HW 6 - SPECIAL RELATIVITY

All problems due April 6th.

1. Rapidity

For a boost of speed v in the z-direction, the Lorentz transformation for the variables $z_\pm \equiv ct \pm z$ can be written $z'_\pm = e^{\mp\eta} z_\pm$, where η is called the rapidity.

Find an equation for η as a function of v.

Show that a boost with rapidity η_1 followed by a boost with rapidity η_2 is equivalent to a single boost of rapidity $\eta_1 + \eta_2$, and using this result derive again the relativistic velocity addition formula.

2. Doppler shift

Show that if an emitter of radio waves is receding from a stationary observer with a constant speed of $v = \beta c$, then the frequency ν_0 of the wave in the reference frame of the emitter and the frequency ν in the reference frame of the receiver are related by

$$\nu = \nu_0 \sqrt{\frac{1-\beta}{1+\beta}}.$$

[Try to construct the derivation yourself without looking it up in a book – think about the time between arriving wavefronts as measured by the observer]

3. Reflection from a moving mirror

A plane mirror moves perpendicular to its plane surface at constant speed $v = \beta c$. If in the lab frame, the angle of incidence of light is θ_1, show that the angle of reflection, θ_2, is given by

$$\sin\theta_2 = \sin\theta_1 \frac{1-\beta^2}{1+\beta^2-2\beta\cos\theta_1}$$

4. Worldlines

The figure shows the *worldlines* of two identical clocks that move from A to B, $x = v_0 t$ and $x = \frac{1}{2}a_0 t^2$ with v_0, a_0 constants. For each clock find the proper time between A and B and state which clock has the smaller proper time.