Electromagnetism HW 5 – dielectrics

due Wed 14th Oct

Exercise 1. A polarizable sphere of radius R, having dielectric constant κ, is filled with free charge of uniform density ρ_f.

1.1 Find the polarization, $\vec{P}(\vec{r})$

1.2 Find the volume polarization charge density and the surface polarization charge density and confirm that the total polarization charge is zero.

Exercise 2. A spherical conductor of radius R_1 is surrounded by a polarizable medium which extends from R_1 to R_2 with dielectric constant κ.

2.1 If the conductor carries a charge Q, find $\vec{E}$ everywhere and the distribution of polarization charge, and confirm that the total polarization charge is zero.

2.2 If the conductor is grounded and the entire system placed in a uniform electric field $\vec{E}_0$, find the potential everywhere and determine how much charge is drawn up from ground to the conductor.

[*Hint:* If the z-axis is chosen to be in the direction of $\vec{E}_0$, the system is independent of ϕ and the potential must take the form

$$\varphi(r > R_2, \theta) = \sum_\ell A_\ell \, r^\ell P_\ell(\cos\theta) + \sum_\ell B_\ell \, \frac{1}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$\varphi(R_1 < r < R_2, \theta) = \sum_\ell C_\ell \, r^\ell P_\ell(\cos\theta) + \sum_\ell D_\ell \, \frac{1}{r^{\ell+1}} P_\ell(\cos\theta).$$

Apply suitable boundary and matching conditions to determine the coefficients.]

Exercise 3. The dielectric constant of the dielectric between the plates of a parallel plate capacitor varies linearly with distance from one plate to the other. If the values at the two plates are κ_1, κ_2, where $\kappa_2 > \kappa_1$, and the plates are separated by a distance, d, show that the capacitance per unit area is

$$\frac{\epsilon_0}{d}\frac{\kappa_2 - \kappa_1}{\log \kappa_2/\kappa_1}.$$

[*Hint:* if the plates carry charge Q and $-Q$ respectively, the capacitance can be defined as $Q/$(difference in potential between the plates)]

Exercise 4. An infinitely long cylindrical shell of dielectric has inner radius a and outer radius b and dielectric constant κ. Suppose this object is placed in a previously uniform electric field of magnitude E_0 with the cylinder axis perpendicular to the field.

4.1 Find the potential and electric field everywhere.

[*Hint:* the most general solution to Laplace's equation in cylindrical coordinates takes the form,

$$\varphi(\rho, \phi) = A + B \log \rho + \sum_{m=1}^{\infty} \left(C_m \, \rho^m + D_m \frac{1}{\rho^m} \right) (E_m \sin m\phi + F_m \cos m\phi),$$

but the $\sin m\phi$ terms aren't required in this case (why not?)]

4.2 Find the polarization surface charge distributions.

4.3 Examine the limiting case that gives a solid dielectric cylinder. Approximately sketch the field lines in this case, and indicate the regions of high and low surface charge density.

4.4 Examine the limiting case that gives a cylindrical cavity in a uniform dielectric.