

1968), Chap. 8; D. Park, *Introduction to the Quantum Theory* (McGraw-Hill, New York, 1964), Chap. 4.

³See, e.g., in Ref. 2, Merzbacher, Chap. 6; Schiff, Chap. 2; Park, Chap. 4.

⁴L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, London, 1958), Chap. III.

⁵B. R. Holstein and A. R. Swift, *Am. J. Phys.* **50**, 829 (1982); **50**, 833 (1982).

⁶There may, of course, exist other branch points in the complex plane, but they need not concern us here.

⁷A. R. Swift, University of Massachusetts Rept. No. UMHEP-168 (1982); N. Froman and P. Froman, *JWKB Approximation* (North-Holland, Amsterdam, 1965).

⁸See, e.g., D. W. McLaughlin, *J. Math. Phys.* **13**, 1099 (1972); M. V. Berry

and K. E. Mount, *Rep. Prog. Phys.* **35**, 315 (1972).

⁹A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1965), Chap. VI, Sec. II; Merzbacher, Ref. 2; D. Park, *Introduction to the Quantum Theory* (McGraw-Hill, New York, 1964), Chap. 4.

¹⁰See, e.g., M. V. Berry and K. E. Mount, *Rep. Prog. Phys.* **35**, 315 (1972).

¹¹Note that contour C_0 must pass through the branch cut twice in order that x_1, x_2 remain on the same sheet. Also, the contour is always ascending on the first sheet, descending on the second in order that $T < 1$.

¹²As an additional exercise it is useful to apply these semiclassical techniques to the potential $V(x) = V_0 \text{sech}^2 ax$, whose exact solution is quoted in Ref. 4.

¹³*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (National Bureau of Standards, Washington, DC, 1964).

Aerodynamic drag crisis and its possible effect on the flight of baseballs

Cliff Frohlich

Institute for Geophysics, University of Texas at Austin, Austin, Texas 78712

(Received 1 April 1983; accepted for publication 12 July 1983)

At Reynolds numbers above about 10^5 the aerodynamic drag force on a sphere drops sharply as the flow begins to become turbulent in the boundary layer. For baseballs, this "drag crisis" may occur at speeds which are typical for pitched or batted balls. The effects of the drag reduction on the behavior of both pitched and batted balls is significant, and may explain several features of the game of baseball which previously have been unexplained or attributed to other causes. In particular, the drag reduction may help to explain why pitched fastballs appear to rise, why pitched curve balls appear to drop sharply, and why home run production has increased since the introduction of the alleged "lively ball." Calculations suggest that aerodynamic forces are as important a factor in fastpitch softball as in baseball, and that they are a critical factor in a number of other ball games.

I. INTRODUCTION

It is now firmly established that aerodynamic forces significantly affect pitched baseballs. Indeed, without these forces it would be impossible to throw a ball that curves or "knuckles," i.e., changes direction sharply sideways after it has left the pitcher's hand. Although once a matter of controversy,¹⁻⁴ both baseball players and physicists now agree that the ball does deviate from a "straight" path,^{5,6} i.e., the path followed by a ball affected only by the forces of gravity and aerodynamic drag. Perhaps the most comprehensive study is that of Briggs,⁷ who showed that baseballs can curve up to about 29 cm from a straight path because the rotation of the ball causes pressure imbalances between the left and right side of the ball. Furthermore, Watts and Sawyer^{8,9} explained that the erratic trajectory of knuckle balls occurs because of asymmetry in the pressure field around the ball caused by the presence of the stitching on the baseball.

The purpose of the present paper is to suggest that the behavior of pitched and hit baseballs may be strongly affected by the change in drag regime which occurs when the Reynolds number exceeds about 10^5 . The Reynolds number R is defined as

$$R = Vd/\nu.$$

Here, d is the diameter of the baseball (7.32 cm), V is its

velocity relative to the air, and ν is the kinematic viscosity of air (about $0.000015 \text{ m}^2/\text{s}$ at 20°C). This "drag crisis"¹⁰ occurs when the laminar flow of air in a boundary layer near the ball begins to separate and become turbulent. The ultimate effect of this turbulence in the boundary layer is to reduce the size of the turbulent wake behind the ball, and hence reduce the drag force. Generally, the drag force F_d is characterized in terms of drag coefficient C_d

$$F_d = -\frac{1}{2}\rho C_d A V^2, \quad (1)$$

where ρ is the fluid density of air (1.29 kg/m^3) and $A = \pi d^2/4$ is the cross-sectional area of the ball. At the drag crisis the drag coefficient C_d may drop by a factor of 2 to 5 as the velocity increases by a factor of less than 2 (see Fig. 1). Although several researchers have discussed the effect of the sudden drag reduction on the behavior of golf balls,¹¹⁻¹⁴ the author is not aware of any previous study which investigates the effect on baseballs.

The Reynolds number at which turbulence occurs in the boundary layer and causes the drag reduction depends strongly on the roughness of the sphere's surface.^{15,16} Generally the drag reduction will occur at lower Reynolds numbers as the surface roughness increases (Fig. 2). For golf balls, this has the effect that balls with roughened or dimpled surfaces can be driven considerably further than balls with smooth surfaces.¹³ This has been known for some time.¹ Indeed, partly because there exist strong eco-

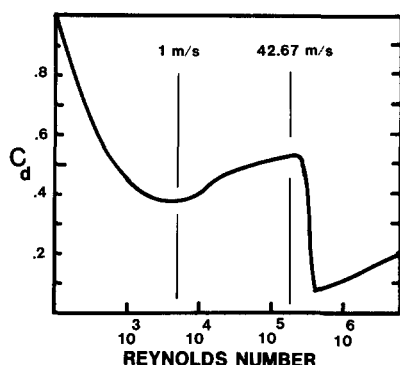


Fig. 1. Drag coefficient C_d of a smooth sphere versus Reynolds number R (Refs. 16, 10). Note especially the "drag crisis," the sharp decrease in drag that occurs at about $R = 4 \times 10^5$. The vertical lines correspond to velocities of 1 m/s and 42.67 m/s for a smooth sphere with the diameter of a baseball. 42.67 m/s is the terminal velocity of a baseball as measured by Briggs,⁷ and is approximately the velocity of the fastest major league pitchers. Most previous investigators have assumed that a baseball is aerodynamically smooth, and that $C_d = 0.5$ for all Reynolds numbers.

nomic incentives for producing a better golf ball, there has been considerable research concerning the effect that the surface has on the drag and lift of golf balls.¹¹⁻¹⁴

The aerodynamic conditions affecting baseballs are similar enough to those affecting golf balls that it is reasonable to expect that the drag crisis affects the behavior of baseballs as well. Baseballs and golf balls in flight have Reynolds numbers of about 1×10^5 to 2×10^5 (see Table I). In addition, since the radial bumpiness on a baseball caused by the seams is roughly $k = 0.5$ mm (Fig. 3), the effective roughness k/d is about 700×10^{-5} . This is slightly less than that for a golf ball ($k/d = 900 \times 10^{-5}$, as reported by Bearman and Harvey¹⁴). However, it is considerably larger than the surface roughness necessary to strongly affect

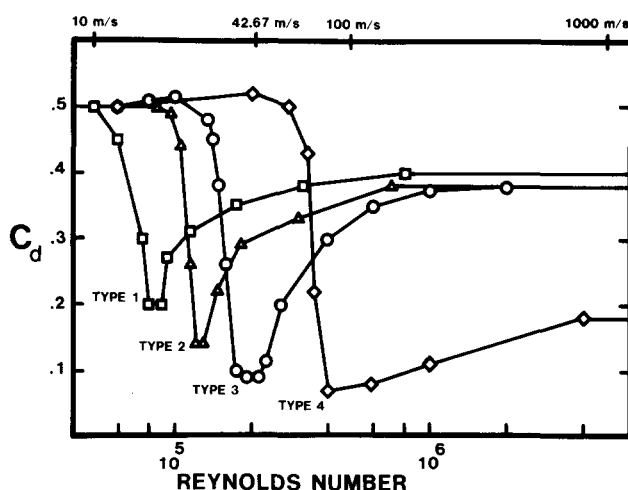


Fig. 2. Effect of the surface roughness of a sphere on drag coefficient C_d at Reynolds numbers near the drag crisis. Surface roughness is parameterized by k/d where k is the height of the roughness elements, and d is the sphere diameter. The roughness for the various spheres shown are type 1 ball— $k/d = 1250 \times 10^{-5}$; type 2 ball— $k/d = 500 \times 10^{-5}$; type 3 ball— $k/d = 150 \times 10^{-5}$; type 4 ball—smooth sphere; and type 5 ball— $C_d = 0.5$ for all R . The top horizontal scale shows the equivalent air speed for a sphere with the diameter of a baseball. The plotted symbols are from Achenbach.¹⁶ The lines connecting measured values show values used for C_d for the calculations reported in this paper.

drag¹⁶ (Fig. 2). Briggs⁷ reported that a baseball could be suspended in the airstream of a vertical wind tunnel when the air speed was about 140 ft/s (42.67 m/s). From Eq. (1), since at terminal velocity the drag force F equals mass of baseball (0.145 kg) times the acceleration of gravity, one finds that $C_d = 0.29$, clearly beyond the onset of the drag crisis for a smooth sphere (see Fig. 1).

Table I. Representative speeds for various balls used in sports, and calculated values of the Reynolds number and ratio a/g of aerodynamic and gravitational forces. For uniformity, even for balls with large reported Reynolds numbers, in this table the author assumes $C_d = 0.5$ when calculating the aerodynamic force. Numbered sources are references from the reference list.

Type of ball	Reported speed (m/s)	Diameter (cm)	Mass (kg)	Reynolds number ($\times 10^5$)	a/g	Source	Comments
Baseball	42.67	7.32	0.145	2.08	1.74	7	Terminal velocity in wind tunnel
Basketball	9.0	24.26	0.600	1.46	0.21	44	Calculated for 25-ft jump shot
Bowling	7.76	21.8	7.27	1.13	0.01	45	Release speed of expert
Golf	61.0	4.26	0.046	1.73	3.80	13	Moderately long drive by pro
Jai alai	67.0	5.08	0.139	2.26	2.30	...	Original source unavailable
Shot put	14.02	11.0	7.27	1.03	0.01	24	World record performance
Soccer	29.1	22.2	0.454	4.31	2.38	55	
Softball	44.2	9.70	0.188	2.86	2.53	40	Ball pitched by very fast professional
Table tennis	4.27	3.8	0.0025	0.11	0.27	39	Forehand drive by expert
Tennis	45.15	6.5	0.058	1.96	3.84	56	Serve of top professionals
Volleyball	30.26	21.0	0.270	4.23	3.86	57	Very hard spike by male college player

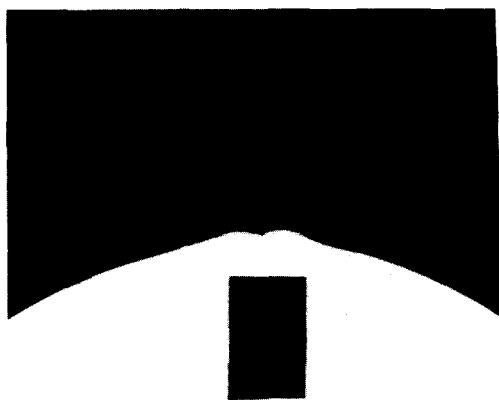


Fig. 3. Shadow cast by the seams of a baseball, showing that the roughness height k is about 0.5 mm. To prepare this figure, the author used an enlarger to cast a shadow of a Rawlings official National League baseball on photographic paper, which was photographed and enlarged. The largest dimension of the black rectangle beneath the seam is 10 mm.

The purpose of the present paper is to speculate on how the observed drag reduction affects the behavior of pitched and hit baseballs. Since no precise measurements of drag coefficient for a baseball are available, the author wrote a computer program to calculate the trajectory of five different types of balls, each with the diameter and mass of a baseball and with drag coefficient which varies in different ways as the Reynolds number increases. For the drag coefficients, the author chose the measurements of Achenbach¹⁶ for spheres of varying roughness. The "type 1" ball had the highest degree of roughness, with the "type 2," "type 3," and "type 4" (smooth sphere) corresponding to spheres of lesser roughness. For the "type 5" ball, C_d was assigned to be constant at 0.5 over the entire range of Reynolds numbers, as has been assumed in a previous analysis of baseball aerodynamics.¹⁷ Because the primary concern in this paper is with the effects of drag reduction on the trajectory of balls, in the calculations we will ignore the effects of rotation of the ball on the trajectory, i.e., we assume that the lift is zero and there is no lateral deflection.

II. NUMERICAL METHODS

From Eq. (1), the components of acceleration in the horizontal and vertical directions can be expressed in terms of the velocity components \dot{x} and \dot{y} :

$$\begin{aligned}\ddot{x} &= -\frac{1}{2} \frac{\rho A}{M} C_d V \dot{x}, \\ \ddot{y} &= -\frac{1}{2} \frac{\rho A}{M} C_d V \dot{y} - g.\end{aligned}\quad (2)$$

These equations are not integrable for most normally encountered functional forms of C_d (Ref. 18) and thus trajectories must be calculated numerically. As noted by Frohlich,¹⁹ the trajectories are relatively robust if the numerical integration is performed at regular intervals of time Δt , and if the third derivative of position is calculated in terms of the velocity components. In particular, if $B = \rho A / 2M$,

$$\begin{aligned}\ddot{x} &= -B \left(\frac{\partial C_d}{\partial V} \dot{v} \dot{x} + C_d \dot{v} \dot{x} + C_d v \ddot{x} \right), \\ \ddot{y} &= -B \left(\frac{\partial C_d}{\partial V} \dot{v} \dot{y} + C_d \dot{v} \dot{y} + C_d v \ddot{y} \right),\end{aligned}\quad (3)$$

where

$$\dot{v} = (\dot{x}\ddot{x} + \dot{y}\ddot{y})/v.$$

Thus if the position (x, y) and the velocity (\dot{x}, \dot{y}) are known at any point in the trajectory then we can calculate the change in velocity and position after a time Δt in terms of a Taylor series expansion

$$\Delta x = \dot{x}\Delta t + \frac{1}{2}\ddot{x}\Delta t^2 + \frac{1}{6}\dddot{x}\Delta t^3,$$

$$\Delta \dot{x} = \ddot{x}\Delta t + \frac{1}{2}\dddot{x}\Delta t^2,$$

and similarly for Δy and $\Delta \dot{y}$.

In each time interval the drag coefficient and its partial derivative were determined using the values reported by Achenbach¹⁶ and a straightforward semilog interpolation scheme. In particular, at a velocity V corresponding to a Reynolds number R between the Reynolds numbers R_1 and R_2 where drag coefficient values C_1 and C_2 are plotted in Fig. 2,

$$C_d = C_1 + (C_2 - C_1) \frac{\ln R - \ln R_1}{\ln R_2 - \ln R_1}$$

and

$$\frac{\partial C_d}{\partial V} = \frac{\partial C_d}{\partial R} \frac{\partial R}{\partial V} = \frac{d}{vR} \frac{C_2 - C_1}{\ln R_2 - \ln R_1}.$$

Using a FORTRAN program written to implement the above scheme, the author obtained satisfactory results with $\Delta t = 0.003$ s. In particular, tests of the effect of Δt on the computed range of baseballs "hit" with the above algorithm showed that changing Δt from 0.003 to 0.001 s never changed the calculated range by more than 1 cm, or less than about 0.01%. For baseballs "pitched" with the above algorithm a Δt of 0.003 s was used.

III. TRAJECTORY CALCULATIONS FOR PITCHED BASEBALLS

To investigate the effect on pitched baseballs of variations in C_d and in initial velocity, the author calculated trajectories for five different functional types of the drag coefficient C_d . Although the distance from the pitcher's rubber to the plate is 60 ft, 6 in. (18.44 m), cinematographic studies by Selin⁴ found that typically the ball is released by the pitcher about 56 ft (17 m) from the plate, and about 7 ft (2.13 m) vertically above the plate (0.48 m pitcher's mound with ball released at 1.65 m level).

Speeds of pitched baseballs vary depending on the pitcher and the type of pitch. However, the range of speeds varies from about 24 m/s (80 ft/s) for changeup or knuckleball pitches,⁴ to 45 m/s (148 ft/s or about 100 mph) which has been reported for a few fastball pitchers, including Bob Feller and Nolan Ryan.²⁰ Baseballs released at these speeds and at the height and distance mentioned above crossed the plate in the strike zone if they were thrown horizontally or at angles slightly above the horizontal.

The calculations performed in this study showed clearly that balls pitched into different drag regimes had trajectories that were significantly different. For example, identically pitched type 2 and type 3 balls with initial velocities of 35.66 m/s arrive at the plate with elapsed times differing by 0.016 s, vertical height differences of 5.3 cm, and velocities differing by 1.99 m/s (Table II). A speed of 35.66 m/s corresponds to a Reynolds number of 1.74×10^5 , which is in the most steeply decreasing part of the C_d drag crisis for a

Table II. Calculated elapsed times, final vertical height, and final speeds for pitched balls of various types released at different initial speeds. The calculations assume that the ball is released horizontally by the pitcher at a height of 1.5 m above the ground, and the calculations describe its trajectory when it crosses the plate 17 m from the point of release and 6.1 m before it crosses the plate. Negative vertical heights indicate that a ball released horizontally would hit the ground before reaching the plate.

Initial speed (m/s)	Ball type	Values as ball crosses plate			6.1 m before crossing plate	
		Final speed (m/s)	Elapsed time (s)	Vertical height (m)	Vertical height (m)	Elapsed time (s)
42.67	1	38.28	0.424	0.651	1.159	0.267
	2	38.31	0.424	0.653	1.158	0.267
	3	41.65	0.406	0.700	1.171	0.260
	4	36.36	0.436	0.620	1.151	0.272
	5	36.58	0.434	0.625	1.152	0.271
35.66	1	32.28	0.506	0.290	1.012	0.319
	2	32.41	0.505	0.292	1.014	0.319
	3	34.40	0.489	0.345	1.028	0.312
	4	30.59	0.521	0.242	1.000	0.325
	5	30.75	0.519	0.247	1.001	0.325
26.21	1	24.53	0.686	-0.728	0.602	0.433
	2	25.10	0.678	-0.696	0.608	0.430
	3	23.22	0.706	-0.817	0.579	0.441
	4	23.13	0.708	-0.825	0.576	0.442
	5	23.20	0.717	-0.821	0.579	0.441

type 3 ball, whereas it corresponds to the increasing part of the C_d curve for the type 2 ball (Fig. 2). Thus as it approaches the plate and slows, the type 3 ball faces drag forces which increase deceleration from $a/g = 0.27$ to $a/g = 0.39$, whereas the type 2 ball faces forces which decrease from $a/g = 0.83$ to $a/g = 0.65$.

In a similar fashion, type 2 and type 3 balls behave in different ways if they are pitched at 26.2 m/s. This corresponds to a Reynolds number of 1.28×10^5 , which is close to the drag minimum for type 2 balls but well before the onset of the drag crisis for type 3 balls. In this case the type 2 ball arrives at the plate 0.028 s before and 12.1 cm above the type 3 ball. Since typical bat speeds are 30–40 m/s (Refs. 21, 22), it is clear that timing differences of 0.028 s will affect a batter's point of contact with the ball. Similarly, since typical bat diameters are 7 cm, and since the ball diameter is 7.32 cm, it is clear that vertical height differences of 12.1 cm can make the difference between a solidly hit line drive and a swing and miss.

Studies of major league batters have shown that the decision to swing at a pitch is made when the ball is about 20 ft (6.1 m) from the plate.²³ Since the pitcher may be able to affect the drag regime by changing its orientation, spin, etc., he would find it especially useful if this affected the ball's trajectory after the batter had begun his swing. The calculations in Table II and Table III show clearly that most of the differences in vertical height between type 2

and type 3 pitches accrues over the final 6.1 m of the balls' trajectories. Indeed, for pitches with initial velocity of 26.21 m/s, 9.2 cm of the total of 12.1-cm vertical height difference between type 2 and type 3 balls developed in the last 6.1 m (Table III). However, at all speeds less than half of the difference in elapsed time occurred in the last 6.1 m.

Note that in the range of Reynolds numbers appropriate for baseball, there is almost no difference between the behavior of a smooth ball (type 4) and a ball with a constant drag coefficient of 0.5 (Table II). It is probably for this reason that previous studies, such as that of Erickson,¹⁷ felt justified in using a constant drag coefficient to calculate baseball trajectories.

IV. TRAJECTORY CALCULATIONS FOR BATTED BASEBALLS

To investigate the effect on batted baseballs of variations of C_d and velocity, the author calculated the minimum values of initial velocity (Fig. 4) that permitted a ball to travel distances ranging from 300 to 500 ft (91.4 to 152.4 m). This is identical to a calculation of the maximum distance that a ball can travel for a particular initial velocity. For all calculations it was assumed that the batter made initial contact with the ball 1.5 m above the ground.

The results (Fig. 4) show clearly that the maximum distance a ball travels depends strongly on the drag regime. For example, a batter can hit a type 2 ball 300 ft when the initial velocity is only 35.2 m/s, but for a type 4 ball the initial velocity must be 44.5 m/s, or 26% higher. To bat a ball 500 ft, a type 3 ball must have an initial velocity of only 46.9 m/s, whereas a type 5 ball must have an initial velocity of 78.9 m/s, or 68% higher.

There is also a striking difference in the initial take-off angle associated with the maximum range (Fig. 5). For example, to reach 300 ft the optimum take-off angle varied from 34.5° (type 2 ball) to 43.5° (type 1 ball). To reach 500 ft, the optimum take-off angle varied from 29.5° (type 3 ball) to

Table III. Amount of vertical height and elapsed time difference between pitched type 2 and type 3 balls which accrue in final 6.1 m of trajectory.

Initial speed (m/s)	Height difference (cm)	Time difference (s)
42.67	3.4	0.007
35.66	3.7	0.007
26.21	9.2	0.011

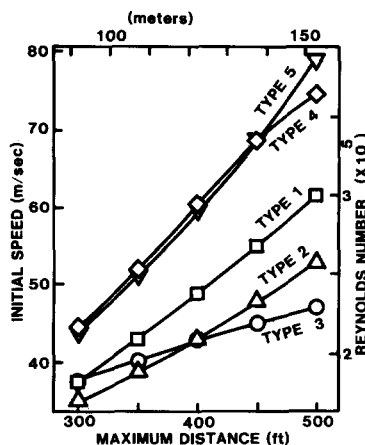


Fig. 4. Calculations of the maximum distance traveled for various types of balls hit at various initial speeds. The calculations assume that the balls begin their trajectory with the speed indicated at a point 1.5 m above home plate. Symbols as in Fig. 2.

43° (type 2 ball). Except for the type 2 ball, over the range considered the take-off angle necessary for optimum performance decreases as the initial velocity increases (Fig. 5). Other investigators have noted that air resistance causes a decrease in take-off angle to be associated with increasing velocity if maximum distance is to be reached.^{19,24}

In general, the effect of the drag crisis is to produce a regime where the aerodynamic drag force actually decreases as the velocity increases. This can produce behavior which at first seems uncharacteristic of a drag force. For example, the optimum release angle for a type 2 ball actually increases as the initial speed increases over the range of values shown in Figs. 4 and 5. Calculations showed that for all the type 2 trajectories in Figs. 4 and 5, the speed of the ball reached the speed of minimum drag or about 25 m/s (see Table IV) slightly before it reached the highest point in its trajectory. Thus it experienced very low drag forces while traveling most of its distances horizontally. For type 2 balls, the optimum batting strategy is to get the ball as high as possible before it enters the horizontal, low drag

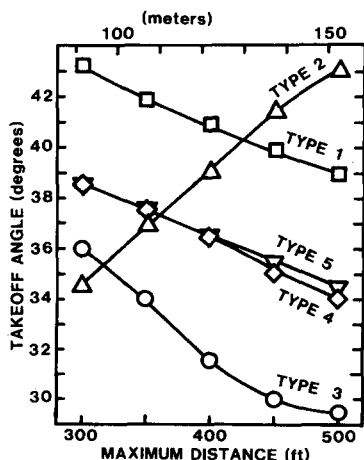


Fig. 5. Optimum take-off angles necessary to obtain given distances with minimum initial speeds for various types of balls, i.e., take-off angles associated with trajectories described in Fig. 4. Symbols as in Fig. 2. For example, Figs. 4 and 5 indicate that to hit a type 3 ball 400 ft, the minimum initial velocity is 42.67 m/s, and the associated take-off angle is 31.5°.

Table IV. Calculated terminal velocity and velocity of minimum drag for various types of ball. Ball diameter is that of a baseball and as discussed in the text; for ball types 1–4 the terminal velocity is stable only if it occurs on the upward-sloping portion of the drag curve. For the type 4 ball, there exist one unstable and two stable terminal velocities.

Ball type	Terminal velocity (m/s)	Reynolds number ($\times 10^5$)	Velocity of minimum drag (m/s)	Reynolds number ($\times 10^5$)
1	37.39	1.82	17.41	0.85
2	38.62	1.88	25.40	1.24
3	52.84	2.58	40.98	2.00
4 (stable)	85.80	4.19	81.96	4.00
(metastable)	81.33	3.97		
(stable)	31.86	1.55		
5 ($C_d = 0.5$)	32.36	1.58		
measured by Briggs	42.67	2.08		

portion of its trajectory. In contrast, because the speed at which the minimum drag occurs is significantly higher for type 3 balls, they experience minimum drag during the initial portions of their trajectory, and not at the apex. For type 4 and type 1 balls, speeds are quite different from the speeds of minimum drag over their entire trajectory, and thus like a type 5 ball, their optimum release angle decreased as the initial velocity increased.

V. DISCUSSION

It is interesting to speculate as to which of the drag curves in Fig. 2 is most appropriate for baseballs. Although the drag curve for golf balls has been reported by several investigators,^{11,14} the author is unaware of a published C_d curve for a baseball. Briggs⁷ data suggest that $C_d = 0.29$ for $R = 2.08 \times 10^5$ when a baseball was supported in a vertical wind tunnel at terminal velocity. Presumably at terminal velocity the airstream supported the baseball at a stable equilibrium where

$$\frac{\partial F_d}{\partial V} < 0$$

because otherwise small fluctuations in the airstream velocity would result in rapid acceleration or deceleration of the baseball. This will nearly always occur when

$$\frac{\partial C_d}{\partial V} > 0.$$

A C_d of 0.29 on the upward sloping part of the C_d curve corresponds roughly to a type 2 ball (see Fig. 2). The calculations of terminal velocity (Table IV) and the measurements of Briggs⁷ suggest that the drag curve for a baseball falls somewhere between a type 2 and a type 3 ball.

However, in 1969 Watts (personal communication) made measurements of C_d for a baseball between the ranges of $R = 0.25 \times 10^4$ and 1.6×10^5 and found no clear evidence of the onset of the drag crisis. In particular, all of the measured C_d were between 0.41 and 0.56 with values between 0.49 and 0.56 occurring for r in the range 3×10^4 to 8×10^4 . Slightly lower values of C_d occurred at higher R . These measurements could be observed if a baseball were a type 3 ball.

Unfortunately, the measurement of C_d for a baseball or any sphere is a more difficult experiment than it might first appear. For spheres, the measured C_d has been shown to depend on the width of the wind tunnel used,¹⁶ on the type of turbulence damping used upstream in the tunnel, and on the type of support structure used to hold the sphere in the tunnel. It also depends on whether the sphere is free to rotate, on the spin velocity of the sphere, and on small changes in the surface roughness.¹⁶ For a baseball, since its seams (Fig. 3) provide a relatively gross and asymmetric surface roughness feature, it also depends on the orientation of the ball.

A. Drag crisis and pitched baseballs

If, as we suggest above, the drag crisis for baseballs occurs in the range of Reynolds numbers appropriate for pitched and batted baseballs (approximately 1×10^5 to 3×10^5), then this may affect and/or explain several features of the game of baseball. In particular, for pitched baseballs, it may help explain the trajectories of fastballs, curves, and knuckleballs.

For fastball pitches, the drag crisis may affect both the trajectory of the ball in space and the time when it reaches the plate. In particular, a type 3 ball thrown at 35.66 m/s will arrive 0.03 s earlier and 9.75 cm higher above the plate than a type 5 ball (Table II). For batters or catchers used to a different drag regime, the fact that a fastball arrives several centimeters above its expected position contributes to its appearance of rising. Indeed, Selin⁴ measured six fastballs with high-speed cameras and found that they arrived at home plate from 9 to 82 cm above the height expected for a ball affected by gravity alone. Generally the apparent rising is attributed to the fact that the ball is released with a significant backspin, which causes it to curve in the upward direction. However, the present study suggests that a significant fraction of the apparent rise that was observed by Selin could be explained by the reduction of drag associated with the drag crisis.

For curve ball pitchers, the original question of whether curves actually curve³ has now been replaced by the question of whether the apparent suddenness of curves is real, or merely an illusion. For example, the measurements reported by Allman⁵ suggest that the ball curves regularly at a constant rate over the entire path, but this is viewed as a sudden curvature by the batter. However, many experienced batters would vehemently contest this, arguing that "slow" curves curve at a regular rate but that sharp curves change paths abruptly. The author of the present article suggests that a curve thrown with a velocity slightly higher than the onset of the drag crisis might slow down along its path, and experience a sudden increase of force as the drag coefficient changed from about 0.1 to 0.5. For a type 3 ball, the ratio a/g changes from 0.14 to 0.41 as the velocity changes from 35.9 to 28.7 m/s. Since presumably the lift forces associated with spinning also change significantly during the drag crisis, the ball's trajectory would be changed by the increases in both the drag and lift forces.

Knuckleball pitches experience the most erratic trajectories of all, and are typically thrown with little or no rotation at velocities of about 25.6 m/s (Ref. 4). Often, neither the catcher nor the pitcher knows which way the pitch will break, or if it will break at all. The present author suggests that the onset of the drag crisis provides the kind of chaotic

turbulence regime that might help to produce a typical knuckleball trajectory. For example, for a type 2 ball a velocity of 25.6 m/s corresponds almost exactly to the minimum of the C_d curve. Tiny changes in the velocity, the wind conditions along the trajectory, and the relative position of the seams with respect to the airstream could affect whether all or only a part of the ball experienced the turbulent forces associated with boundary layer separation, or the high drag forces associated with laminar flow in the boundary layer.

In his study of flow past roughened spheres, Achenbach¹⁶ noted that near the onset of the drag crisis, spheres behaved in a fashion that sounds distinctly like knuckleball behavior:

"Finally, an unexpected result should be mentioned. At critical flow conditions the sphere was pushed by lift forces towards the wall of the tube [wind tunnel]. This movement grew more and more intense as the critical Reynolds number was approached The phenomenon described could not be studied in detail. However, an attempt will be made to explain its initiation. In the critical flow range a small variation in the Reynolds number causes a drastic change in the drag coefficient. This change is due to a downstream shift of the boundary separation point accompanied by a recovery of the static pressure at the rear of the sphere. The flow state is rather unstable and therefore any slight asymmetry of the geometry due to eccentricity of individual surface roughness would cause a premature transition of the boundary layer. The resulting three-dimensional pressure distribution yields a force perpendicular to the mean flow direction which drives the sphere towards the wall."

Clearly, a pitcher who wished to use the drag crisis to advantage would like to be able to change the aerodynamic properties of the baseball so that different pitches could be affected by entirely different drag curves (as in Fig. 2). In fact, there is undeniable evidence that pitchers go to a lot of trouble to change the aerodynamic properties of the ball for different pitches. The most well-known example is the so called "spitball" which was outlawed in 1920, but which numerous pitchers are accused of throwing each season.²⁵ For example, in the 1982 season Gaylord Perry of the Seattle Mariners was suspended for ten days because he "doctored the ball." In practice a ball's performance characteristics can be altered by cutting or scuffing part of the surface, by applying grease to a part of the surface, or by applying saliva or perspiration. There are also legal ways of changing the character of the ball, such as working up the surface of part of the ball with the palms and fingers, and roughening the strings along part of the seams with the fingernails.

In practice, baseball pitchers trying to throw particular pitches always grip the ball in the same orientation relative to the seams. Thus each time the pitch is released the spin magnitude and axis orientation is uniform. This insures that the torques and forces applied by the air to the ball are always the same, at least over the initial portion of the ball's trajectory. By this means a pitcher can control when a pitch will "break" because as it travels towards the plate the spin axis will change in a regular manner due to precession or due to aerodynamic torques. This continues until the orientation of the strings and spin axis relative to the airstream changes; separation stops occurring in the boundary layer.

This causes a sudden change in the forces affecting the ball, which means that the pitch “breaks.”

Presumably it is by controlling the spin axis that a pitcher makes a curve or slider break in the same way each time that it is thrown. Because the pitcher can control the spin axis, he is in effect controlling how and when different aerodynamic forces affect the ball. In this way it is possible for him to throw different pitches which have the same initial speed but which are affected by significantly different drag regimes.

It is not clear to the author whether the pitcher controls the spin axis primarily by using aerodynamic torques, or rather by utilizing the natural free precession of the ball which will occur if it is slightly aspherical. An interesting experiment would be to measure the principal axes and moments of inertia of a collection of unused baseballs to determine if nonspherical features of the balls had the same magnitude and orientation relative to the trademark and seams. If so, this would help to explain the remarkable degree of control that some pitchers have over the trajectory of pitched baseballs.

Finally, it is worth noting that if gross changes were made on the surface of the regulation baseball, this would undoubtedly change the game significantly even if the ball's mass and diameter were unchanged. For example, if the ball's cover were seamless and smooth like a type 4 ball, it would be significantly more difficult to hit home runs (Fig. 4). However, a seamless ball would curve less, making it easier to hit. Were the ball to become smooth, knuckleball pitchers would disappear from the major leagues overnight. The net result would be that significantly more singles would be hit and more runs scored, but there would be fewer extra base hits and home runs. As discussed by James,^{26,27} this would create a game favoring the steal and the sacrifice bunt.

B. Drag crisis and batted balls

The existence of the drag crisis clearly affects the initial velocity that is necessary for a batter to hit a home run—a ball that travels on the fly over the outfield fence. In major league stadiums at the present time, the distance to the outfield fence ranges from 302 to 355 ft (92 to 108 m) along the foul lines to between 400 and 440 ft (122 to 134 m) in the center of the outfield.

Unfortunately, the author has been unable to find any paper which reports measurements of the initial velocity of baseballs hit as home runs. Bryant *et al.*²⁸ measured the velocity of line drives hit to the outfield by batters taking pitches from a pitching machine. For pitches of 25.3 m/s speed, the initial velocity off the bat was 39.6 m/s. Briggs⁷ noted without explanation that some batted balls must be hit with initial velocities considerably in excess of the terminal velocity (42.67 m/s). This is undoubtedly so, as we can see if we model the bat/ball interaction as a simple collision. For example, suppose a ball with a velocity of 40 m/s interacts with a 0.91-kg (32-oz) bat moving with a speed of 30 m/s in such a fashion that the coefficient of restitution is 0.3. Then if the ball is hit straight back at the pitcher its speed will be 53 m/s. However, this is a relatively conservative calculation, as bat speeds of 40 m/s have been measured for a small sample of college players,²² and the coefficient of restitution of a baseball has been measured to be between 0.5 and 0.6.^{20,29–31} If the bat and ball

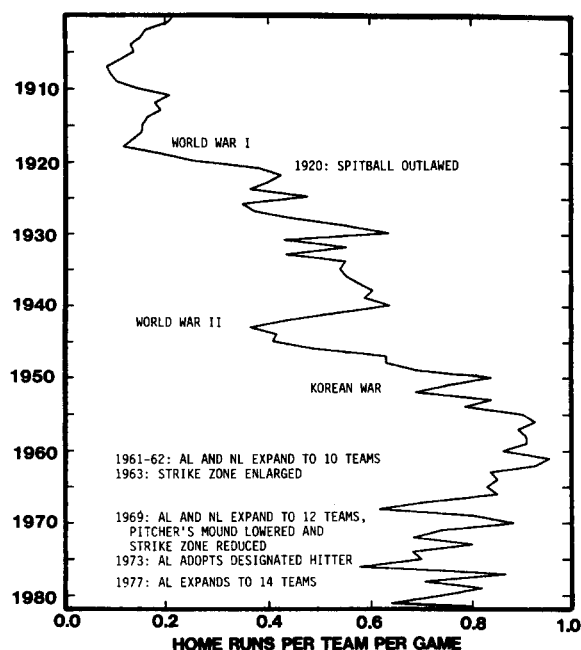


Fig. 6. Number of home runs per team per game hit in the major leagues each year since 1900.

both have velocities of 40 m/s and if the coefficient of restitution is 0.5 it would be hit straight back at the pitcher with a speed of 78 m/s. Of course, balls hit at angles different from horizontal will have lower speeds.

The author's calculations (Fig. 4) show that the distance that a ball travels depends strongly on the drag regime in effect, i.e., on the type of ball hit. For example, a type 4 ball (smooth sphere) with initial velocity near the terminal velocity of 42.67 m/s will travel a maximum distance of 87.6 m, whereas a type 3 ball would travel 122.1 m, or about 39% further. In a vacuum, the sphere would travel 187.3 m. These calculations suggest that if a batter desiring home runs can hit a ball hard enough to “punch through” the drag crisis, he can hit the ball considerably further than would be expected if C_d were constant. In effect, small increases in initial velocity can produce a large increase in distance. Although the precise velocity of batted home runs is not known, it is clear from the above discussion that balls of the type 2 and type 3 variety must be commonly hit at speeds exceeding the minimum in the drag curve (Table IV).

The question of the effect of material changes in the baseball on the ease of hitting home runs has produced a controversy which has concerned some baseball enthusiasts for more than 50 years. The key problem in the “lively ball controversy” concerns why the number of home runs hit in the major leagues increased so sharply beginning about 1920 (Fig. 6), and has varied so greatly since that time. Many knowledgeable baseball experts^{32,33} firmly believe that the coefficient of restitution of the baseball has been changed numerous times, either unintentionally by the ball manufacturers or intentionally at the request of the major leagues. Both the ball manufacturers and the major leagues have denied that the ball has changed significantly, and instead attribute the obvious changes in the number of home runs to other factors, such as the presence of generally faster pitchers, the existence of bigger, stronger, better trained hitters, and changes in managerial strategy which

encourage home runs. Although the “lively ball” proponents do not deny that the physical characteristics of ball-players and baseball strategy have changed since 1920, they consider it impossible that these changes could affect home run production so significantly. Unfortunately, measurements of the characteristics of baseballs, and especially the coefficient of restitution, have been made only irregularly.^{29-31,34-37}

With one exception²⁹ none of the participants in the lively ball controversy has realized that the aerodynamic properties of the ball will affect the hitting of home runs as greatly as does the coefficient of restitution. As previously noted, for baseballs hit at velocities near the onset of the drag crisis, a small increase in velocity will make the ball travel much further. Thus the drag crisis means that slightly stronger athletes will be disproportionately more effective at hitting home runs, or that slightly faster pitchers will give up disproportionately more home runs. In addition, it is possible to change the distance that baseballs travel simply by making subtle changes in the ball’s aerodynamic properties without affecting the coefficient of restitution. For example, if the surface were slightly roughened or if the seams were raised slightly, the velocity of the onset of the drag crisis would be reduced and a ball would travel further for a given initial velocity. Any investigators wishing to resolve the lively ball controversy with experiments should carefully study the characteristics of the cover and seams as well as the ball’s mechanical properties.

One of the most comprehensive reports concerning the mechanical properties of baseballs was commissioned by *Popular Mechanics* in 1961. These investigators analyzed the composition of both old and new balls, studied the effects of aging on baseball properties, and measured seam heights as well as coefficients of restitution on balls manufactured in 1927, 1930, and 1961. They found that seam heights did indeed differ for different baseballs, and that even for baseballs manufactured in the same year coefficients of restitution could vary more than 20%. Simple measurements of the drag coefficient suggested that the 1961 balls had drag coefficients 10% to 15% lower than the older balls. Changes in the compositions and the manufacture of the baseball occur quite regularly,³⁸ indeed, the cover of the official baseball was changed in 1974 from horsehide to cowhide. Clearly, any scientist wishing to investigate the aerodynamic properties of the baseball should be prepared to study the differences between balls as well as the properties of a particular ball.

C. Relative importance of aerodynamic effects in various ball games

To evaluate how aerodynamic forces influence a particular ball game, it is useful to calculate both the Reynolds number R and the ratio a/g of aerodynamic forces to gravitational forces in typical situations (Table I). In addition, the surface roughness of the ball is a factor as it affects at what Reynolds number the drag crisis begins. The comparisons in Table I are only approximate. Although information concerning the dimensions and masses of balls are available in any encyclopedia, measurements of velocities are difficult to find. It is sometimes difficult to determine whether the reported values are measurements or guesses, whether they are for typical or exceptional players, and whether they are for typical or exceptional situations.

When careful measurements of ball speed are available, they sometimes differ markedly from commonly quoted speeds. For example, whereas Ortiz³⁹ quotes a table tennis magazine to the effect that ping pong balls in play may travel “in excess of 70 mile per hour” (31.3 m/s), her own measurements of the speed of typical returns for expert players do not exceed 4.27 m/s. Comparisons between reported measurements are difficult, as many reported measurements are not accompanied by information about how the measurements were made, or what portion of the trajectory was measured. This is a critical subtlety, because calculations suggest that at the higher reported speeds the initial, average, and final speeds may differ significantly. For example, a type 3 softball pitched with an initial speed of 29.6 m/s and released 12.6 m from the plate will cross the plate at a speed of 25.7 m/s, and its average speed over the entire path would be 27.4 m/s. A table tennis ball with an initial speed of 31.3 m/s will cross a standard 8-ft (2.43-m) table with final and average speeds of about 22 and 26 m/s, respectively.

A potentially useful research project that might appeal to undergraduate physics students would be to determine accurately the speeds of balls used in various sports by players at different skill levels under various conditions. To be most valuable, measurements should be made as nearly as possible under actual game conditions. It probably would be possible to determine speeds reasonably accurately using only a movie camera, if care were taken in the selection of film and if the observer took notes concerning the approximate distance, etc. between the camera and the subject.

One ball game which must be influenced strongly by aerodynamic effects is fastpitch softball. The rules of softball are quite similar to those of baseball, however, the ball’s diameter is 9.7 cm and its mass is 0.188 kg, about one-third larger than the diameter and mass of a baseball. Softball pitchers throw the ball from a distance of 46 ft (14.0 m) with an underhand motion and without any pitcher’s mound. However, the few available measurements suggest they achieve speeds comparable to those of baseball pitchers. Miller and Shay⁴⁰ measured the speeds of five male college softball pitchers and four male town league pitchers, and found that the average speed between release and the plate was as high as 29.6 m/s. They review the results of previous studies of professional players which report initial velocities as high as 44.2 m/s. Popular articles report even higher speeds.⁴¹

The softball and baseball illustrate an interesting paradox concerning the aerodynamic effects on objects of similar shape but different sizes. Since the Reynolds number is proportional to the diameter, for any given speed

$$\frac{R_{\text{softball}}}{R_{\text{baseball}}} = 1.33,$$

and so the softball will be affected by the drag crisis at significantly lower speeds than a baseball. If we rewrite Eq. (1) in terms of the Reynolds number R , and recall that $A = \pi d^2/4$, we find that the drag force is independent of the ball diameter and depends only on the Reynolds number, i.e.,

$$F_d = -(\pi/8)\rho v^2 C_d R^2.$$

Since a softball is more massive than a baseball, for the same Reynolds number its trajectory will be less strongly

influenced by aerodynamic forces. Because a softball looks just like an overgrown baseball, we assume that the drag coefficients are the same, although the author is unaware of any measurements whatsoever of the drag coefficients of a softball. In other words, for the same Reynolds number R

$$\frac{(a/g)_{\text{softball}}}{(a/g)_{\text{baseball}}} = \frac{M_{\text{baseball}}}{M_{\text{softball}}} = 0.77.$$

However, if a softball and a baseball have the same *speed* and are in a regime where their drag coefficients are about the same, e.g., well below or well above the drag crisis, then

$$\frac{(a/g)_{\text{softball}}}{(a/g)_{\text{baseball}}} = \frac{(d^2/M)_{\text{softball}}}{(d^2/M)_{\text{baseball}}} = 1.36.$$

Thus at the same speed the softball's trajectory will be affected more strongly by aerodynamic forces than a baseball's. Thus one can justify the argument that softballs are affected more strongly, and also affected less strongly than baseballs by aerodynamic forces. However, athletes who are experienced batters in both baseball and softball suggest that aerodynamic effects are probably more pronounced for pitched softballs than baseballs (Jack Caldwell, personal communication).

In spite of the difficulties with the data in Table I, it is quite clear that aerodynamic forces are especially important in several other sports, including table tennis, golf, volleyball, jai alai, and tennis. However, the amount of research available concerning the importance of aerodynamic forces differs greatly for these sports. Although there exist several papers on golf^{1,12,14,42} and a few on baseball,^{4,5,7,8} the author is unaware of any published work on the others. With jai alai, for example, the author has been unable even to find any original source where the speeds of balls in play were measured, although the value of 150 mph used in Table I is quoted in several encyclopedias. Mehta and Wood⁴³ suggest that aerodynamic forces are important in cricket. In contrast, aerodynamic forces are less significant in sports such as basketball⁴ and bowling.^{45,46} Surprisingly, papers estimating aerodynamic effects have been published for the shot put^{24,47} and for the hammer,^{48,49} even though the relative aerodynamic effects for these events are remarkably small. This may be because there has been considerable interest in the effect of aerodynamics on other field events, such as discus^{19,50-52} and javelin.^{53,54}

There also exist significant differences concerning the extent that lift caused by rotation of the ball is important to various sports. For golf, it is all important; indeed, much of the distance obtained from driving is permitted only because of the lift caused by the ball's rotation. In spite of their dislike of the hook and slice, golfers would drive the ball significantly less far in a vacuum than is possible in air. For baseball and softball, the fact that the trajectory is affected significantly by spins applied by the pitcher is responsible for much of the character of the game, especially its relatively low scoring. For tennis and table tennis, the use of spin is a pervasive strategy because it allows players a larger selection of trajectories for which balls are hit in bounds.

VI. SUMMARY

(1) The trajectories of baseballs traveling at air speeds appropriate for both thrown or batted balls are likely to be influenced by the "drag crisis," the decrease in aerodynamic

drag which occurs at Reynolds numbers above about 10^5 . Previous investigators have ignored the effects of the drag crisis on baseballs, probably because for very smooth spheres the crisis occurs at air speeds higher than typical speeds of pitched and batted balls.

(2) Unfortunately, detailed measurements of the aerodynamic drag force on a baseball are not available. Such measurements are difficult because the drag coefficient may depend critically on the orientation of the ball and the spin as well as the air speed.

(3) The author speculates that baseball pitchers can control the air velocity where the drag crisis occurs by changing the spin of the ball, the orientation of the ball, or by other means. If so, this may affect the arrival time of the ball at the batter by several hundredths of a second, and the vertical height of the ball upon arrival by several cm.

(4) The erratic behavior of so-called knuckleball pitches can be explained if knuckleballs are pitched with a speed near the onset of the drag crisis.

(5) Calculations show that the distance traveled by batted baseballs depends strongly on the velocity of the onset of the drag crisis. Unlike most projectiles undergoing aerodynamic drag, in some cases the batter obtains maximum distance by increasing the take-off angle as initial ball velocity increases.

(6) For more than half a century, baseball enthusiasts have argued about the significance of changes in the number of home runs hit by major league baseball players. Generally, they have ignored the possible influence of the aerodynamic drag crisis on the number of home runs hit. Because of the drag crisis, small changes in the initial velocity of the ball can result in a disproportionately large change in the distance traveled. Also, for any particular initial velocity, small changes in the characteristics of the surface of the baseball could make large changes in the distance traveled.

(7) Calculations suggest that aerodynamic forces play an important role in several ball games and sports, including baseball, softball, and golf. However, little or no research is available concerning the importance of aerodynamics on tennis, ping-pong, or jai alai. Surprisingly, a number of studies are available concerning field events such as shot put, hammer, discus, and javelin.

ACKNOWLEDGMENTS

The author thanks Robert G. Watts of Tulane University and Dale Sawyer of the University of Texas for their critical review of this paper. The manuscript was also improved by suggestions from Bill James of Lawrence, KS; James Hay of the University of Iowa; and Peter Brancazio of Brooklyn College. Finally, I am indebted to Jack Caldwell of Littleton, CO for his personal observations of the trajectories of pitched balls. This is University of Texas Institute for Geophysics Contribution No. 579.

¹P. Tait, Trans. R. Soc. Edinburgh 37, 427-440 (1893).

²R. M. Bache, Sci. Am. 41, 42 (1904).

³F. L. Verwiebe, Am. J. Phys. 10, 119-120 (1942).

⁴C. Selin, Res. Q. 30, 232-240 (1959).

⁵W. F. Allman, Science 82 3, 32-39 (1982).

⁶W. F. Allman, Science 83 4, 92-93 (1983).

⁷L. J. Briggs, Am. J. Phys. 27, 589-596 (1959).

- ⁸R. G. Watts and E. Sawyer, *Am. J. Phys.* **43**, 960–963 (1975).
- ⁹R. Weaver, *Am. J. Phys.* **44**, 1215 (1976).
- ¹⁰L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Addison-Wesley, Reading, MA, 1959).
- ¹¹J. M. Davies, *J. Appl. Phys.* **20**, 821–828 (1949).
- ¹²D. Williams, *Q. J. Appl. Math.* **12**, 387–392 (1959).
- ¹³A. Cochran and J. Stobbs, *The Search for the Perfect Swing* (Lippincott, New York, 1968).
- ¹⁴P. W. Bearman and J. K. Harvey, *Aeronaut.* **27**, 112–122 (1976).
- ¹⁵S. Goldstein, *Modern Developments in Fluid Dynamics* (Clarendon, Oxford, 1938).
- ¹⁶E. Achenbach, *J. Fluid Mech.* **65**, 113–125 (1974).
- ¹⁷A. L. Erickson, "Aerodynamics of a baseball," Master's thesis, Johns Hopkins University, 1961.
- ¹⁸G. W. Parker, *Am. J. Phys.* **45**, 606–610 (1977).
- ¹⁹C. Frohlich, *Am. J. Phys.* **49**, 1125–1132 (1981).
- ²⁰J. G. Hay, *The Biomechanics of Sports Techniques*, 2nd ed. (Prentice-Hall, Englewood Cliffs, NJ, 1978).
- ²¹P. Kirkpatrick, *Am. J. Phys.* **31**, 606–613 (1963).
- ²²D. R. McIntyre and E. W. Pfautsch, *Res. Q.* **53**, 206–213 (1982).
- ²³J. L. Breen, "What makes a good hitter?," *J. Health, Phys. Educ., Recreation* **38**, 36–39 (1967).
- ²⁴D. B. Lichtenberg and J. G. Wills, *Am. J. Phys.* **46**, 546–549 (1978).
- ²⁵W. Leggett, *Sports Illus.* **18** (22), 18–21 (1963).
- ²⁶B. James, *1981 Baseball Abstract* (published by author, Lawrence, KS, 1981).
- ²⁷B. James, *Sports Illus.* **57** (11), 30–34 (1982).
- ²⁸F. O. Bryant, L. N. Burkett, S. S. Chen, G. S. Krahenbuhl, and P. Lu, *Res. Q.* **48**, 505–509 (1977).
- ²⁹Anonymous, *Pop. Mech.* **116**, Oct., pp. 134–137; Nov., pp. 100–101; Dec., pp. 82–84 (1961).
- ³⁰T. C. Brody, *Sports Illus.* **19** (7), 12–15 (1963).
- ³¹L. Keith, *Sports Illus.* **46**, 23–27 (13 June 1977).
- ³²E. Cook, *Percentage Baseball*, 2nd ed. (M.I.T. Cambridge, MA, 1966). Cook states: "Diagnosis of the home run syndrome requires decision between improbable muscular dystrophy in the Ruthian era and the increased elasticity of the baseball... It is just too much to ask anyone to believe that more animal power exists in the game today than occurred in the era of Ruth, Gehrig, Foxx, Hornsby, Greenberg, Wilson & Co. when all major batting and slugging records were established with or without rabbit genes in the baseball. These things ... enforce the suspicion that erratic changes of home runs in endemic proportions moving always upward to 1961 may be related more to some human agency than to natural evolution, adaptation to environment or survival of the fittest. When impotent pitchers and bantam-weight infielders begin dunking the old apple into the bargain seats, something other than a new race of superman is responsible for the plethora of round-trip tickets to the plate. ... If the Elders prefer to move farther in this direction [of increasing resilience], a small increment of resilience could be relied upon to place the baseball permanently in orbit and save those extravagant space agencies a lot of shekels firing silly foul balls at the moon."
- ³³E. Walker, *The Sinister First Baseman and Other Observations* (Celestial Arts, Millbrae, CA, 1982).
- ³⁴L. S. Treadwell, *Sci. Am.* **141**, 310–312 (1929).
- ³⁵R. C. Boyle, *Sports Illus.* **15** (9), 14–17 (1961).
- ³⁶H. Weiskopf, *Sports Illus.* **33**, 20–25 (20 July 1970).
- ³⁷R. Fimrile, *Sports Illus.* **39**, 24–29 (27 August 1973).
- ³⁸Bill James, author and publisher of *Baseball Abstract*, responded to the author's questions concerning the composition of the baseball as follows: "Changes in the composition of the baseball occur every year, according to Tom Seaver. He started early in his career taking two or three baseballs a year and cutting them apart. About '77 a change was made in the covering that lined the cork center of the ball. Before '77 that was covered with a hard plastic, quite a bit like a ping pong ball; after that it was covered with a little softer, more pliable rubber. Another year they changed the type of glue that was used to fasten the cover to the "string" part of the ball. Just a couple of years ago, they changed from using American-made baseballs to using baseballs made in Haiti or some place; maybe ten years ago, a switch was made from winding the string part of the baseball by hand to doing it by machine. A baseball has got dozens of different components, if you look carefully enough. I believe there are four different types of glues that are used altogether. How many grades of cork are there? I can't believe it's just one. The claim that "they've" juiced up the baseball is just paranoia. But all the materials that make up a baseball become scarce, become more expensive, become awkward to use, and are changed. The people who manufacture the balls might do tests to insure that these things don't change the resiliency of the ball, but the problem is that the "test" to which the players put the balls, which is to take thousands and thousands of them and throw them at great rates of speed and whop hell out of them with heavy clubs, is so much more severe and extreme than the tests performed by the engineers that one can easily see how results that don't show up at all in the initial tests could stand out sharply in the eventual test. So—the ball changes, the results change. Always has, always will."
- ³⁹C. Ortiz, "A cinematographic analysis of the forehand counterdrive, forehand loop, and forehand topspin serve in table tennis," Master's thesis, Texas Woman's University, Denton, TX, 1972.
- ⁴⁰R. G. Miller and G. T. Shay, *Res. Q.* **35**, 433–437 (1964).
- ⁴¹J. McCallum, *Sports Illus.* **50** (22), 92–106 (1979).
- ⁴²H. Erlichson, *Am. J. Phys.* **51**, 357–362 (1983).
- ⁴³R. Mehta and D. Wood, *New Sci.* **87**, 442–447 (1980).
- ⁴⁴P. J. Brancazio, *Am. J. Phys.* **49**, 356–365 (1981).
- ⁴⁵Y. Murase, M. Mihashita, H. Matsui, S. Mizutani, and H. Wakita, "Biomechanics of bowling," in *Biomechanics IV*, edited by R. C. Nelson and C. A. Morehouse (University Park Press, Baltimore, MD, 1974).
- ⁴⁶R. L. Huston, C. Passerello, J. M. Winget, and J. Sears, *J. Appl. Mech.* **46**, 937–943 (1979).
- ⁴⁷B. P. Garfoot, *Track Tech.* **32**, 1003–1006 (1968).
- ⁴⁸H. Payne, *Hammer Throwing* (Am. Athletic Assoc., London, 1969).
- ⁴⁹J. Dapena and M. A. Teves, *Res. Q.* **53**, 78–81 (1982).
- ⁵⁰J. A. Taylor, *Athletic J.* **12**, 9–10, 45–47 (1932).
- ⁵¹R. V. Ganslen, *Athletic J.* **44**, 50–52, 68, 88–89 (1964).
- ⁵²T. Soong, *J. Appl. Mech.* **43**, 531–536 (1976).
- ⁵³J. Terauds, *Track & Field Q. Rev.* **7**, 88–95 (1974).
- ⁵⁴T. Soong, *J. Appl. Mech.* **42**, 257–262 (1975).
- ⁵⁵S. Plagenhoef, *Patterns of Human Motion* (Prentice-Hall, Englewood Cliffs, NJ, 1971).
- ⁵⁶S. Plagenhoef, *Fundamentals of Tennis* (Prentice-Hall, Englewood Cliffs, NJ, 1970).
- ⁵⁷R. C. Nelson, *Res. Q.* **35**, 83–84 (1964).