Modeling Unemployment as an Inventory: 
A Multicointegration Approach\textsuperscript{1}

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Abstract

We examine the dynamic phenomenon of unemployment as a constantly changing inventory of unemployed individuals. We focus on the possibility raised by Elsby, et al. (2009) of an innate “inseparability” between the flows into and out of unemployment. Multicointegration, introduced by Granger and Lee (1989), offers a natural way to model the level of unemployment as an inventory. We find that there is multicointegration between inflows into and outflows from unemployment and the level of unemployment itself. By identifying this multicointegrating relationship, we are able to specify an error correction model for unemployment, improving forecasting ability.

JEL Classification Numbers:  E24, E2, J64, J2, C22.

Key Words: unemployment, employment, multicointegration, labor market flows, inventory, unemployed workers, labor force
I. Introduction

The U.S. labor market is characterized by significant movements of workers switching between employers as well as between different states of labor market participation. Research on unemployment has recognized this fluid nature of the labor market and investigated the role of worker flows in bringing about the observed changes in aggregate unemployment. In this paper, we contribute to the literature on unemployment flows by using the econometric concept of multicointegration to estimate the long-run interactions between the stock of unemployment and the flows into and out of unemployment.

Much of the literature on unemployment flows models the inflows and outflows of unemployment as separate determinants of unemployment and investigate the effect each component has on aggregate unemployment. However, as Elsby et al. point out in their 2009 paper, “inflows and outflows may be inherently inseparable”, indicating that a common factor may exist among the flows into and out of unemployment. According to this view, the inflow rate influences the outflow rate through the former’s impact on the stock of unemployment as well as by direct changes in the level of outflow. For example, in certain situations an increase in the flow into unemployment raises the stock of unemployed, in turn generating a decrease in the hazard rate of exiting unemployment even though the number of people leaving unemployment remains constant. This interpretation of the labor market implies that cyclical unemployment is determined not only by the relationship between the flows and the stock of unemployment but also the relationship between the inflow and the outflow.

The econometric technique of multicointegration, developed by Granger and Lee (1989, 1990), offers a natural way to model an inventory through modeling the special cointegrating relationships between the stock and flow variables as well as among the flow variables. In this
case we use multicointegration to model the stock of unemployed workers as an inventory while taking into account the inseparable nature of the flows into and out of unemployment. This concept of multicointegration (detailed in Section III) introduces a deeper form of cointegration among variables particularly useful in modeling stock-flow relationships. In the case of unemployment, this multicointegrating relationship identifies a long-run relationship between not only the flow into and out of unemployment, but also between these flow variables and the stock of unemployment itself.

In this paper, we show that a multicointegrating relationship does in fact exist between the flows into and out of unemployment as well as between those flows and the level of unemployment. By identifying these relationships, we are able to specify an error correction model for unemployment, capturing the long-term and short-term dynamics of the interaction between the flow of individuals through the labor market and changes in the stock of unemployed individuals. In addition, estimation of the cointegrating parameter measuring the impact accumulated inflow has on accumulated outflow yields useful information regarding these relationships of the flow into and out of unemployment. Our estimated value of one for this cointegrating parameter indicates that on average over the entire sample period the inflows into unemployment moved at a similar pace to the outflows from unemployment. Figure 1 shows the movement of the flows into and out of unemployment over the sample period. This empirical evidence supports our expected result of a cointegrating parameter close to unity in that while short-term deviations may occur between the inflows and outflow of unemployment, in the long-run frictional unemployment and the economic needs of a society dictate these cointegrated movements of the flows into and out of unemployment. Furthermore, we show that incorporating the long-run relationship between inflows and outflows of unemployment

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improves the forecasts of the unemployment level over a relatively stable ex-post forecast period when compared to ARIMA models and models of unemployment based on other economic indicators. Overall, the behavioral relationships we identify through this research provide us with a framework for future research in order to incorporate the varying cyclical trends within the flow activity.

The structure of this paper is as follows. Section II reviews the current literature and defines the unemployment stocks and flows used in this research. Section III provides a brief review of the technique of multicointegration. Section IV gives a detailed description of the data used in this analysis. Section V presents the results of tests for multicointegration among the flow variables for unemployment, as well as the empirical results of the error correction models. Section VI provides a comparison of forecasts for the unemployment level from four models over an ex-post forecast period. Finally, Section VII offers the conclusions, possible policy implications, and areas for future research.

II. Literature and Methodology

a. Relevant Literature

Modern literature on the cyclical behavior of unemployment has been dominated by the flow approach which emphasizes the role of inflows into and outflows out of unemployment in generating the aggregate unemployment patterns. One of the earliest contributions to this literature was made by Blanchard and Diamond (1990) whose research emphasized the substantial variation in worker flows underlying the changes in aggregate unemployment. Blanchard and Diamond (1989, 1990) develop a basic model to examine not only the stocks, or levels, related to unemployment but also the flows between states of unemployment and
employment. Using the monthly gross flow series from the Current Population Survey (CPS) as adjusted by John Abowd and Arnold Zeller (1985), they estimate the joint movements of the labor force (L), unemployment (U) and vacancies (V) via vector autoregressions. Since the new CPS series for disaggregated flows were not yet available, Blanchard and Diamond (1990) regress the Abowd-Zellner corrected gross flows on a constant, a time trend, and the current and lagged values of employment, unemployment and vacancies. Thus, rather than using these disaggregated worker flows to forecast unemployment, the authors use employment, unemployment and vacancies to interpret and estimate these disaggregated worker flows. In contrast, our research uses the interaction between these disaggregated worker flows to estimate and interpret the unemployment level as well as the worker flows themselves.

In their review of the literature, Davis, Faberman and Haltiwanger (2006) distinguish between “supply-side” flows as workers move between labor force states through events such as retirement, labor force entry and re-entry, and job changes known hereafter as worker flows, and “demand-side” flows through job creation and destruction known hereafter as job flows. On the “supply-side,” worker flows can be measured from the employer perspective using data on hires, separations, quits and layoffs as well from the worker perspective using survey data such as the Current Population Survey. While much of their previous work, particularly by Davis and Haltiwanger (1990), focuses on the job flow data, or rather establishment data, Davis, Faberman and Haltiwanger (2006) provide an estimate of the “supply-side” worker flows as measured from the employer perspective by regressing each variable (hires, separations, quits and layoffs) on job creation and destruction. While they provide various conclusions regarding the impact of this worker flow behavior on unemployment inflows and outflows, they stop short of providing a forecast for the unemployment level based on these variables. In our research, we use the worker
flows measured from the worker perspective and provided by the Current Population Survey to provide a forecast for the unemployment level.

As in our research, Bleakly, Ferris and Fuhrer (1999) also use the “supply-side” disaggregated worker flows as measured by the Current Population Survey to forecast unemployment of the late 1990’s, specifically addressing why Okun’s Law consistently underestimated unemployment forecasts. They augment the Okun’s Law regression to include measures separating the outflow from unemployment to employment (UE) versus the outflow from unemployment to outside to the labor force (UN). Their research reveals that the inclusion of these disaggregated flows as well as measures for unemployment duration into an augmented Okun’s law regression significantly improve the forecast for unemployment compared with forecasts from the conventional Okun’s Law equation. However, use of Okun’s Law, whether in its traditional form or in the Bleakly, Ferris and Fuhrer (1999) augmented version, depends on accurately estimating potential GDP. In our research, by establishing a model depending solely on the relationship between the flows into and out of unemployment through multicointegration, we do not face the potential problem of underestimating GDP.

As mentioned in the introduction, building particularly on the work of Shimer (2007), Elsby, Michaels, and Solon (2009) provide an intuitive statistical decomposition of unemployment to assess the individual impacts that inflow and outflow have on unemployment while offering insight into the cyclical nature of unemployment inflows and outflows. They illustrate the flows into and out of unemployment and their impacts on the stock of unemployed individuals with the metaphor of a queue forming at a traffic intersection awaiting a green light. In their metaphor, each cycle allows five cars to pass through the green light before turning red again. Under normal conditions, a relatively constant number of cars enter the queue. However,
if an extraneous event, such as construction on an alternate route, increases the inflow of cars into the intersection with no change in the flow out of the intersection, then the stock of cars in the queue increases. While the number of cars exiting the intersection remains five, the outflow rate decreases due to the increased number of cars in the queue. Thus, as Elsby, et al. (2009) point out, using their original decomposition one would surmise that the increase in unemployment is based on a decrease in the exit rate, when in fact there is no change in the outflow process.

Our point of departure in this study is the possibility indicated by Elsby, et al. (2009) that “inflows and outflows may be inherently inseparable.” Their traffic metaphor demonstrates that the outflow rate may be endogenous with the inflow rate and this relationship between the two flows together may critically determine their impacts on the unemployment pool. As pointed out by Elsby et al. (2009), the potential interactions between inflows and outflows of unemployment has received little attention in the literature as most research has primarily treated the outflow rate as depending only on exogenous variables².

The econometric method of multicointegration allows us to include this interaction between flows in estimating the relationship between the flows and stocks. In this particular case, the use of a multicointegrated error-correction model to estimate unemployment levels incorporates the relationship between unemployment inflows and outflows into our model of unemployment.

b. Unemployment Stocks and Flows

In this research we examine the movement into and out of unemployment as an inventory stock-flow problem with the number of unemployed individuals constantly shifting. For various

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² Elsby et al. conjecture that the potential endogeneity of outflow rates has not received much attention because of the literature’s emphasis on search and matching models, which imply that the outflow rate is exogenous.
reasons, such as voluntary or involuntary separation from current employment, the reentry into
the labor force for personal or economic reasons, or even the coming of age 16 and beginning the
initial search for a job, thus entering the labor force, each month people flow into the stock of
unemployed from outside the labor force as well as from the ranks of those formerly employed.
Likewise, each month people flow out of the stock of unemployed into employment by finding a
job. In addition, individuals may flow out of the stock of unemployed by leaving the labor force
for personal or even economic reasons as in the case of discouraged workers. Figure 2 offers an
illustration of unemployment as inventory flow of individuals continually flowing into and out of
the inventory of unemployed for a variety of reasons. These variables of flow into and out of the
stock of unemployment capture much more information than merely recording the number of
people in an economy currently unemployed.

Through this paper, we utilize the technique of multicointegration in modeling the
relationship existing between the stock of unemployed individuals and the flows into and out of
unemployment. Examining unemployment as an inventory stock-flow model allows us to study
these flows into and out of unemployment as a multicointegration problem similar to work by
Granger and Lee (1989, 1990) on goods and housing inventories.

In order to employ multicointegration, we must first identify the stock and flow variables
relevant to unemployment. Each month the Bureau of Labor Statistics, using data from the
Current Population Survey, reports the number of jobs created or lost in the U.S. economy, the
rise or fall in the number of unemployed individuals, the unemployment rate, and a host of other
labor market statistics designed to help gauge the health of the economy. Perhaps the most often
reported statistic is the unemployment rate

\[ u_t \equiv \frac{U_t}{L_t} \times 100 \]  

(2. a)
defining the unemployment rate as the percentage of individuals in the labor force ($L_t$) considered unemployed ($U_t$) in time period $t$. Based on Equation 2.a, a change in the unemployment rate ($u_t$), between time periods may result from a change in the unemployment level ($\Delta U_t$), a change in the labor force ($\Delta L_t$), or a combination of both.

Since the early 1950’s the U.S. unemployment rate has ranged from 2.5% to almost 11% in late 1982, and averaging around 5.7%\(^3\). As Figure 3 shows, unemployment rates consistently rise during recessionary periods (shaded areas in Figure 3), visually confirming the aggregate negative co-movement between the changes in the unemployment rate and the growth of GDP at the onset of a recession. The inverse relationship between changes in the unemployment rate and the growth of GDP is known as Okun’s Law. Though historically one of the “most robust” statistical relationships, recent data reveal evidence of a divergence from Okun’s Law (Elsby, Hobijn, and Sahin 2010). Additionally, Figure 3 shows the asymmetry over time existing between the rapid increases in unemployment at the onset of a recession followed by the slow decrease in the unemployment rate during recovery. Therefore, numerous questions remain regarding the accurate depiction of the relationship between movements of unemployment levels and aggregate economic indicators.

Another aggregate statistic, the unemployment level is defined as the difference between the number of people in the labor force ($L_t$) and the number of people employed ($E_t$) in time period $t$, as given by Equation 2.b.

\[
U_t \equiv L_t - E_t
\]

Through equation 2.b we identify three relevant labor market states: Unemployed (U), Employed (E), and Labor Force (L). At any given time, an individual is either “in the labor force” (L) or “not in the labor force” (N). For individuals in the labor force, a person is classified as either

\(^3\) Reported by St. Louis Federal Reserve Economic Database (FRED).
employed (E) or unemployed (U). Movements between the three statuses, unemployed (U),
employed (E) and not in the labor force (N), dominate the focus of much research on the
dynamics of unemployment. As the U.S. labor market continually creates and eliminates jobs,
workers flow into and out of the stock of unemployed individuals as illustrated in Figure 2.
Building on equation 2.b and using the notation from the BLS ‘Labor Market Transition Matrix’
in Figure 4 we characterize the unemployment level (Ut) in time t as the unemployment level in
the previous period plus the difference between the flow into and out of unemployment from the
other statuses in the labor market such that

\[ U_t = U_{t-1} + (NU + EU) - (UN + UE) \]  \hspace{1cm} (2. c)

or equivalently, the change in the unemployment level is

\[ \Delta U_t = (NU + EU) - (UN + UE) \]  \hspace{1cm} (2.d)

Thus, we have identified the stock variable of unemployment (U), and four flow variables: NU-
the movement from not in the labor force into unemployment; EU-the movement out of
employment into unemployment; UN-the movement from unemployment to not in the labor
force; and UE-the movement from unemployment into employment for our stock-flow
interpretation of unemployment4.

III. Multicointegration

Based on the nature of macroeconomic data, cointegration has developed as an influential
concept in time series econometric analysis. Cointegration identifies a dynamic long run
relationship existing between integrated (non-stationary) variables moving together through time

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4 The NU flow variable used here is actually the NU (not in the labor force to unemployment) gross flow plus the
OU (other to unemployment) gross flow from the BLS gross flow data. The OU gross flows are individuals turning
16 and moving into unemployment. This OU flow is relatively small, and has no impact on tests for
multicointegration
based on an enforcement mechanism determined by economic forces, known as an error correction mechanism (Granger 1991). Since most macroeconomic data is non-stationary, integrated of order one, I(1), cointegration typically entails a special linear combination of two non-stationary I(1) variables yielding a stationary, I(0), residual (Engle and Granger 1987). This special linear combination defines a dynamic equilibrium path over time. Engle and Granger (1987) formally introduce the idea of error-correction models (ECMs) that capture and combine the long-run and short-run components of the underlying relationships between cointegrated variables.

Building on the cointegration work of Engle and Granger (1987), Granger and Lee (1989, 1990) introduce the possibility of a deeper form of cointegration known as multicointegration. In a multicointegration relationship, integrated variables are held together not only by the original cointegrating relationship, but also by a second linear relation between the accumulated sum of the residuals and the original variables (Granger and Lee 1989). Visually, this idea can be illustrated as a water tank with water continually flowing in through an input pipe and at the same time water flowing out through a separate output pipe. The water level is an inventory slowly changing over time depending upon the inflows and outflows. Thus, the concept of multicointegration is especially useful in modeling stock and flows as two distinct cointegrating relationships reflect both the equilibrium forces among the flow variables as well as a separate relationship existing between the stock and flow forces (Engsted and Haldrup 1999).

Cointegration, as introduced by Engle and Granger (1987), occurs if two, non-stationary variables \( x_t \) and \( y_t \) are combined into a unique linear relationship

\[^5\text{I(1) means that the variable must be first differenced one time to become stationary. I(0) means that the variable is stationary (first differenced zero times).}\]
such that $z_t$ is stationary, meaning integrated of order zero or $I(0)$. Further, according to Granger and Lee (1989), it follows that the accumulated sum of these residuals

$$Q_t = \sum_{j=0}^{t} z_{t-j}$$

will be integrated of order one, $I(1)$. If another linear relationship exists such that $Q_t$ is cointegrated with either, or both, of the original variables, $x_t$ or $y_t$, then a multicointegrating relationship exists between $x_t$ and $y_t$. This concept, thus, defines a relationship between flow and stock variables. This multicointegrating relationship suggests two different levels of cointegration between the two flow variables (Granger and Lee 1989). Engsted, Gonzalo, and Haldrup (1997) show that the presence of multicointegration among variables will cause a misspecification of the standard error correction model, thus invalidating hypothesis tests and leading to biased estimators in a typical cointegrated system. Therefore, identifying this multicointegrating relationship is necessary to introduce the appropriate specification into the error correction models for forecasting and hypothesis testing (Engsted, Gonzalo and Haldrup 1997).

The two-step multicointegration test introduced by Granger and Lee (1989) first estimates the proposed cointegrating relationship among the original variables as shown in Equation 3.a, $x_t = Ay_t + z_t$. The order of integration of the residual, $z_t$, is tested using the Augmented Dickey Fuller (ADF) test such that

$$\Delta \hat{z}_t = \rho_0 \hat{z}_{t-1} + \sum_{j=1}^{p} \rho_j \hat{z}_{t-j} + \nu_t$$

This may also be expressed as $x_t = Ay_t + z_t$ as it would be in a cointegrating regression. In this case the $z$'s are the residuals from a cointegrating regression.
A t-test on $\rho_0$ assesses the null hypothesis of a unit root, $(H_0: \rho_0 \geq 0)$ using a Dickey-Fuller distribution. If $\rho_0$ is significantly negative and the results of the ADF test reject the null hypothesis of a unit root, thus concluding that $z_t$ is stationary, then a separate multicointegrating regression is proposed between the accumulated sum of the residuals, $Q_t = \sum_{j=0}^{t} z_{t-j}$, and either of the original variables such that

$$w_{1t} = Q_t - B y_t \quad (3.d)$$

or

$$w_{2t} = Q_t - B x_t \quad (3.e)$$

Once again the residuals, $w_{1t}$ or $w_{2t}$, are tested using the ADF test in Equation 3.c. However, Granger and Lee (1989) recommend imposing the more stringent critical values from Engle and Yoo (1987) rather than the standard Dickey-Fuller test critical values. In this case, a rejection of the null hypothesis of a unit root implies not only stationarity of the residuals, but also multicointegration between $x_t$ and $y_t$ (Granger and Lee 1989). One recognized limitation of the Granger-Lee two step method for multicointegration is that the first cointegrating relationship must be known a priori. In other words, $Q_t$ must be available directly from a data source rather than estimated from the first cointegrating regression (Granger and Lee 1989) in order to prevent errors from the original regression from infecting the second cointegrating regression (Engle and Yoo 1991). In such cases where the first cointegrating relationship is not known beforehand, a one step procedure is necessary to accurately test for multicointegration (Engsted, Gonzalo and Haldrup 1997).

Granger and Lee (1990) show that in the presence of multicointegration the flow error correction model (ECM) may be estimated such that

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7 In regression form: $Q_t = B y_t + w_{1t}$ or $Q_t = B x_t + w_{2t}$
\[ \Delta x_t = \alpha + \gamma_1 z_{t-1} + \gamma_2 w_{t-1} + lagged(\Delta x_t, \Delta y_t) + residual \quad (3.f) \]

\[ \Delta y_t = \alpha + \gamma_1 z_{t-1} + \gamma_2 w_{t-1} + lagged(\Delta x_t, \Delta y_t) + residual \quad (3.g) \]

where \( z \) is the residual from the first cointegrating relationship between \( x \) and \( y \), and \( w \) is the residual from the cointegrating relationship between the accumulated sum of the residuals, \( Q \) and one of the original flow variables \( (x_t \text{ or } y_t) \). In addition, Lee (1996) presents a stock adjustment form of the error correction model such that

\[ \Delta Q_t = \alpha + \gamma_1 z_{t-1} + \gamma_2 w_{t-1} + \mu_1 \Delta y_t + lagged(\Delta x_t, \Delta y_t) + residual \quad (3.h) \]

where \( z \) is the residual from the first cointegrating relationship between \( x \) and \( y \), \( w \) is the residual from the cointegrating relationship between the accumulated sum of the residuals \( Q \) and one of the original variables \( (x_t \text{ or } y_t) \), and \( \Delta y_t \) is the concurrent first difference of \( y_t \). Equations 3.f and 3.g represent the error correction models (ECMs) for the flow variables, while equation 3.h presents the ECM for the stock variable.

The Engsted, Gonzalo, and Haldrup (1997) paper identifies an alternative single-step method to test for multicointegration among variables. By formulating the test as an I(2) system, Engsted, Gonzalo and Haldrup (1997) introduce the favorable statistical properties of systems integrated of order two identified by Johansen (1995) as well as the ability to test for multicointegration in instances where the original cointegrating relationship is not known a priori. Additionally, this single step method yields a super-super consistent cointegrating parameter estimate as well as the ability to include trend components in the multicointegrating relation. The super-super consistency of the cointegrating parameter estimates as well as the ability to incorporate linear trends into the model, aid in the empirical analysis of these models (Engsted, Gonzalo, and Haldrup 1997).
However, since most macroeconomic variables are integrated of order one, it is necessary to transform the two flow variables into I(2) variables to utilize the one step method. Engsted, Gonzalo and Haldrup (1997) propose the adoption of the I(2) cointegration analysis from Johansen (1995) by considering the accumulated sum of each of the I(1) flow variables such that, if $x_t$ and $y_t$ are both variables integrated of order one, then $\sum_{j=1}^{t} x_j = X_t$ and $\sum_{j=1}^{t} y_j = Y_t$ are each integrated up to order two, I(2). Using these transformed variables, the one-step method given by Engsted, Gonzalo and Haldrup (1997) may be used to test for multicointegration among the variables using the relation

$$Y_t = \alpha_0 + \delta t + \kappa_0 X_t + \kappa_1 \Delta X_t + \kappa_2 \Delta Y_t + \omega_t$$

(3.i)

where $t$ is the linear trend component. Once again, the ADF test in Equation 3.c is used to identify the order of integration of $\omega_t$ from Equation 3.i. Critical values for $\rho_0$ from this test are based on the distribution derived and reported in Haldrup (1994). According to Haldrup (1994), the distributions are determined by the number of I(1) regressors in the model, $(m_1)$ and the number of I(2) regressors, $(m_2)$. If, based on this ADF test, $\omega_t$ is stationary, the flow variables are multicointegrated.

Leachman, et al. (2005) use both the one and two step techniques to analyze the sustainability of fiscal budgeting using multicointegration to identify suitable policy response mechanisms. They find that the technique of multicointegration among government spending and revenue provides a useful test for sustainability in fiscal practices. Earlier work by Leachman and Francis (2000) also used the technique of multicointegration in analyzing the sustainability of foreign debt. Both papers employ the concept of multicointegration based on the one and two step methods identified above, providing a valuable example for empirical studies using multicointegration tests and analysis.
IV. Data and Relationships

a. Definitions

In Section II, we identified three stock variables: unemployment (U), employed (E), not in the labor force (N), and four flow variables impacting the stock of unemployment: unemployment to employment (UE), unemployment to not in the labor force (UN), employment to unemployment (EU), and not in the labor force to unemployment (NU) for this research. All data for unemployment (U), employed (E), and labor force (L) are based on the conventional definitions from the Bureau of Labor Statistics (BLS) focusing on the private, civilian, non-institutionalized population aged 16 and over (U.S. Bureau of Labor Statistics 2009). Within the civilian, non-institutionalized population 16 and over, an individual is either in the labor force (L) or not in the labor force (N). According to BLS statistics, persons without a job, who have actively searched for work within the past four weeks, and are available for work are considered unemployed (U). Individuals not matching these definitions for either employed or unemployed are considered “not in the labor force” (N). A particular subset of individuals “not in the labor force” are discouraged workers. Discouraged workers are individuals desiring to work, available for work, who have looked for work within the past 12 months, but have not looked for employment in the past 4 weeks. The number of discouraged workers may become particularly important during times of economic turmoil as they are likely to re-enter the workforce as prospects for employment improve. From these definitions, it is obvious that movements in the number of individuals in the labor force may have a significant, and sometimes, unexpected impact on unemployment measures specifically in regards to discouraged workers (U.S. Bureau of Labor Statistics 2009).
An accurate understanding of each labor market status is imperative for the definition of flows between these statuses and ultimately the unemployment inflow and outflow. We define inflow into unemployment and outflow from unemployment as

\[ inflow = EU + NU \]
\[ outflow = UE + UN \]

The inflow is the combination of the individuals moving from an employed status in time t-1 to an unemployed status in time t (EU) and individuals moving from outside the labor force in time t-1 to an unemployed status in time t (NU). Likewise, the outflow is the combination of individuals moving from an unemployed status in time t-1 to an employed status in time t (UE) and individuals moving from an unemployed status in time t-1 to out of the labor force in time t (UN). Using the inflows into and outflows from unemployment, the change in unemployment in period t can be defined as

\[ \Delta U_t = inflow_t - outflow_t \quad (4.a) \]

b. Data for Empirical Tests

We use seasonally adjusted, monthly data for the unemployment level, the unemployment rate, and the gross flows reported by the BLS based on the Current Population Survey. The gross flow data capture the movement of individuals between one labor market status to another from month-to-month (U.S. Bureau of Labor Statistics 2008) as summarized in the flow matrix in Figure 4. As a result of discrepancies in the data, referenced in Blanchard and Diamond (1990), BLS researchers implemented changes to this data series incorporating seasonal adjustments to correct many measurement errors that previously existed. This new data set of gross flows is available from February 1990 forward (Frazis, et al. 2005). Therefore, our analysis dataset covers the period from February 1990 through April 2010, and consists of 243 observations.
Figure 1 shows the gross inflow, gross outflow and the unemployment level over this time period. All data are available at www.bls.gov and are reported monthly in terms of thousands of people.

V. Multicointegration Applied to Unemployment

a. The Multicointegration Tests

In this section we will test for a multicointegrating relationship between the inflows into unemployment \( x_t \) and the outflows from unemployment \( y_t \) using both the Granger-Lee two-step method and the one-step method proposed by Engsted, Gonzalo, and Haldrup (1997). According to Equation 4.a, the change in the unemployment level in time \( t \) \( (\Delta U_t) \) equals the inflows into unemployment \( x_t \) minus the outflows from unemployment \( y_t \). Using the Augmented Dickey-Fuller test (ADF), we first test the level of integration of each separate variable, the unemployment level \( U_t \), the inflow into unemployment \( x_t \), and the outflow from unemployment \( y_t \). Table 1 displays the Dickey-Fuller test results indicating all three variables are indeed integrated of order one.

Similar to the inventory data analyzed by Granger and Lee (1989), the change in unemployment \( (\Delta U_t) \) is available directly from the data and does not have to be estimated from the cointegrating regression. Therefore, it is valid to use the two-step method to test for multicointegration. Using this method, we first test for a cointegrating relationship between the inflows and outflows of unemployment using a proposed cointegrating regression of

\[
x_t = \alpha_0 + \alpha_1 y_t + z_t
\]  

(5.a)

where \( x_t \) is the inflow into unemployment in time \( t \), \( y_t \) is the outflow from unemployment in time \( t \), and \( z_t \) is the residual. Intuitively, a cointegrating relationship with a coefficient close to 1.0
between the inflow into unemployment, $x_t$, and the outflow from unemployment, $y_t$, is consistent with economic theory as well as with empirical evidence regarding unemployment. While short term deviations between the inflow, $x_t$, and outflow, $y_t$, of unemployment may occur, in the long run cointegration is expected between these two variables based on factors including frictional unemployment, economic stability, and the natural rate of unemployment hypothesis. Frictional unemployment, the constant movement of workers between jobs, prevents the unemployment level from reaching zero at any given time, thus providing a downward constraint precluding the inflow from remaining at levels lower than the outflow from unemployment for long periods of time. Likewise, if the inflow into unemployment persists at long run levels higher than the outflow from unemployment, then the unemployment rate would gradually approach 100%. The economic needs of a society will prevent such an occurrence. These upper and lower constraints force the economy to its natural rate of unemployment, its equilibrium rate of unemployment, based on the rates of job separation, inflow into unemployment, and job finding, outflow from unemployment (Barro 1997). Hence these upper and lower constraints among the flow into and out of unemployment force a cointegrating relationship between the two variables. Using the ADF to test the residual $z_t$ for stationarity, we find that $x_t$ and $y_t$ are, in fact, cointegrated with a 5% significance level, see Table 3.A1, and the coefficient is estimated to be 1.01.

Given $U_t = \Sigma z_t$ and the unemployment level ($U_t$) available from the empirical data set, we now proceed to the second portion of the Granger and Lee two-step method testing for a cointegrating relationship between the unemployment level ($U_t$) and each of the original flow variables, $x_t$ and $y_t$. According to Granger and Lee (1989), there are four potential cointegrating regressions, any of which would indicate multicointegration between variables $x_t$ and $y_t$. The four cointegrating regressions tested are
All residuals were tested using the ADF test and are reported in Table 2. According to Engle and Yoo (1987), for a sample size of 200 or greater, the critical value for rejection of the null hypothesis of a unit root is -3.25 at the 5% significance level. Table 2 shows that all four cointegrating regressions produce stationary residuals. Thus, we have found multicointegration among these variables. The cointegrating relationship from Equation 5.e is the most robust to changes in time period and the most significant in estimating the ECM. Therefore, this equation is chosen as our second cointegrating relationship.

In addition to the two-step Granger-Lee method, the one-step method introduced by Engsted, Gonzalo, and Haldrup (1997) is also used to test for a multicointegrating relationship. As detailed in Section III, in order to test for multicointegration using the one step method, we first transform the flow variables into I(2) variables by cumulating the I(1) flow variables such that $\sum_{j=1}^{t} x_j = X_t$ and $\sum_{j=1}^{t} y_j = Y_t$. Under a null hypothesis that no multicointegration is present, we may now test for the presence of a multicointegrating relationship according to the one-step method as given in equation 3.i. As Haldrup (1997) indicates, the cumulation of these flow variables does in fact produce a deterministic trend. Therefore, a trend component is included in the multicointegrating regression. Using the transformed values of $Y_t$ and $X_t$, and a trend component ($t$), we find the multicointegrating regression to be

$$Y_t = \delta_0 + \delta_1 t + \delta_2 X_t + \delta_3 X_t + \delta_4 Y_t + \eta_t$$

(5.f)
where $X_t = \sum_{j=1}^{t} x_j$, $Y_t = \sum_{j=1}^{t} y_j$, $t$ is the trend, $\Delta X_t$ is the inflow in time $t$, and $\Delta Y_t$ is the outflow in time $t$.\(^8\) Results for this regression are summarized in Table 3.B. Again using the Augmented Dickey-Fuller test, we test the residuals $\eta_t$ under the null hypothesis of a unit root. The ADF test statistic for the residuals is -5.13. Using the distribution of the test statistics derived and reported in Haldrup (1994), the critical value for one I(1) variable (the trend) and one I(2) variable with a sample size of 240 is -4.19 at a 5% significance level. Based on this critical value, we can reject the null hypothesis of a unit root and conclude that the residual $\eta_t$ is stationary (~I(0)), and we reject the null hypothesis that no multicointegration is present. Once again we find multicointegration between these variables.

According to Engsted and Haldrup (1999), the coefficient on $X_t$, $\kappa_0$ is a super-super consistent estimate of the magnitude of the change in one accumulated flow variable to the other. As expected, in order to prevent unemployment from rising or falling without bound, the $\kappa_0$ coefficient on $X_t$ is very close to one. The magnitude of $\kappa_0 = 1$ reveals that on average across the sample period of 1990 through 2010 the accumulated outflow from unemployment moved at a similar pace to the accumulated inflow into unemployment.

Thus, both the two-step and one-step methods (results summarized in Table 3) offer strong evidence of a multicointegrating relationship between the inflow into unemployment and outflow from unemployment, implying that two long-run equilibrium relationships exist between the two flow variables and the stock variable, the level of unemployment. Additionally, these two long-run equilibrium relationships can be modeled using error correction models (ECMs).

---

\(^8\) Note that in the use of the accumulated notation that $\Delta X_t = x_t = \Delta x = \Delta X$ the inflows into unemployment in time $t$ and $\Delta Y_t = y_t = \Delta y = \Delta Y$ the outflows from unemployment in time $t$
b. **Error Correction Models**

Using the results from the two-step test, we estimate error correction models for both the stock and flow variables. Since the presence of multicointegration leads to a misspecification of the standard error correction model (Engsted, Gonzalo, and Haldrup 1997), the strong evidence of multicointegration necessitates the formulation of a multicointegration ECM to appropriately include both cointegrating relationships. Granger and Lee (1990) show that in the presence of multicointegration the flow ECM may be estimated such that

\[
\Delta x_t = \alpha_0 + \gamma_1 z_{t-1} + \gamma_2 \xi_{t-1} + \text{lagged}(\Delta x_t, \Delta y_t) + \text{residual} \quad (5.g)
\]

and

\[
\Delta y_t = \alpha_1 + \delta_1 z_{t-1} + \delta_2 \xi_{t-1} + \text{lagged}(\Delta x_t, \Delta y_t) + \text{residual} \quad (5.h)
\]

where \( z_{t-1} \) is the residual from the first cointegrating relationship between \( x_{t-1} \) and \( y_{t-1} \) (from equation (5.a)) and \( \xi_{t-1} \) is the residual from the cointegrating relationship between unemployment (U) and y (from equation (5.e)). In addition, Lee (1996) presents a stock adjustment error correction model based on a constant, the lagged residuals from both cointegrating relationships, the first difference of the concurrent outflow (\( \Delta y_t \)), and lagged values of \( \Delta x_t \) and \( \Delta y_t \) such that our unemployment model is

\[
\Delta U_t = \alpha_3 + \beta_1 z_{t-1} + \beta_2 \xi_{t-1} + \mu_1 \Delta y_t + \text{lagged}(\Delta x_t, \Delta y_t) + \text{residual} \quad (5.i)
\]

A time trend component is included in each ECM to account for the continual population growth and the consequent growth in the labor supply over the sample period. Results for all three ECMs are given in Table 4. The number of included lagged differences of the flow variables is based on the minimization of the Akaike Information Criterion (AIC) as well as for correction of serial correlation within the model. As detailed in Table 4, LM tests indicate no significant serial correlation exists within the error correction models. Additionally, any issues with
heteroskedasticity are remedied using the White Standard Error Corrections, again noted in Table 4.

The results summarized in Table 4 indicate that the ECM for outflow ($\Delta y_t$) is much stronger in terms of significance and $R^2$ than the ECM for inflow ($\Delta x_t$) suggesting outflow may be more responsive to the cointegrating relationship. Furthermore, the inclusion of the concurrent first difference of outflow in the stock ECM significantly improves the forecasting ability and robustness for modeling the change in unemployment. Granger and Lee (1989) found similar results among the ECMs of production, inventory, and sales, attributing the stronger ECM for $\Delta p_t$ in part to evidence that “production is a controllable variable, and the control mechanism may well react to the value of the previous $z_t$.” These similarities are noted for future research regarding the controllability aspects of the flow variables. If additional research shows outflow is the more controllable variable contributing a stronger control mechanism to unemployment, these findings could be valuable to focus policy regarding unemployment.

VI. Forecasting

In order to test the forecasting ability of the error correction models specified in this research, we completed an out-of-sample ex-post forecast using four separate models for the unemployment level. Due to the economic volatility beginning in late 2007, we selected an ex-post sample period of 2006.01 through 2007.06. Results of our forecasting exercise are given in Appendix I. Four models were used to calculate the unemployment level over the forecast period. The four models tested were: (1) the stock error correction model (ECM) specified in this research in Table 4 using forecasts for inflow and outflow based on the flow error correction models (ECM) from this research also in Table 4, (2) the stock error correction model (ECM)
from this research in Table 4 with forecasts for inflow and outflow based on various aggregate economic indicators, (3) a model for the unemployment level itself based on aggregate economic indicators, and finally (4) an ARIMA model on the unemployment level. Each model was estimated over the sample period 1990.02 through 2005.12. We then calculated forecasts of the unemployment level using each model over the ex-post forecast period of 2006.01 through 2007.06. These forecasts for the unemployment level were then compared to the actual unemployment level, and the root mean squared forecast error (RMSFE) and root mean squared percent error (RMSPE) were calculated for each model at the 6-month, the 12-month and the 18-month forecast periods. All results are summarized by Table A.1 and Figure A.1 in the Appendix.

As illustrated in Table A.1 and Figure A.1 in the Appendix, over the ex-post forecast period, the use of the error correction models from this research improve the forecasts for the unemployment level compared to forecasts of the unemployment level from the aggregate economic indicator model for unemployment and the ARIMA model. Improved forecasts were evident through smaller RMSFEs and RMSPEs in both forecasts using the stock error correction model specified in this research.

VII. Conclusion

Unemployment is a dynamic and complex phenomenon that cannot be explained solely through the aggregate statistics. A true representation of changes in unemployment activity depends on understanding the movement in the underlying flows through the labor market, a point highlighted by Blanchard and Diamond (1990). Through this research we have shown that a multicointegrating relationship exists between the flows into and out of unemployment and
with the level of unemployment. We have captured the long-run relationship between these variables through an error correction model which naturally incorporates the inseparability of the flows into and out of unemployment mentioned by Elsby, et al. (2009). We have also shown that incorporating these long run relationships into a model for the unemployment level improves the forecasts of the unemployment level over a relatively stable ex-post forecast period when compared to ARIMA and another linear model for unemployment based on various economic indicators. Thus, through the identification of multicointegration and the use of multicointegration techniques, we have provided a method to integrate the dichotomy of stocks and flows into an econometric model for the unemployment level.

The identification of these relationships among the stock and flow variables is pivotal to explaining unemployment. However, additional work remains. As indicated through various studies, the impact of economic conditions on these flow variables varies in magnitude, intensity, direction, duration, and possibly even controllability. Particularly in the identification of the impact of various aggregate economic indicators on the flows into and out of unemployment, subsequent work on the structural models of the flow equations could improve the forecasting ability of this error correction model. Research particularly focused on the concept of controllability of the outflow may also prove useful in policy initiatives and directions. Overall, the behavioral relationships we identified through the technique of multicointegration provide a framework for future research. Using this research as a foundation, we hope to improve this model through simultaneously estimating the inflow into unemployment, the outflow out of unemployment and their relationship with changes in the unemployment level using the specified error correction model. Building on the foundations established here, we hope to learn more about the symptoms of unemployment for early detection and perhaps even prevention.
References


FIGURE 1: Graph of Gross Flows and Unemployment Level from 02/1990 through 04/2010 (in thousands)\(^9\)

Unemployment and Flows (in thousands)

\[\text{Data from Bureau of Labor Statistics from Current Population Survey.}\]
FIGURE 2  Diagram of Inventory of Unemployed Workers as a Stock and Flow Process

- Quit
- Layoff
- Fired
- Enter Labor Force Population (16+)
- Encouraged Worker
- Enter/Re-enter Labor Force

INFLOW

UNEMPLOYMENT (U)

OUTFLOW

- Individual finds a job
- Becomes discouraged
- Retire
- Leave workforce to care for family, return to school, illness, etc
- Death
FIGURE 3  Graph of U.S. Unemployment Rate 1948-2010

Graph from St. Louis Federal Reserve Economic Database, (FRED), May 2010.
FIGURE 4  LABOR FORCE STATUS FLOWS

<table>
<thead>
<tr>
<th>Status in prior month</th>
<th>Status in current month</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Employed</td>
</tr>
<tr>
<td>Employed</td>
<td>EE</td>
</tr>
<tr>
<td>Unemployed</td>
<td>UE</td>
</tr>
<tr>
<td>Not in labor force</td>
<td>NE</td>
</tr>
</tbody>
</table>

Labor force status flows\(^1\)

1 Matrix provided by the Bureau of Labor Statistics, the first letter of each flow gives the status of the individual in the previous month followed by the second letter giving the status in the current month. Consistent with the notation used by the Bureau of Labor Statistics as well as much of the previous work using these labor force status flows, the transition matrix illustrates the relevant labor force status flows. In this matrix, individuals’ labor market status in the previous period are represented in the left vertical column, while their labor market status in the current period is represented across the top row. As an individual moves from one labor market status to another labor market status, they are included in the appropriate “flow” statistic. For example, if an individual was unemployed in the previous period (t-1) but is hired by an employer for pay during period t, that individual is included in the flow statistic, UE, meaning that he moved from unemployment in period t-1 to employment in period t. In such a case, he is considered as an outflow from unemployment.
**TABLE 1: ADF Unit Root test results for unemployment level, inflow and outflow**

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF Test Statistic in Levels</th>
<th>ADF Test Statistic in 1st Differences</th>
<th>Conclusion⁷</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment Level (Uₜ)</td>
<td>-.042</td>
<td>-4.893***</td>
<td>Unit Root Exists in levels only; Uₜ~I(1)</td>
</tr>
<tr>
<td>Inflow (xₜ)</td>
<td>.137</td>
<td>-25.477***</td>
<td>Unit Root Exists in levels only; xₜ~I(1)</td>
</tr>
<tr>
<td>Outflow (yₜ)</td>
<td>.0499</td>
<td>-26.756***</td>
<td>Unit Root Exists in levels only; yₜ~I(1)</td>
</tr>
</tbody>
</table>

⁷Critical Values Dickey-Fuller Statistics = -2.873 based on α=.05 significance level and CV =-3.457 based on α=.01 significance Level; ***significant at .01 significance level

**TABLE 2: ADF tests for residuals of 2nd proposed cointegrating relationship**

<table>
<thead>
<tr>
<th></th>
<th>ωₜ</th>
<th>uₜ</th>
<th>vₜ</th>
<th>ξₜ</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF test statistic</td>
<td>-4.47</td>
<td>-4.00</td>
<td>-4.87</td>
<td>-4.46</td>
</tr>
</tbody>
</table>

Engle and Yoo 1987 indicate critical value = -3.25 based on α=.05 significance Level. All residuals found to be stationary.
TABLE 3. Multicointegration Tests based on Two-Step and One Step Methods

A. Granger and Lee Two-Step Procedure

Step 1: Inflow\(_t\) = -.755 + 1.01* Outflow\(_t\) + \(z_t\)
Adj. \(R^2\) = .810
DW = 1.61
ADF\((z_t)\)\(^a\) = -4.89**

Step 2: Outflows\(_t\) = 2150.742 + .2018* Unemployment\(_t\) + \(\xi_t\)
Adj. \(R^2\) = .848
DW = 1.218
ADF\((w_t)\)\(^a\) = -4.46**

B. Single Equation Procedure from Engsted, Gonzalo, and Haldrup (1997)

\[\sum \text{OUTFLOW}_t = 15185.00 + 1.001* \sum \text{INFLOW}_t - 2.747* \text{INFLOW}_t - 1.718* \text{OUTFLOW}_t - 2.187* \text{TIME} + \eta_t\]

Adj. \(R^2\) = .99999
DW = .734
ADF\((v_t)\)\(^a\) = -5.13**

---

\(^a\)ADF: Augmented Dickey Fuller test statistic; \(z_t\), \(w_t\), and \(v_t\) are residual values from OLS regressions each tested under the null hypothesis of a unit root.

\(^a\) \(\alpha = .05\) Critical Values are -3.25 for two-step procedure based on Engle and Yoo (1987) and -4.19 for the one-step procedure based on Engsted, Gonzalo and Haldrup (1997) for \(m_1 = I(1)\) regressors = 1 and \(m_2 = I(2)\) regressors = 1.

**Significant at 5% significance level.
## TABLE 4: Multicointegration Error Correction Models

<table>
<thead>
<tr>
<th></th>
<th>Inflow ECM</th>
<th>Outflow ECM</th>
<th>Stock ECM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta x ) (Change in Inflow)</td>
<td>( \Delta y ) (Change in Outflow)</td>
<td>( \Delta U ) (Change in Unemployment)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>-46.101</td>
<td>-1.71</td>
<td>-11.919</td>
</tr>
<tr>
<td></td>
<td>(-1.84)*</td>
<td>(.086)</td>
<td>(-.473)</td>
</tr>
<tr>
<td><strong>ECT1_{t-1} = z_{t-1}</strong>  (flow error correction term)</td>
<td>-160</td>
<td>.382</td>
<td>.875</td>
</tr>
<tr>
<td></td>
<td>(-1.49)</td>
<td>(5.092)***</td>
<td>(7.853)***</td>
</tr>
<tr>
<td><strong>ECT2_{t-1} = \xi_{t-1}</strong>  (stock error correction term)</td>
<td>-273</td>
<td>-2.214</td>
<td>-2.79</td>
</tr>
<tr>
<td></td>
<td>(-2.67)**</td>
<td>(-2.879)***</td>
<td>(-2.649)***</td>
</tr>
<tr>
<td>( \Delta y_t )  (first difference outflow)</td>
<td></td>
<td></td>
<td>-1.078</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-12.446)***</td>
</tr>
<tr>
<td><strong>Time trend (\delta)</strong></td>
<td>.467</td>
<td>.145</td>
<td>.482</td>
</tr>
<tr>
<td></td>
<td>(2.56)**</td>
<td>(.997)</td>
<td>(2.628)***</td>
</tr>
<tr>
<td><strong>\Delta x_{t-1}</strong>  (lagged change in inflow)</td>
<td>-460</td>
<td>-.071</td>
<td>-472</td>
</tr>
<tr>
<td></td>
<td>(-4.358)***</td>
<td>(-1.136)</td>
<td>(-4.450)***</td>
</tr>
<tr>
<td><strong>\Delta x_{t-2}</strong>  (lagged change in inflow)</td>
<td>-.173</td>
<td>-.330</td>
<td>.122</td>
</tr>
<tr>
<td></td>
<td>(-2.112)**</td>
<td>(-5.454)***</td>
<td>(-2.155)***</td>
</tr>
<tr>
<td><strong>\Delta y_{t-1}</strong>  (lagged change in outflow)</td>
<td>.263</td>
<td>-.174</td>
<td>.122</td>
</tr>
<tr>
<td></td>
<td>(3.331)***</td>
<td>(-3.066)***</td>
<td>(1.763)*</td>
</tr>
<tr>
<td><strong>\Delta y_{t-2}</strong>  (lagged change in outflow)</td>
<td>.129</td>
<td>-.130</td>
<td>-.131</td>
</tr>
<tr>
<td></td>
<td>(1.883)*</td>
<td>(-2.195)**</td>
<td>(-2.487)**</td>
</tr>
<tr>
<td><strong>\Delta y_{t-3}</strong>  (lagged change in outflow)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>\Delta y_{t-4}</strong>  (lagged change in outflow)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Adj R^2</strong></td>
<td>.264</td>
<td>.563</td>
<td>.513</td>
</tr>
<tr>
<td><strong>Std Error</strong></td>
<td>154.17</td>
<td>115.66</td>
<td>154.67</td>
</tr>
<tr>
<td><strong>LMserial corr</strong> (( \chi^2 ))(^1)</td>
<td>5.218</td>
<td>4.854</td>
<td>5.52</td>
</tr>
<tr>
<td><strong>LMhetero</strong> (( \chi^2 ))(^2), (( \chi^2 ))(^3)</td>
<td>45.08</td>
<td>62.815(^d)</td>
<td>51.092</td>
</tr>
<tr>
<td><strong>n</strong></td>
<td>240</td>
<td>238</td>
<td>240</td>
</tr>
</tbody>
</table>

*significant at \( \alpha=.10 \);  ** significant at \( \alpha=.05 \);  *** significant at \( \alpha=.01 \)

\(^1\)Critical Value for (\( \chi^2 \))\(^1\).05=9.49
\(^2\)Critical Value for (\( \chi^2 \))\(^2\).05=43.77
\(^3\)Critical Value for (\( \chi^2 \))\(^3\).05=55.76
\(^4\)LM test statistic based on White test indicates heteroskedasticity exists, thus the White standard error correction used; all t-statistics are reported using the White standard error correction.
APPENDIX I—Ex-Post Forecast

TABLE A.1: Comparison of Ex-Post Forecast Models

Root Mean Squared Forecast Error (RMSFE) and Root Mean Squared Percent Error (RMSPE)

RMSFE and RMSPE error are calculated based on formulas:

\[
\text{RMSFE} = \sqrt{\frac{1}{T} \sum_{t=1}^{T} [(U_t^f - U_t^a)^2]}, \quad \text{and} \quad \text{RMSPE} = \sqrt{\frac{1}{T} \sum_{t=1}^{T} \left( \frac{(U_t^f - U_t^a)}{U_t^a} \right)^2}
\]

where \( T \) is the number of forecasted periods, \( U_t^f \) is the forecasted unemployment level and \( U_t^a \) is the actual unemployment level. Based on the four models, the RMSFE and RMSPE forecast error for the ex-post forecast period 2006.01 through 2007.06 are displayed below.

<table>
<thead>
<tr>
<th>RMSFE and RMSPE Percent Forecast Error</th>
<th>ECM Stock/ECM Flow</th>
<th>ECM Stock/Structural Flow</th>
<th>Structural Stock</th>
<th>ARIMA(3,1,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T=6 )</td>
<td>RMSFE: 122.84, RMSPE: 1.74%</td>
<td>RMSFE: 102.52, RMSPE: 1.45%</td>
<td>RMSFE: 220.25, RMSPE: 3.13%</td>
<td>RMSFE: 128.02, RMSPE: 1.82%</td>
</tr>
<tr>
<td>( T=12 )</td>
<td>RMSFE: 124.50, RMSPE: 1.79%</td>
<td>RMSFE: 131.24, RMSPE: 1.90%</td>
<td>RMSFE: 311.59, RMSPE: 4.45%</td>
<td>RMSFE: 157.48, RMSPE: 2.29%</td>
</tr>
<tr>
<td>( T=18 )</td>
<td>RMSFE: 127.33, RMSPE: 1.83%</td>
<td>RMSFE: 128.62, RMSPE: 1.87%</td>
<td>RMSFE: 342.01, RMSPE: 5.00%</td>
<td>RMSFE: 147.59, RMSPE: 2.15%</td>
</tr>
</tbody>
</table>
FIGURE A.1: Forecast Comparison (Ex-Post Forecast 2006.01 through 2007.06)