1 Simple Projectile Motion

In the simplest case (with no air resistance) the 2D motion of a projectile is describe by a system of equations

$$x_f = x_i + (v_0 \cos\theta + v_{ship})t \tag{1}$$
$$y_f = y_i + (v_0 \sin\theta)t - \frac{gt^2}{2}, \tag{2}$$

where x and y are the projectile positions, v_0 is the projectile initial speed, θ is the initial shooting angle, and v_{ship} is the speed of a ship. From the system above it is straightforward that

$$t = \frac{x_f - x_i}{v_0 \cos\theta + v_{ship}} \tag{3}$$

Then from equations (1-3) follows that

$$y_i - y_f + v_0 \sin\theta \frac{x_f - x_i}{v_0 \cos\theta + v_{ship}} - \frac{g}{2}\left(\frac{x_f - x_i}{v_0 \cos\theta + v_{ship}}\right)^2 = 0 \tag{4}$$

Solving this non-linear equation for the angle θ would give the right shooting angle to hit a target.

The problem can be solve analytically when $y_i = y_f$ and v_{ship}=0. Then from equation (4) follows

$$v_0^2 \sin\theta \cos\theta = \frac{g(x_x - x_i)}{2} \tag{5}$$

or using identity $2\sin\theta\cos\theta = \sin 2\theta$ one has

$$\theta = \frac{1}{2}\text{asin}\left(\frac{g(x_f - x_i)}{v_0^2}\right) \tag{6}$$