Electromagnetism HW 4 – conductors

due Mon 5th Oct

Exercise 1. A spherical conducting shell of radius b is concentric with and encloses a conducting spherical ball of radius a.

1.1 Suppose the shell is grounded and a charge Q is on the ball. How much charge lies on the inner and outer surfaces of the grounded shell?

1.2 Suppose the ball is grounded and there is charge Q on the shell. How much charge lies on (a) the ball, (b) the inner surface of the shell and (c) the outer surface of the shell ?

Exercise 2. Two parallel infinite conducting planes are held at zero potential at $z = -d$ and $z = d$. An infinite sheet with uniform charge per unit area, σ, is placed between them (lying parallel) at a position, z'.

2.1 Show that the electric field for $z \leq -d$ and $z \geq d$ must be zero and explain on which surfaces of the plates is charge induced.

2.2 Find the charge density induced on each grounded plate.

2.3 Show that a force per unit area of $\frac{\sigma^2}{2\epsilon_0 d} z' \hat{z}$ is felt by the sheet of charge.

Exercise 3. A perfect conductor contains a vacuum cavity of arbitrary shape. We showed in class that $\vec{E} = 0$ in the cavity. Here we'll consider a different proof of the same result.

First we'll derive *Earnshaw's theorem*: "The scalar potential in a finite, charge-free region of space, V, takes its maximum or minimum values only on the boundary of V."

3.1 Suppose $\phi(\vec{r})$ has a local minimum at some point P inside V. Then $\hat{n} \cdot \vec{\nabla}\phi > 0$ at all points on a small surface surrounding P, and it follows that

$$\int_S d\vec{S} \cdot \vec{\nabla}\phi > 0.$$

Show that this equation implies that $\text{div}\vec{E} \neq 0$ in contradiction to the statement that region R is charge-free. It follows that there cannot be a minimum of $\phi(\vec{r})$ in V.

3.2 Explain how Earnshaw's theorem, along with the statement that $\vec{E} = 0$ in a perfect conductor, ensures that $\vec{E} = 0$ in a charge-free cavity.

Exercise 4. The electric field strength at the surface of a *real* conductor falls from its external value of E_0 to zero within the conductor in a finite distance, δ. There is a corresponding volume density of charge, $\rho(x)$ near the surface of the conductor. Show that the quantity

$$\int_0^\delta dx\, \rho(x),$$

which we may associate with the surface charge density in a perfect conductor, has value $\epsilon_0 E_0$ as we'd expect.

Exercise 5. A spherical metal shell carries a charge Q. Suppose this shell is cut in half and the two halves pulled infinitesimally apart.

5.1 Find the force of repulsion between the two hemispheres.

5.2 If we place a point charge at the origin we can prevent the hemispheres from flying apart. What charge would achieve this?