the armada to the front with a speed of 0.950c relative to S.

How long does the trip take as measured

(a) in the messenger's rest frame

(b) in the armada's rest frame

(c) by an observer in frame S

(a) In the messenger's rest system (called S_m), the velocity of the armada is

$$v' = \frac{v - v_m}{1 - vv_m/c^2} = \frac{0.80c - 0.95c}{1 - (0.80c)(0.95c)/c^2} = -0.625c \ .$$

The length of the armada as measured in S_m is

$$L_1 = \frac{L_0}{\gamma_{v'}} = (1.0\,\text{ly})\sqrt{1 - (-0.625)^2} = 0.781 \text{ ly} \ .$$

Thus, the length of the trip is

$$t' = \frac{L'}{|v'|} = \frac{0.781\,\text{ly}}{0.625c} = 1.25 \text{ y} \ .$$

(b) In the armada's rest frame (called S_a), the velocity of the messenger is

$$v' = \frac{v - v_a}{1 - vv_a/c^2} = \frac{0.95c - 0.80c}{1 - (0.95c)(0.80c)/c^2} = 0.625c \ .$$

Now, the length of the trip is

$$t' = \frac{L_0}{v'} = \frac{1.0\,\text{ly}}{0.625c} = 1.6 \text{ y} \ .$$

(c) Measured in system S, the length of the armada is

$$L = \frac{L_0}{\gamma} = 1.0\,\text{ly}\sqrt{1 - (0.80)^2} = 0.60 \text{ ly} \ ,$$

so the length of the trip is

$$t = \frac{L}{v_m - v_a} = \frac{0.60\,\text{ly}}{0.95c - 0.80c} = 4.0 \text{ y} \ .$$

A new look at momentum

If we continue to define the momentum as $p = mv$
then total momentum is **not** conserved for the observers in different inertial frames.

Relativistic momentum

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv$$

A new look at energy

In chemistry – conservation of mass

Einstein – mass can be considered to be another form of energy!

The law of conservation of energy is really the law of conservation of mass-energy

A new look at energy

Mass energy (or rest energy)

$$E_0 = mc^2$$

Total energy

$$E = E_0 + K = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

Kinetic energy

$$K = E - mc^2 = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)$$

A new look at energy

The total energy of an isolated system can't change

Nuclear or chemical reaction with change of mass

$$Q = M_i c^2 - M_f c^2$$

Matter and Antimatter -> annihilation

Relativity in our life

There is … but not as much as we may expect.
(a story about Rutherford)

Newtonian mechanics is an approximation but a very practical one

- ✓ Nuclear energy
- ✓ GPS technology
- ✓ Radioactive elements in technology
- ✓ Radioactive elements in medicine
- ✓ …

What if the speed of light was about 55 mph?

General Theory of Relativity

Accelerated frames
Gravitation

Time and gravitation

Distortion of space - Bending of light

Black holes (gravitation can trap light)