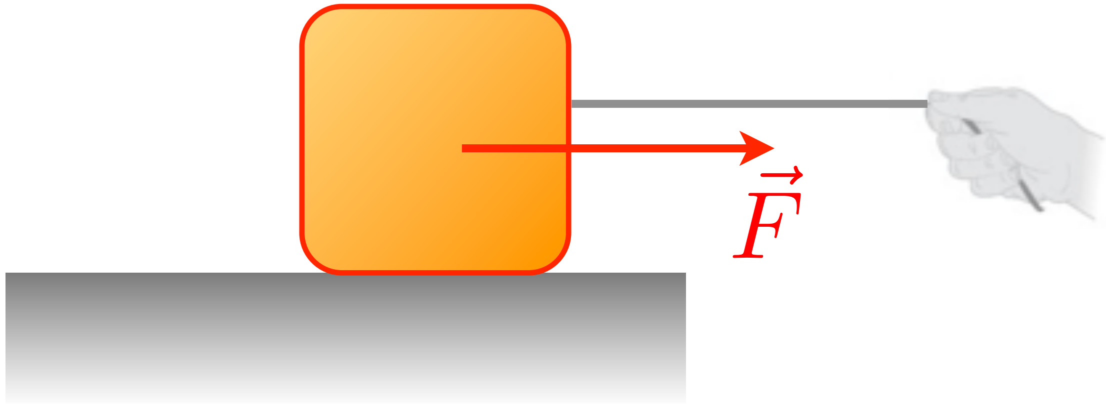
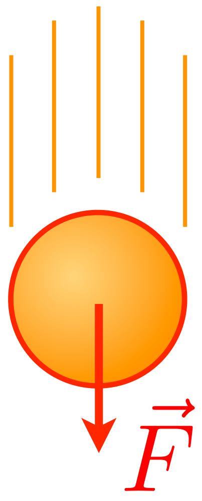
# forces & Newton’s laws of motion

# forces (examples)


a **push** is a force



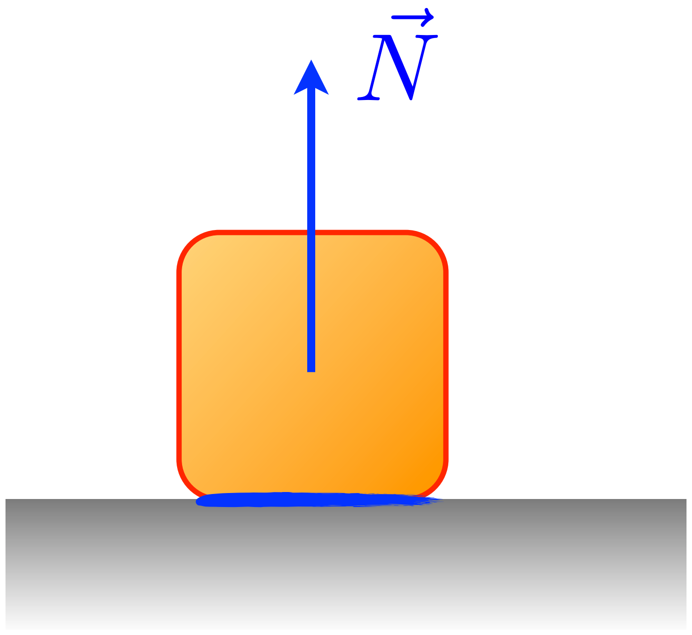

a **pull** is a force

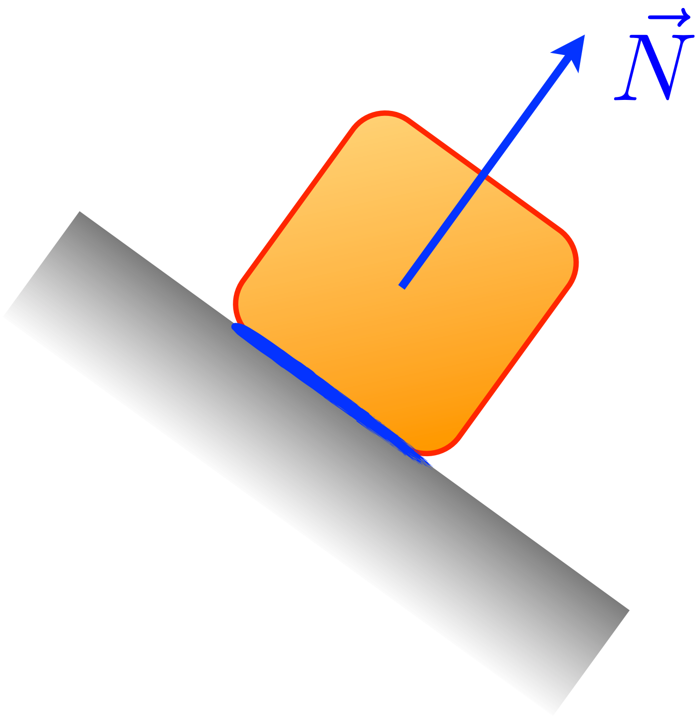



**gravity** exerts a force between all massive objects (without contact)

(the force of attraction from the Earth is called the **weight** force)

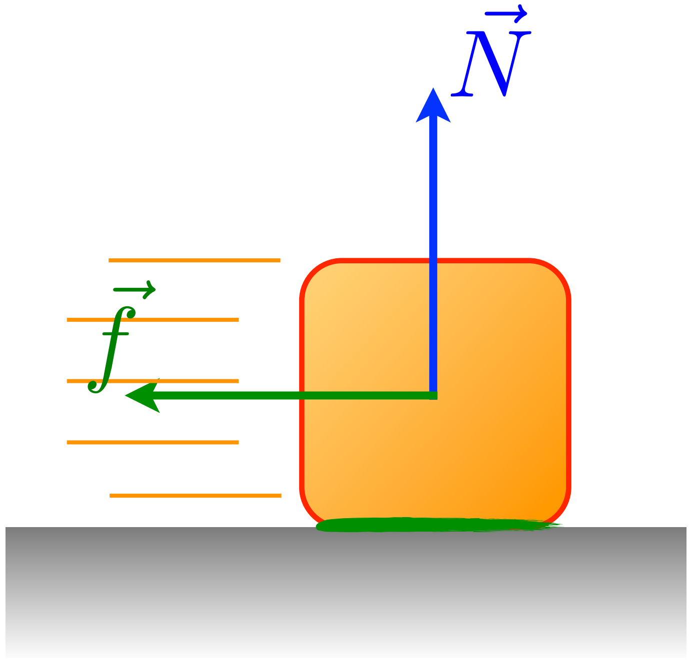
# contact forces




a **normal** force occurs when an object pushes on a surface

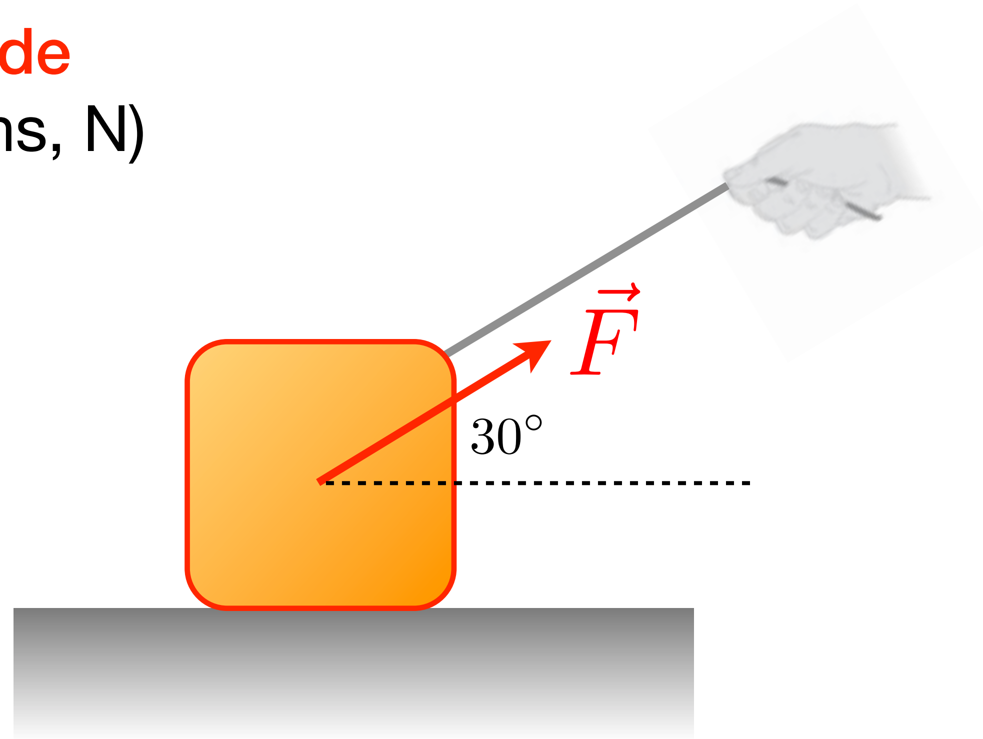
the force is perpendicular to the surface



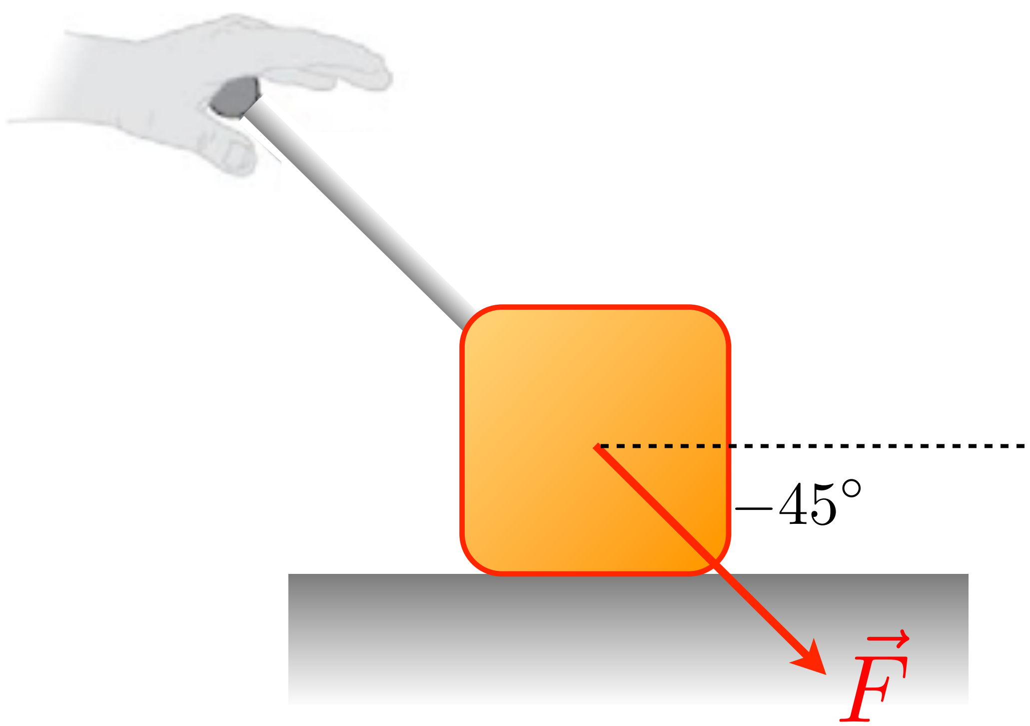

a **friction** force can occur parallel to the surface of contact

# force vectors

forces have magnitude
(measured in Newtons, N)
and direction



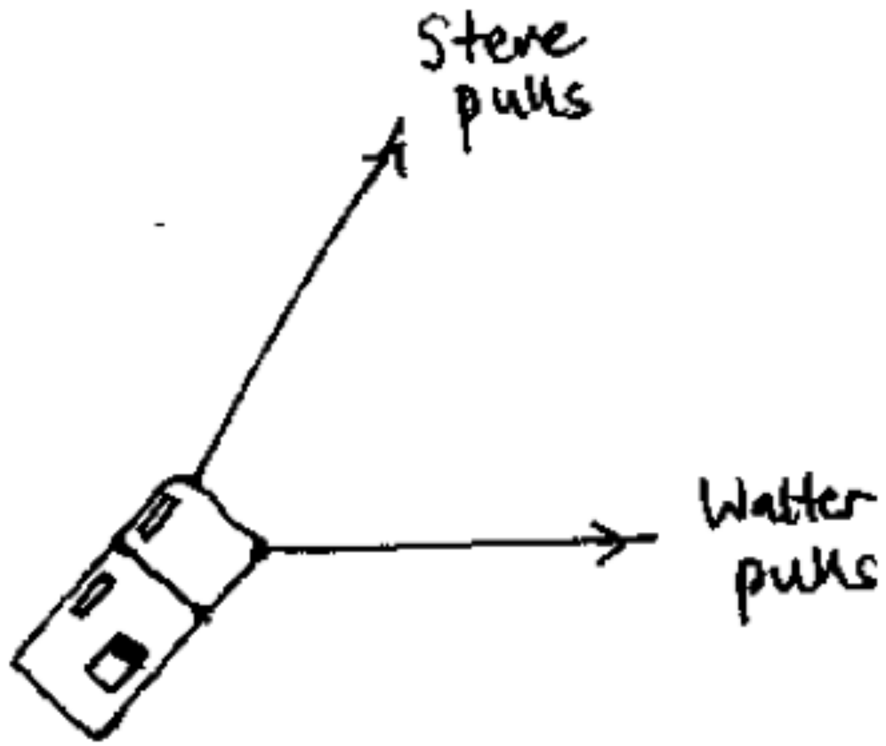

e.g. $\left|\vec{F}\right| = 10\,\mathrm{N}$

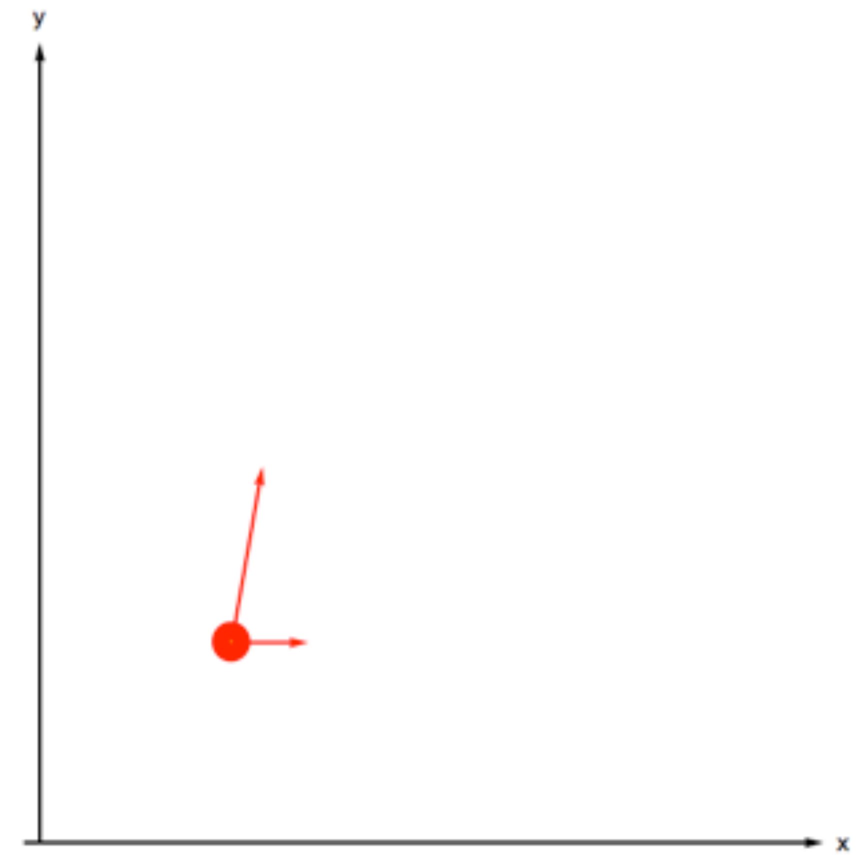


# pulling a fridge - resultant force

two guys are moving a fridge by pulling on ropes attached to it

Steve is very strong, Walter is much weaker



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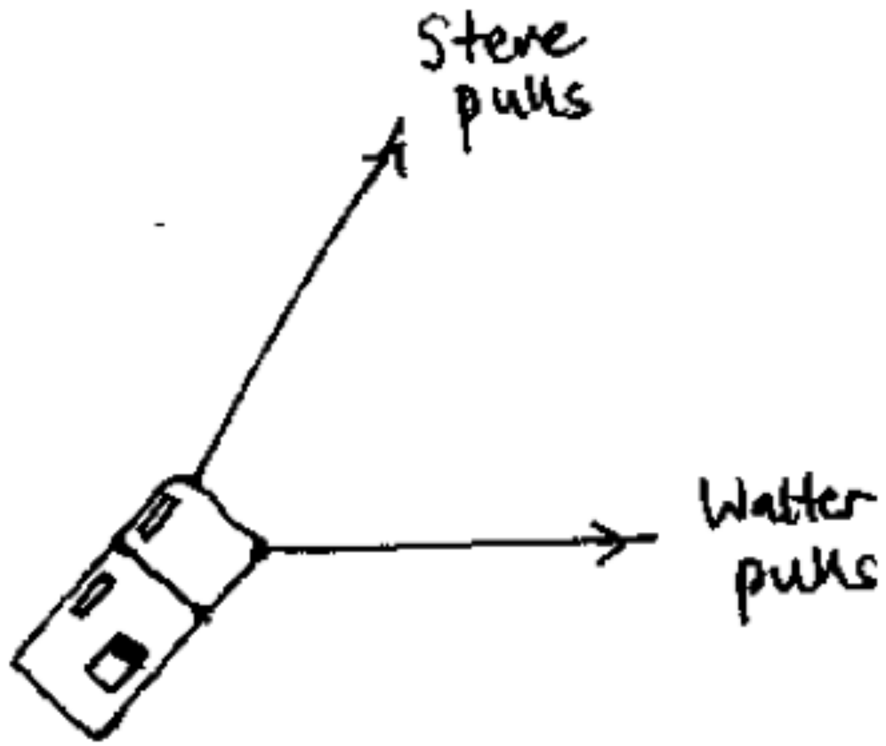
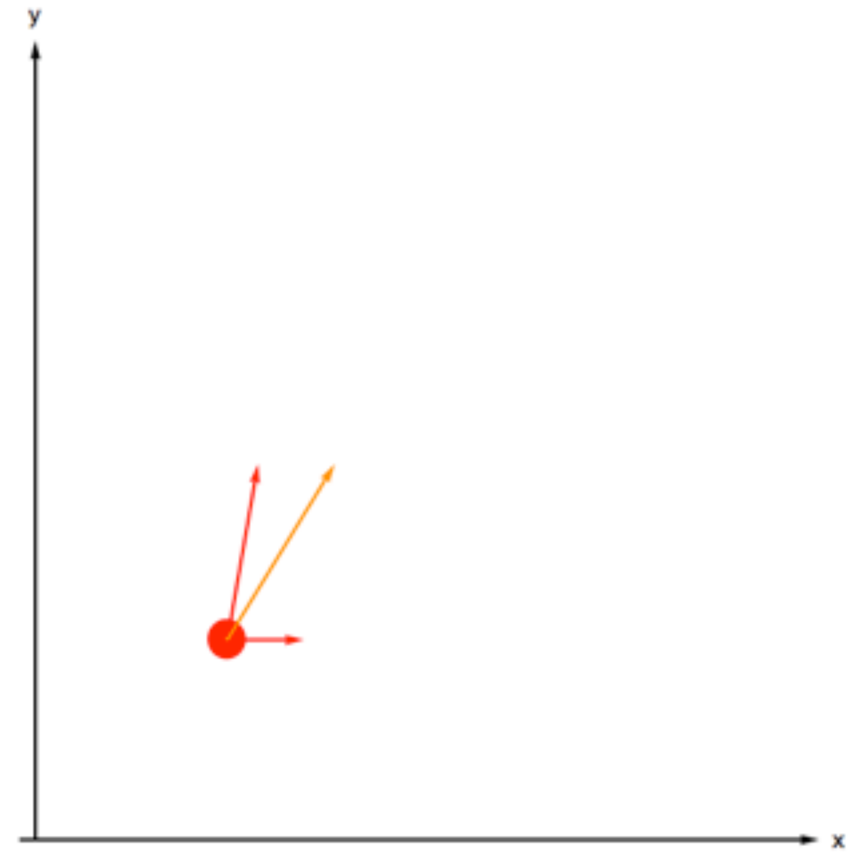
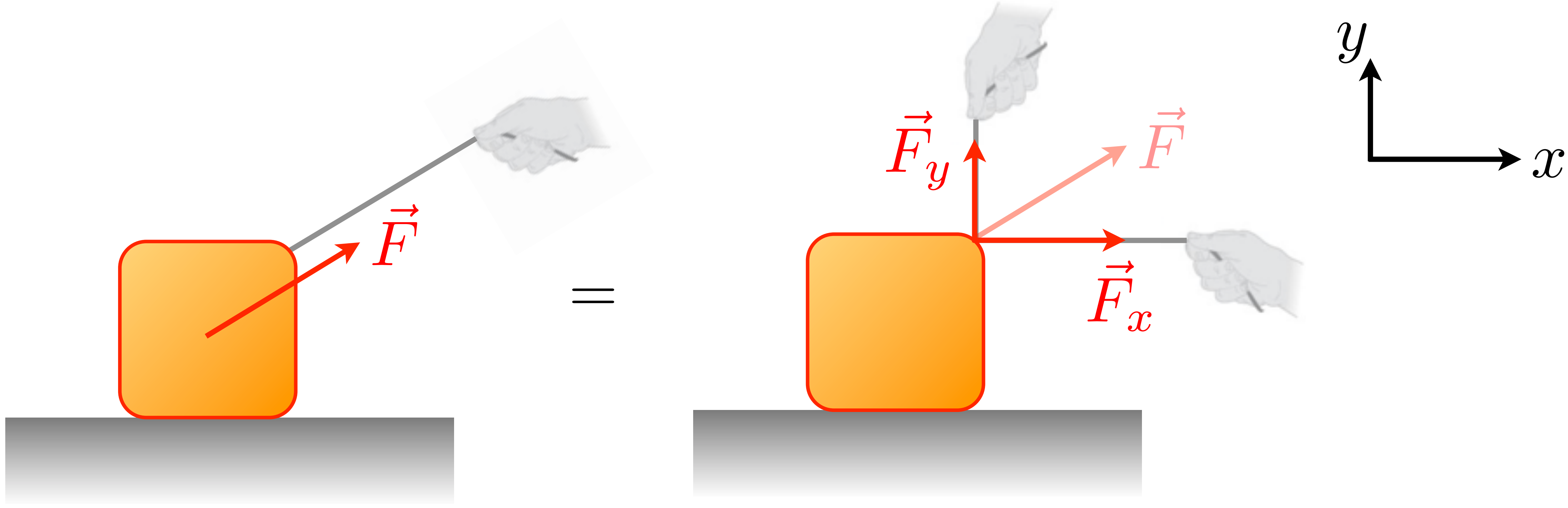


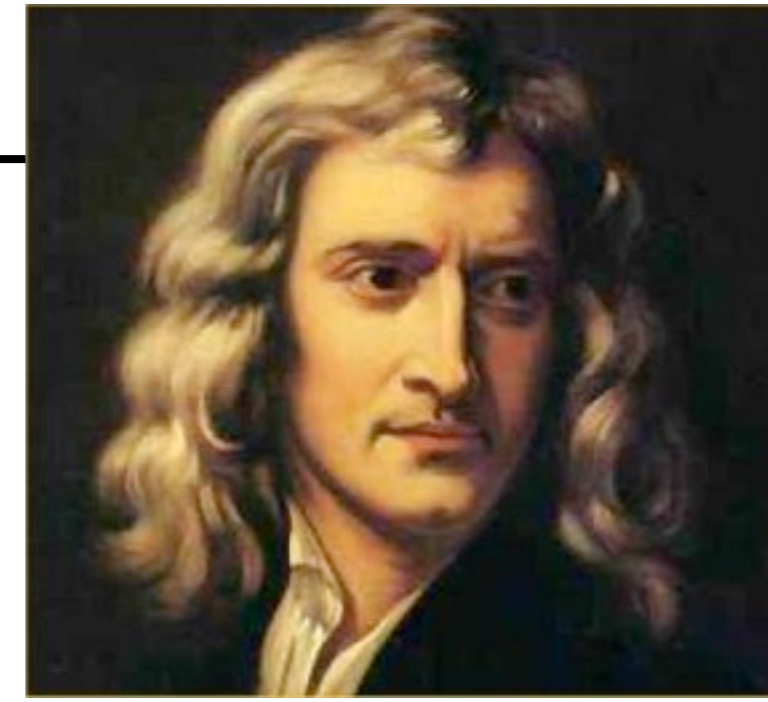

the pair of forces can be replaced by one single force in the direction of the sum of forces with the magnitude of the sum

$$\vec{F}_{\rm tot} = \vec{F}_{\rm S} + \vec{F}_{\rm W}$$

# components of a force



# Newton’s first law

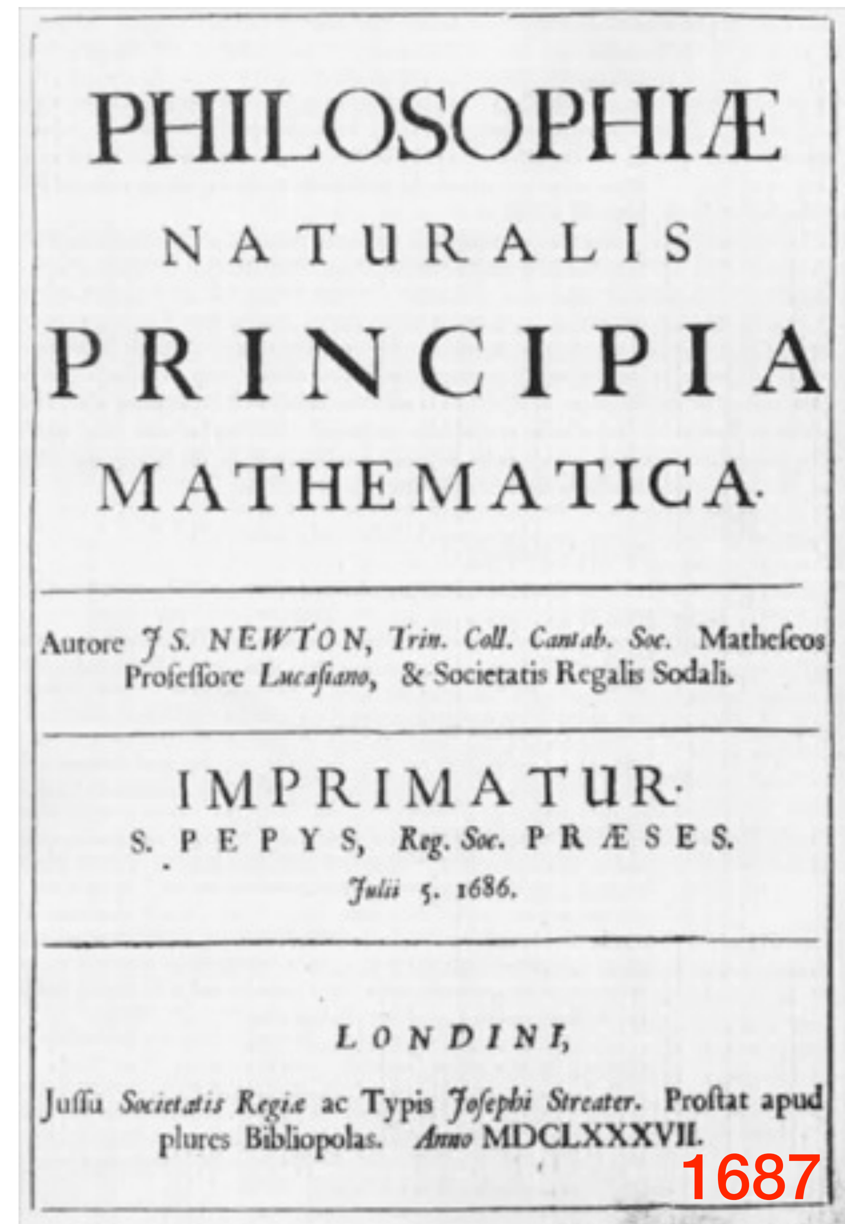


➜ Isaac Newton first proposed the following law of nature to attempt to describe objects in motion

**“ Every object continues either at rest or in constant motion in a straight line unless it is acted upon by a net force “**

a.k.a **‘inertia’**

the statement about objects at rest is pretty obvious, but the “constant motion” statement doesn’t seem right according to our everyday observations



PHILOSOPHIÆ

NATURALIS

PRINCIPIA

MATHEMATICA.

Autore *J S. NEWTON, Trin. Coll. Cantab. Soc.* Mathefeos Profeffore *Lucafiano*, & Societatis Regalis Sodali.

IMPRIMATUR.

S. PEPYS, *Reg. Soc.* PRÆSES.

*Julii* 5. 1686.

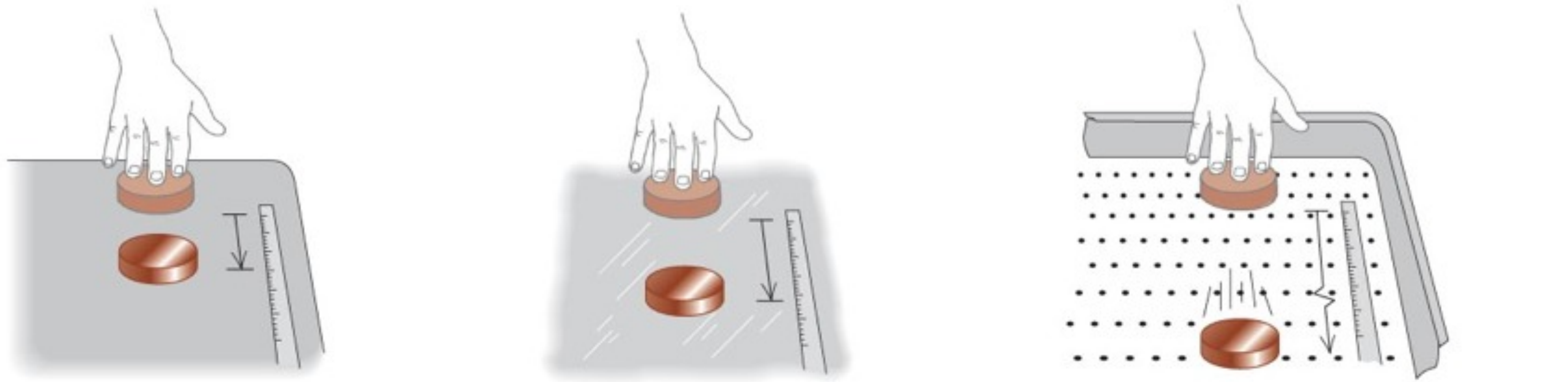
*LONDINI,*

Juffu *Societatis Regiæ* ac Typis *Jofephi Streater*. Proftat apud plures Bibliopolas. *Anno* MDCLXXXVII.

1687

# Newton's first law & friction

" Every object continues either at rest or in constant motion in a straight line unless it is acted upon by a net force "



(a) Table: puck stops short

(b) Ice: puck slides farther

(c) Air-hockey table: puck slides even farther

the problem is we can't easily test the law, because we can't set up a situation where there is no force

friction is ubiquitous

we can try to minimise friction though

# curling & inertia

# Newton's second law

➜ The first law describes what happens when **no force** acts on an object

➜ The second law describes the response of the object to a force being applied

we know that different objects respond
differently to the same magnitude of force

➜ push a shopping cart
➜ push a freight train

➜ very different responses

**" The acceleration of an object is directly proportional to the resultant force acting on it and inversely proportional to its mass. The direction of the acceleration is the direction of the resultant force. "**

or, in a much more compact notation, $\sum \vec{F} = m\vec{a}$

# Newton's second law

" The acceleration of an object is directly proportional to the resultant force acting on it and inversely proportional to its mass.  The direction of the acceleration is the direction of the resultant force. "

$$\sum \vec{F} = m\vec{a}$$

➜ OK, so to move an object at rest we need to accelerate it $\vec{a} \neq 0$

means there must be a net force acting on the object

➜ to change the velocity of an object, we need to accelerate it

means there must be a net force acting on the object

the first law is just when $\sum \vec{F} = 0$ so $\vec{a} = 0$

➜ the mass of an object determines how much acceleration you get for a given force

$$|\vec{a}| = \frac{|\vec{F}|}{m}$$

➜ push a shopping cart (*m* is small) ➜ big acceleration
➜ push a freight train (*m* is big) ➜ small acceleration

# “the” unit of force

➜ the Newton, N, is defined to be the force required to impart an acceleration of 1 $m/s^2$ to a 1 kg mass

there are other units, e.g. pounds (lb) we could use, but let’s stick to SI (“metric”) for now

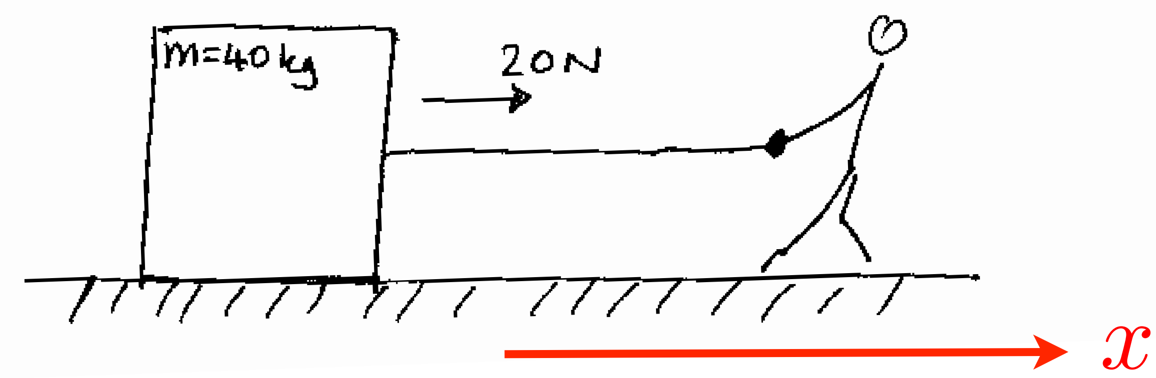
$$F = ma$$

$$1\,\mathrm{N} = 1\,\mathrm{kg\,m/s^2}$$

# using F=ma

➜ a worker with spikes on his shoes pulls with a constant horizontal force of magnitude 20 N on a box of mass 40 kg resting on the flat, frictionless surface of a frozen lake. What is the acceleration of the box ?

$$\sum F_x = +20\,\mathrm{N}$$




$$ma_x = F_x$$

$$a_x = \frac{F_x}{m} = \frac{20\,\mathrm{N}}{40\,\mathrm{kg}}$$

$$= 0.50\,\frac{\mathrm{kg\,m/s^2}}{\mathrm{kg}}$$

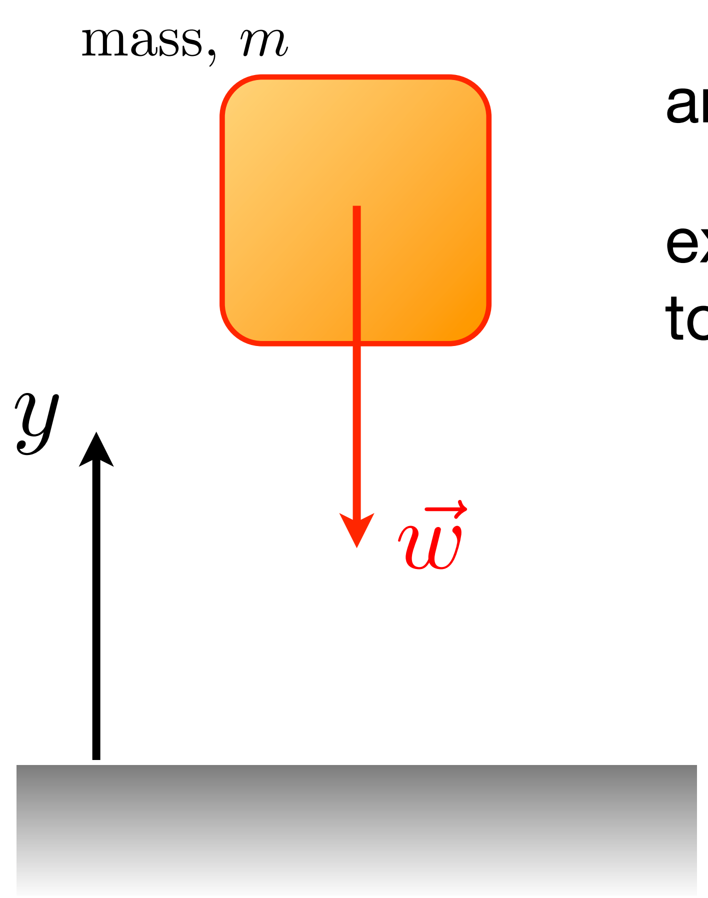
$$a_x = 0.50\,\mathrm{m/s^2}$$

# mass & weight

➜ we need to be careful to distinguish these terms

➜ **mass** is related to the amount of matter ("stuff") in an object

➜ **weight** is specifically the **force** on an object from the gravitational attraction of the Earth




are mass and weight related ?

experimentally we found that all objects in free-fall accelerate toward the Earth with acceleration of magnitude $g = 9.80\,\mathrm{m/s^2}$

$$F_y = ma_y$$

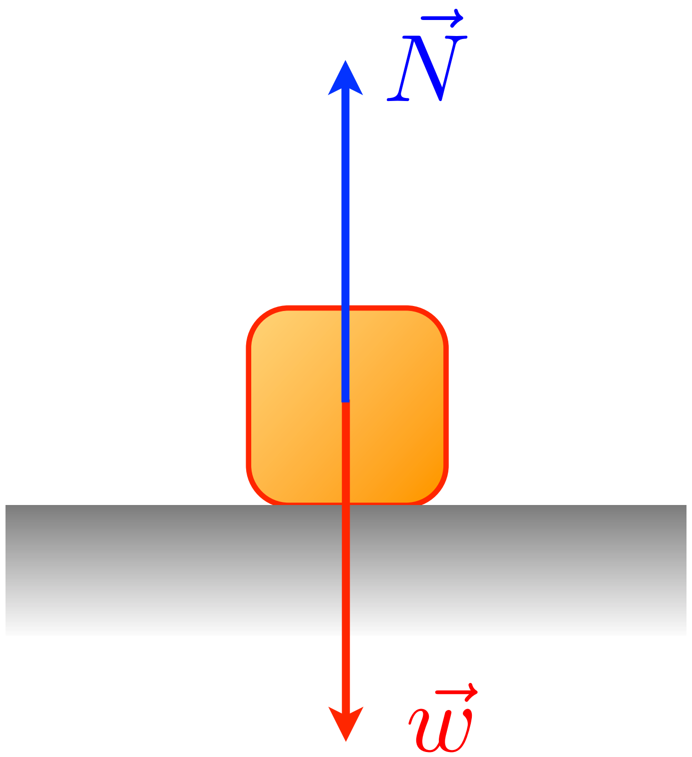
$$-w \qquad m \times (-g)$$

$w = mg$ true for all objects

but the value of **g** is a property of the Earth, it wouldn't be the same value on the Moon

# normal force (at equilibrium)

➜ consider the forces on a box sitting **at rest** on the floor




➜ the weight force points down

*but this can't be the only force on the box*
*- if it was it would accelerate downwards !*

➜ the **normal** force points upward

*the box compresses the surface of the floor at the microscopic level and the floor pushes back*

$$\left.\begin{array}{c} \sum F_y = N + (-w) = N - w \\ m a_y = 0 \end{array}\right\} N = w$$

# Newton's third law

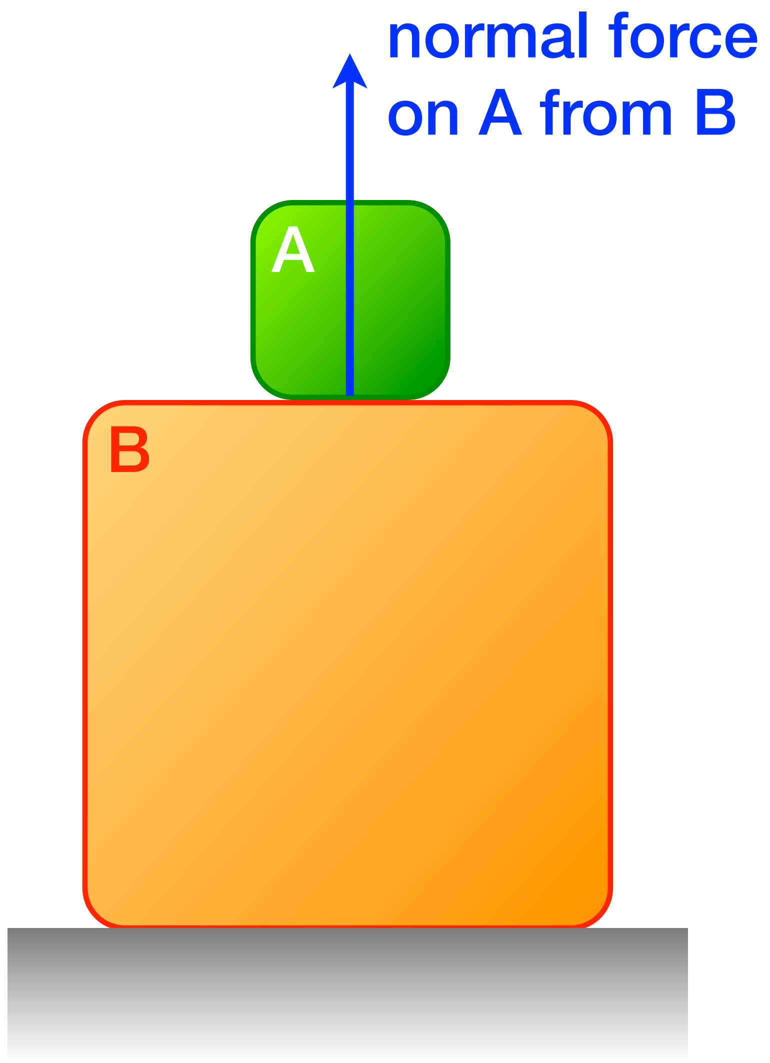
“ To every action there is always opposed an equal reaction ”

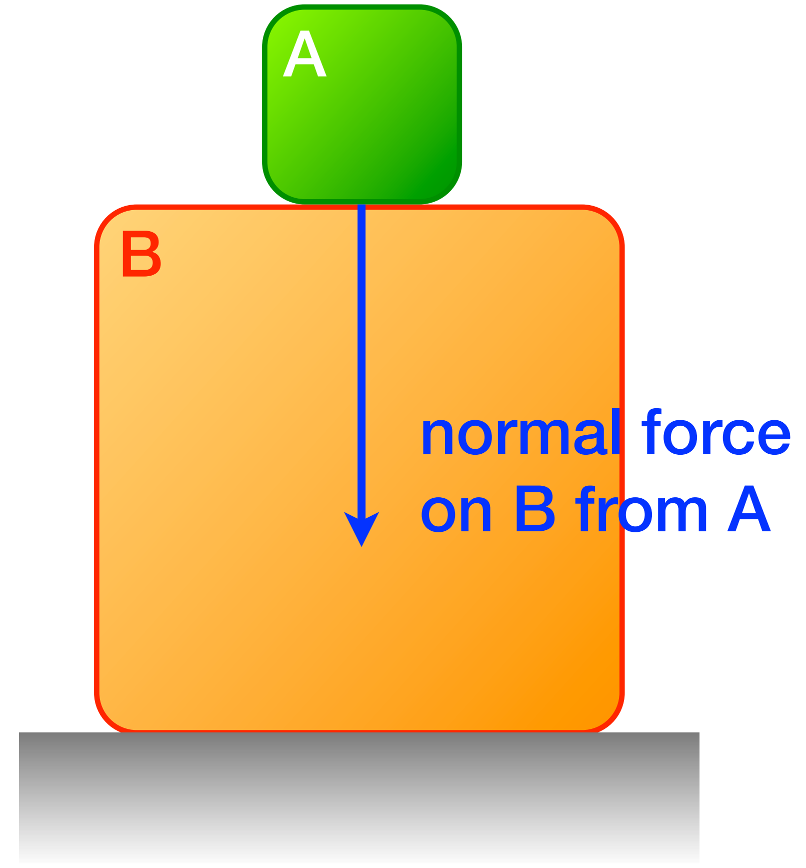
if an object **A** exerts a force on object **B**, object **B** exerts a force of equal magnitude and opposite direction on **A**

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

# normal forces & Newton's third law

➜ box **A** sits on top of box **B** at rest

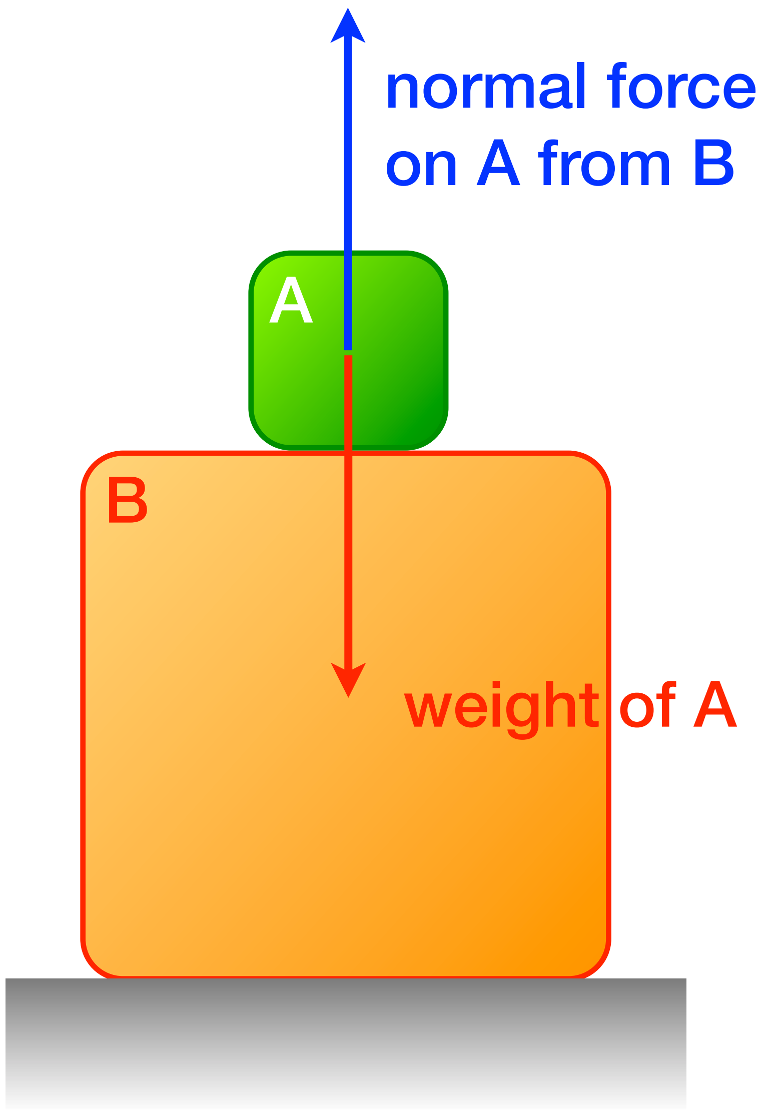


# all the forces

➜ box **A** sits on top of box **B** at rest

what forces act on box **A** ?

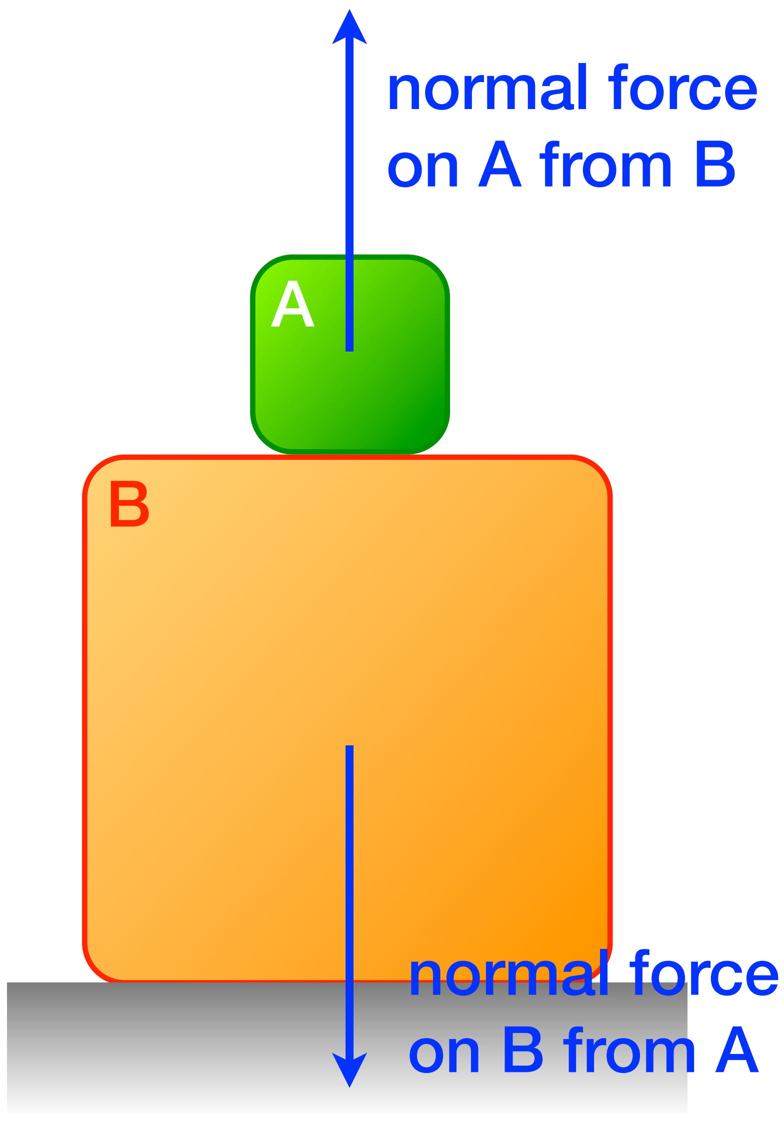



equal & opposite because the box is at rest - the **first** law

# all the forces

➜ box **A** sits on top of box **B** at rest

what is the third law reaction force to the normal force on **A** from **B** ?

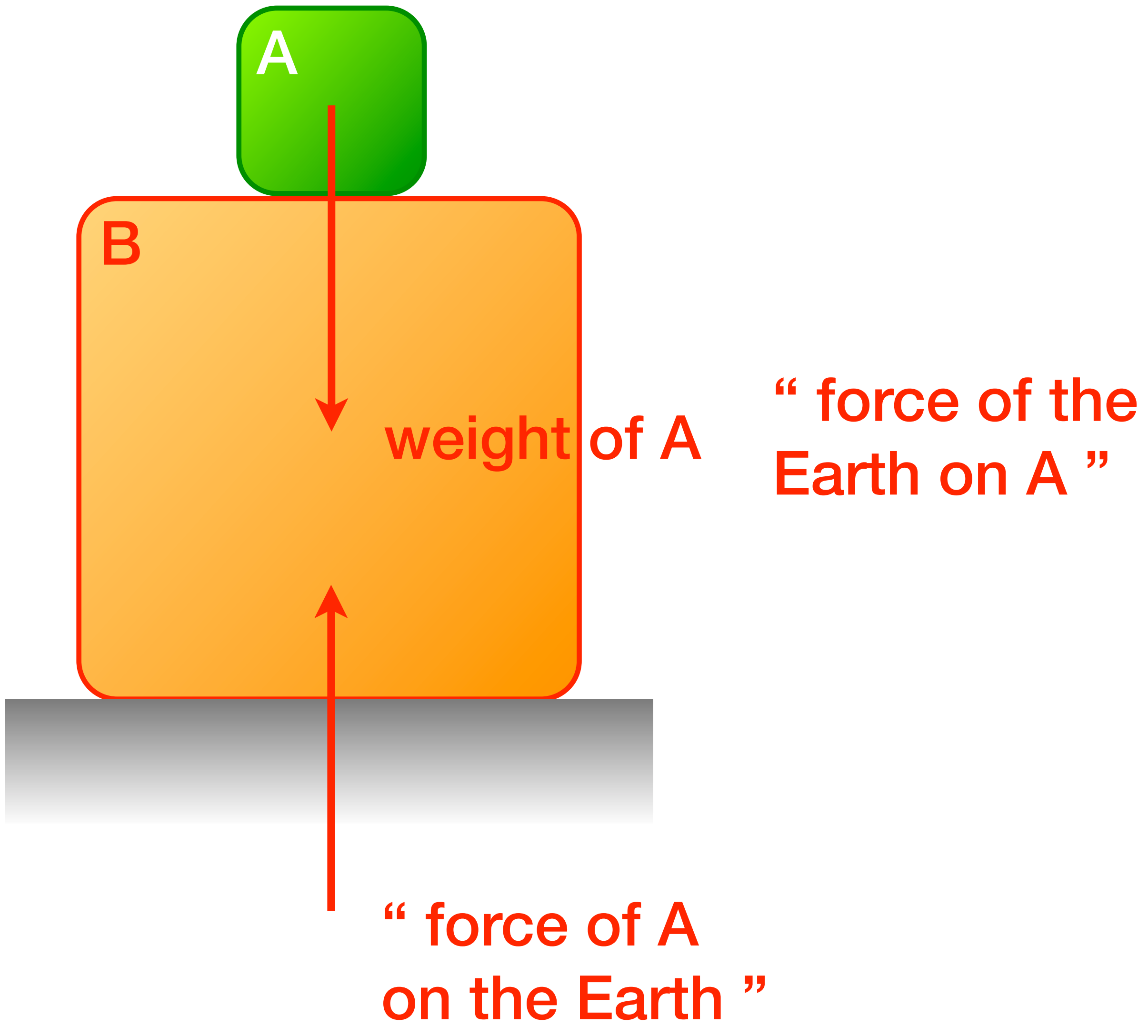



equal & opposite because of the **third** law - N.B. forces act on **different** objects

# all the forces

➜ box **A** sits on top of box **B** at rest

what is the third law reaction force to weight of **A** ?



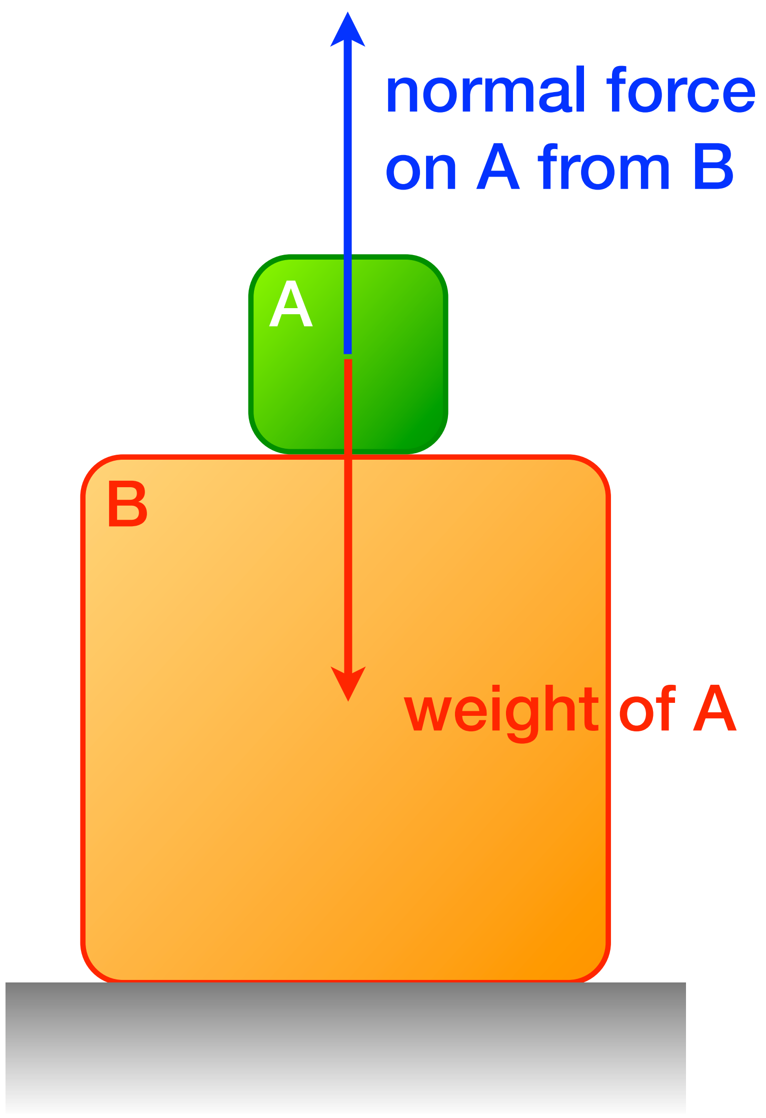

so just as the Earth attracts you toward it, you attract the Earth toward you

# all the forces

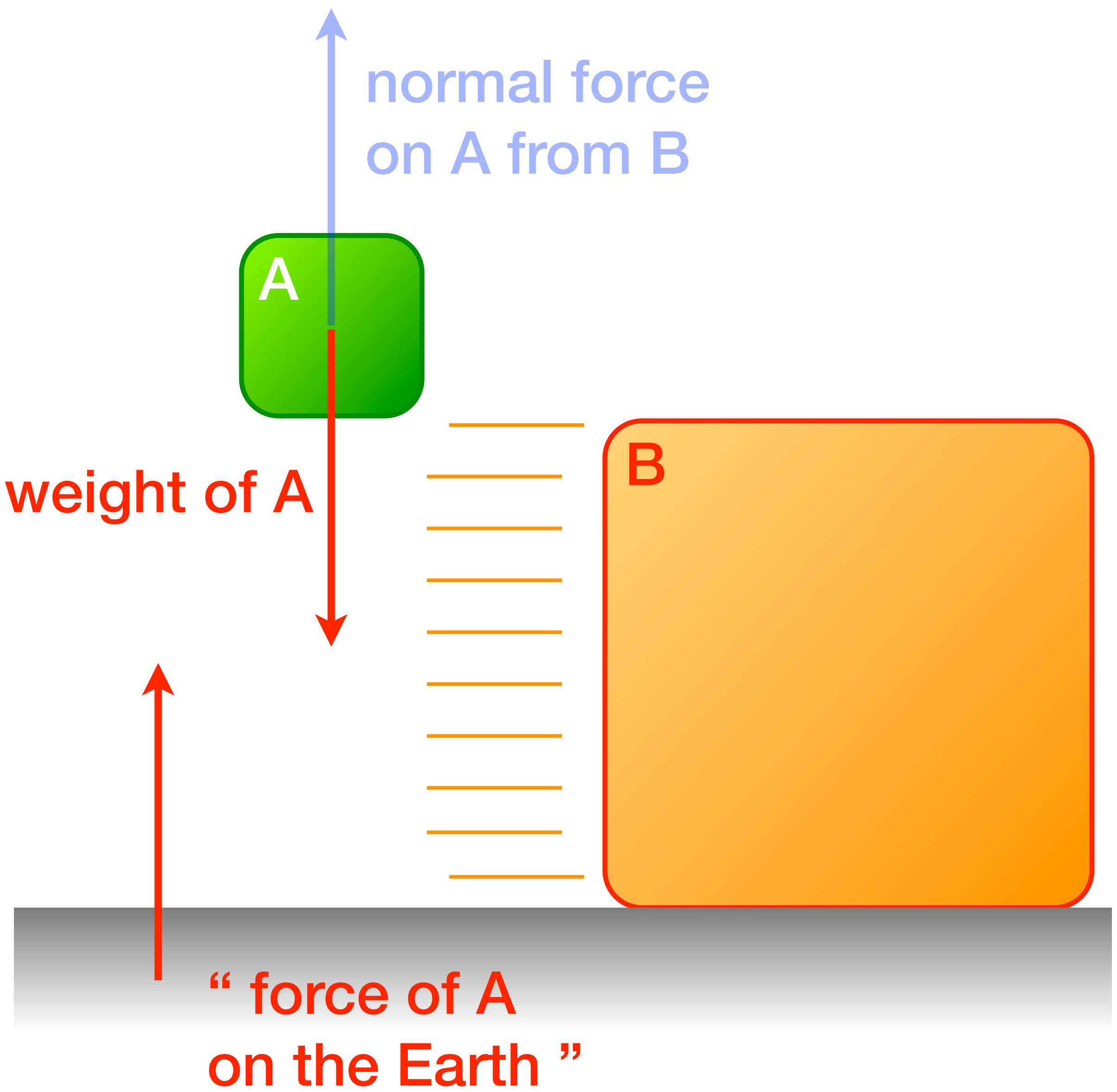
➜ box **A** sits on top of box **B** at rest

is the normal force on **A** the reaction force to the weight of **A** ?

**NO !**



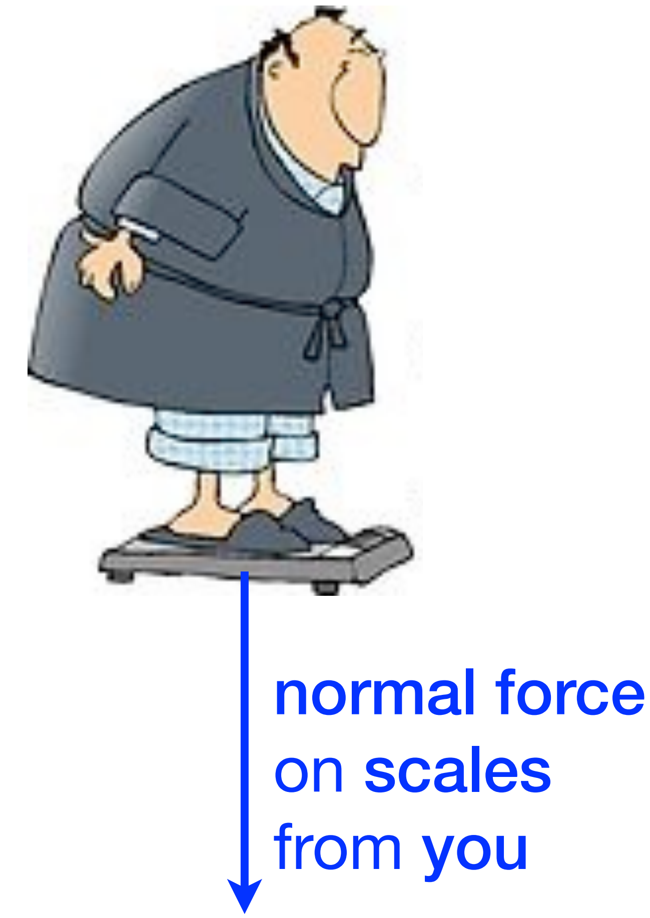

they are equal & opposite,
but they aren't an action-reaction pair



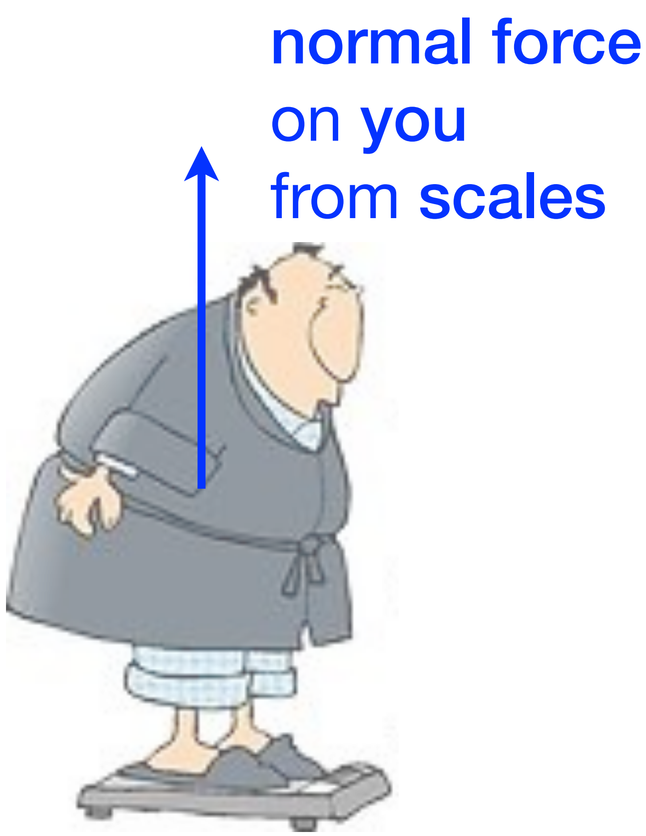

remove box **B** & the normal force goes away

# bathroom scales

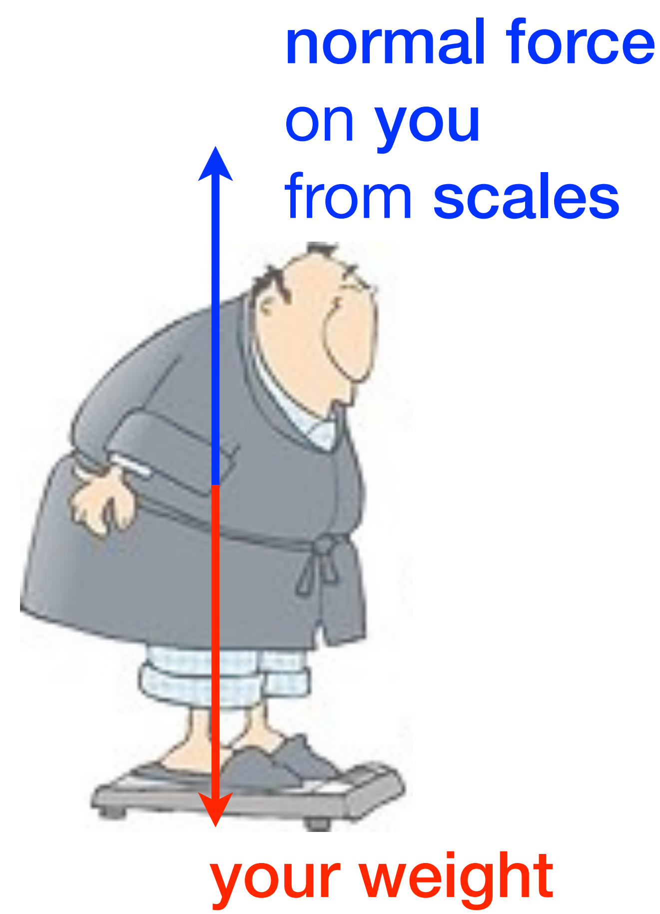
➜ your bathroom scales measure the force **you** exert **on the scales**




the reaction force of
this is the normal force
of **the scales** on **you**




you also have your
weight force



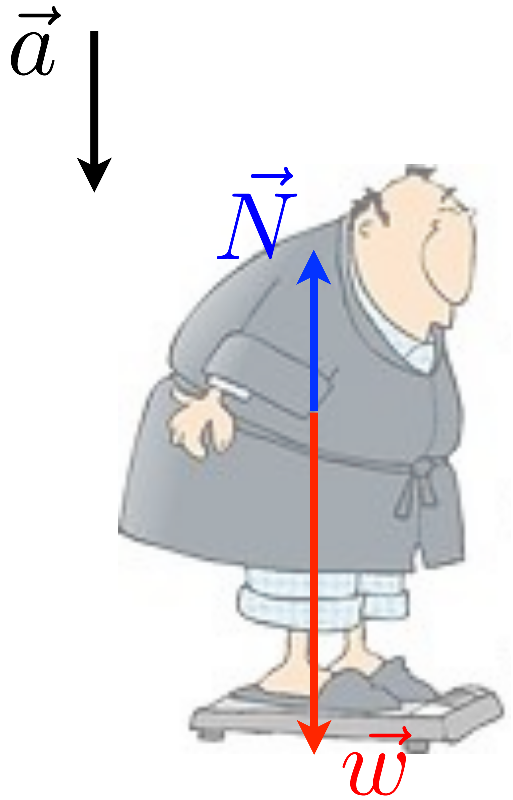

but if you're at rest &
remain at rest, your
acceleration is zero

& thus the scales
measure your weight

# losing weight the easy way

➜ now suppose you stand on bathroom scales while riding an elevator

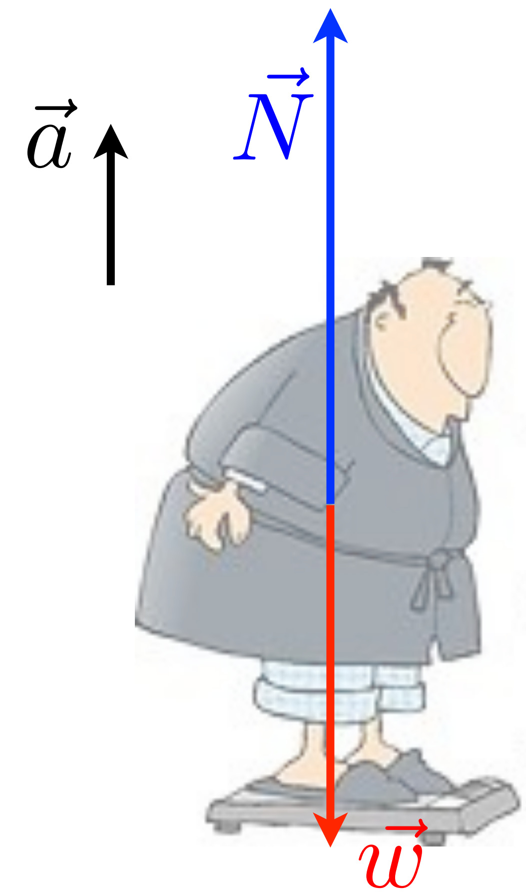
★ elevator accelerating **down**




**smaller** reading on the scales

you “lost weight”

★ elevator accelerating **up**




**larger** reading on the scales

you “gained weight”

# ropes & tension

➜ consider a uniform rope whose ends are being pulled on

➜ look at a small section with mass *m*


Newton's 2nd law applied to this section of rope

$$T_R - T_L = ma$$

➜ rope in equilibrium (not accelerating), $T_R = T_L$ & tension same throughout

➜ rope is massless, $T_R = T_L$ & tension same throughout

we will usually assume ropes to be effectively massless or in equilibrium such that the tension is the same throughout the rope

# systems in equilibrium in more than one dimension

➜ here by 'in equilibrium', we mean at rest or moving with constant velocity

➜ in that case $\vec{a} = \vec{0}$ and by Newton's second law $\sum \vec{F} = \vec{0}$

➜ or "all the forces on an object must balance"

really just Newton's first law

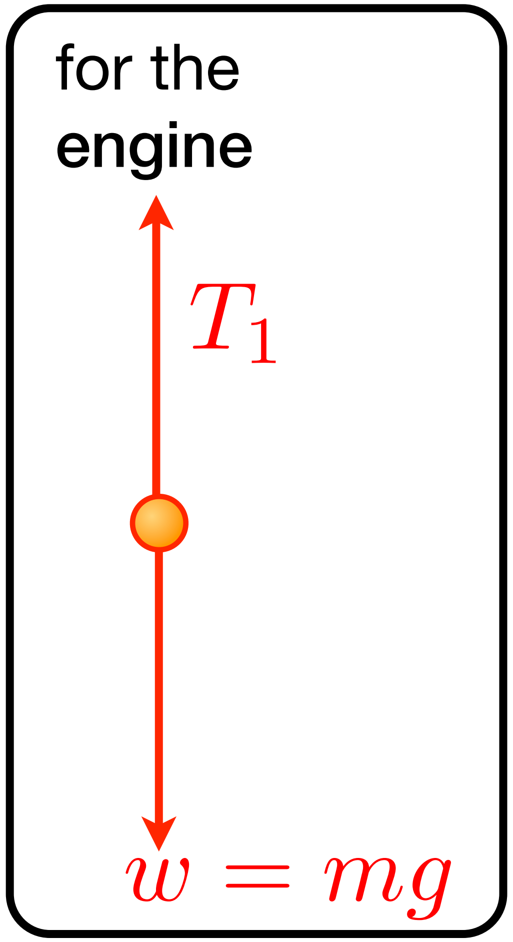
➜ it is often helpful to split the problem up into components

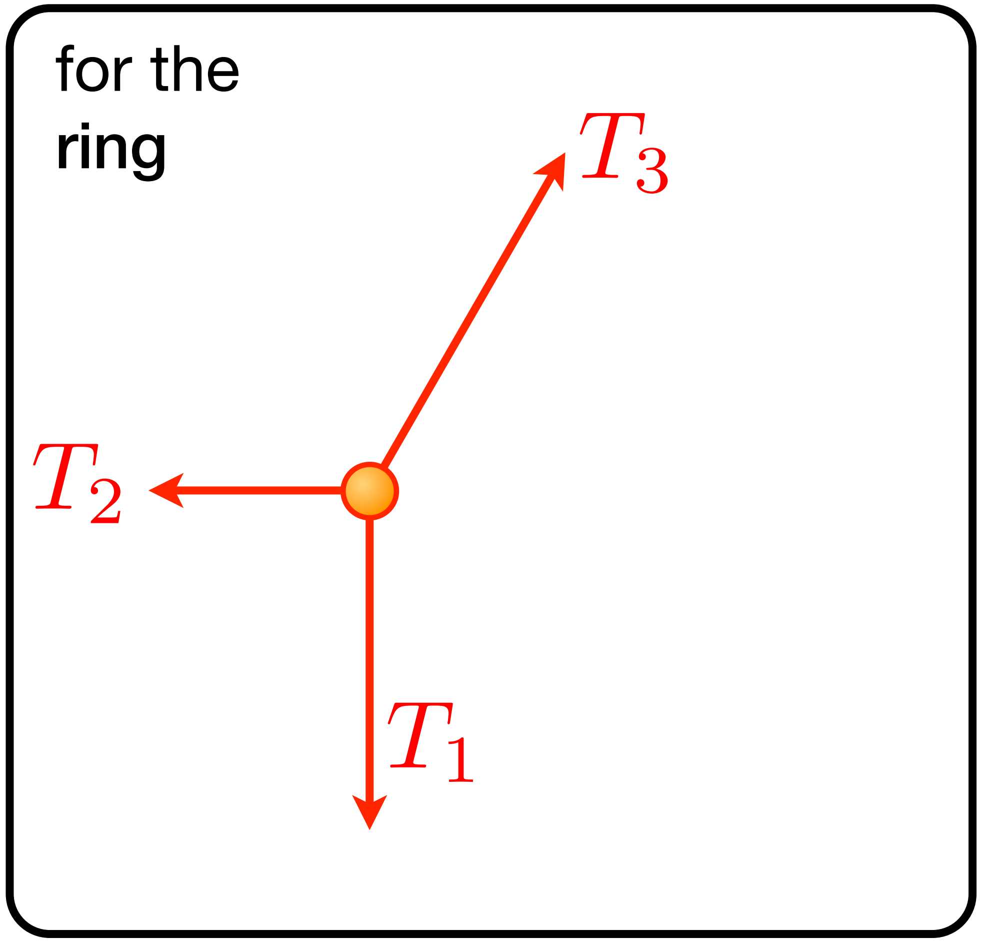
$$\sum F_x = 0$$

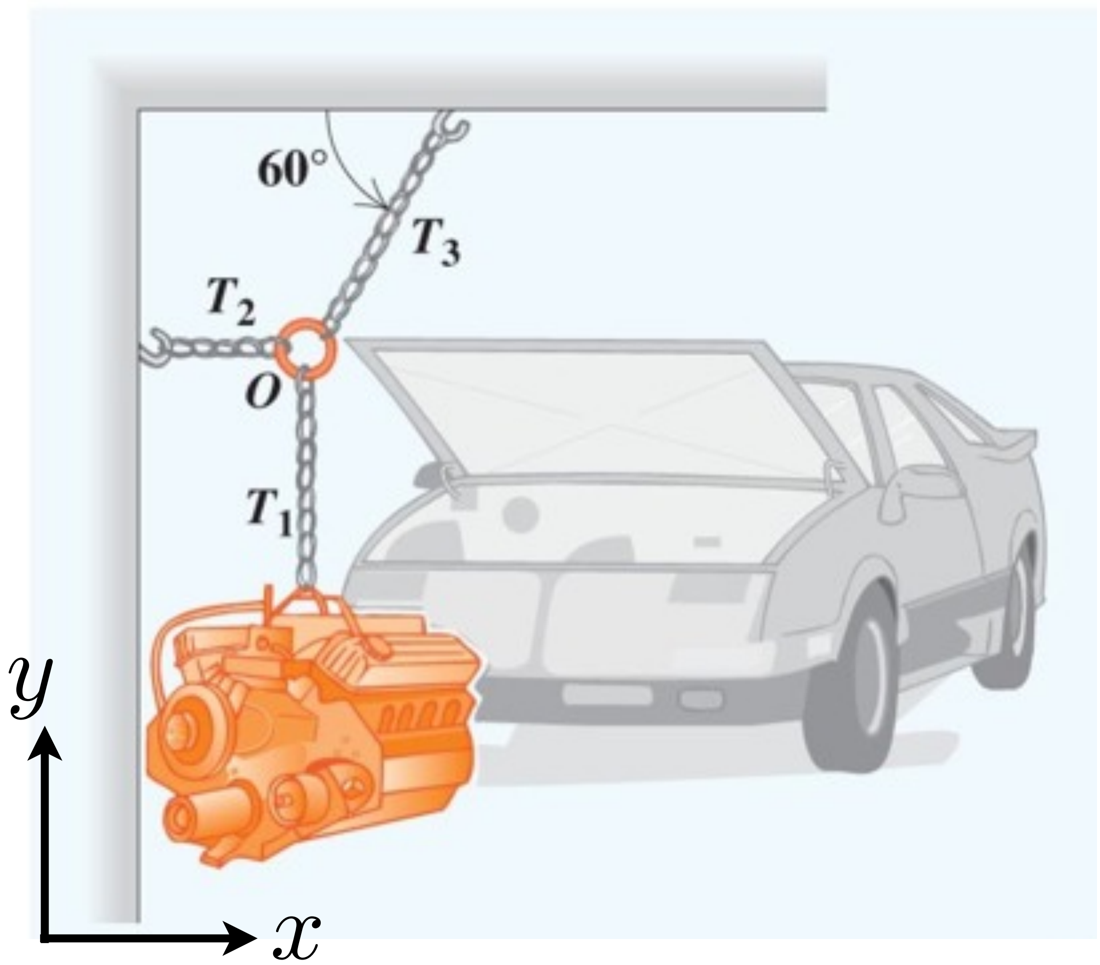
$$\sum F_y = 0$$

# two-dimensional equilibrium

➜ a car engine of mass 500 kg hangs at rest from a set of chains as shown. Find the tension in each chain, assuming their masses are negligible.



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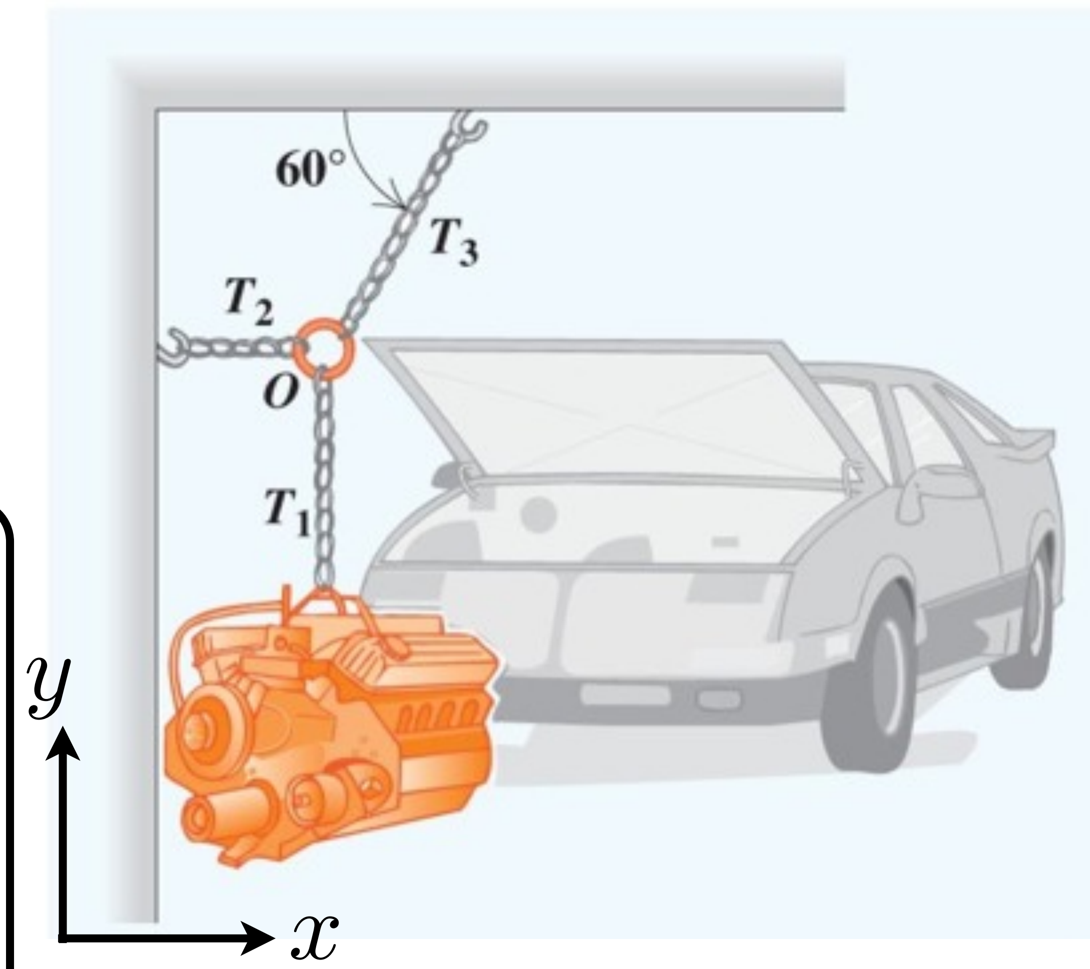
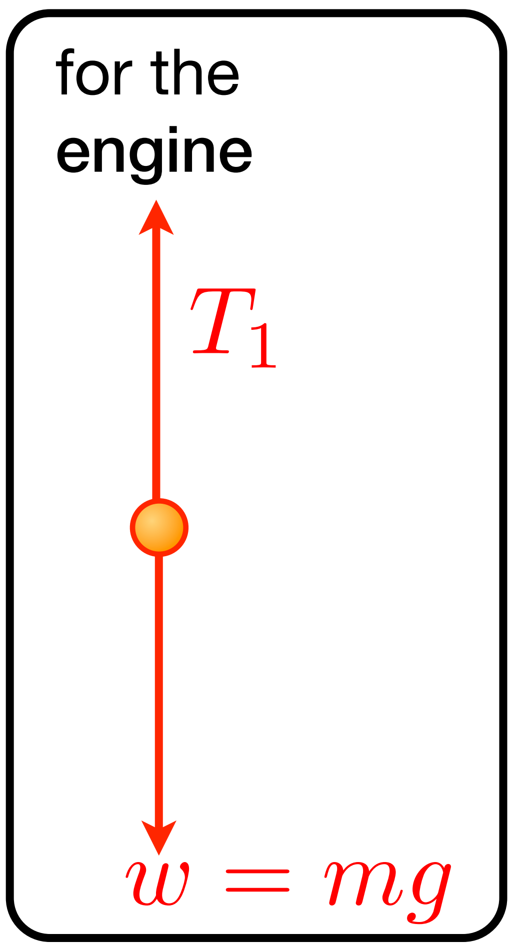
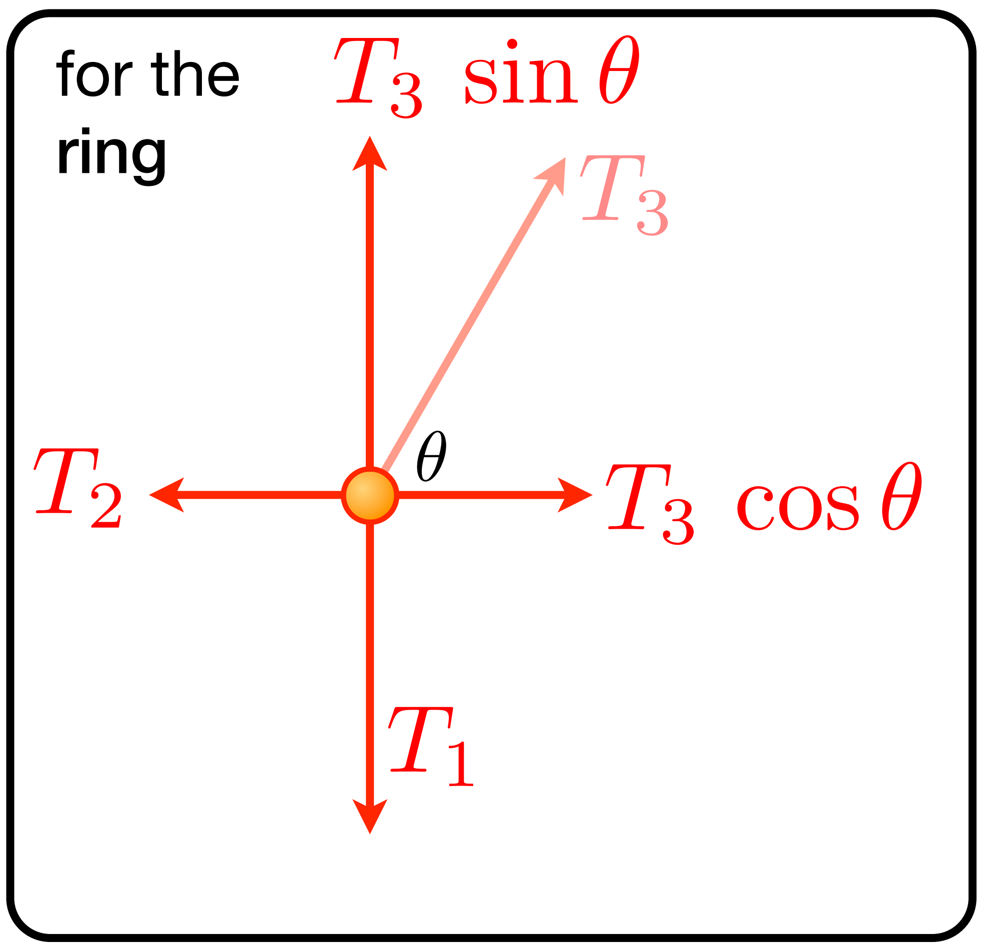



$$\theta = 60^\circ$$

$$\sum F_y = T_1 - w = 0$$

$$\implies T_1 = w$$

$$\sum F_y = T_3 \sin\theta - T_1 = 0$$

$$\implies T_3 \sin\theta = w$$

$$\implies T_3 = w / \sin\theta$$

$$\sum F_x = T_3 \cos\theta - T_2 = 0$$

$$\implies T_2 = T_3 \cos\theta$$

# systems out of equilibrium

➜ here by 'in equilibrium', we mean at rest or moving with constant velocity

➜ so systems out of equilibrium have an acceleration

➜ and we must use Newton's second law in full $\sum \vec{F} = m\vec{a}$

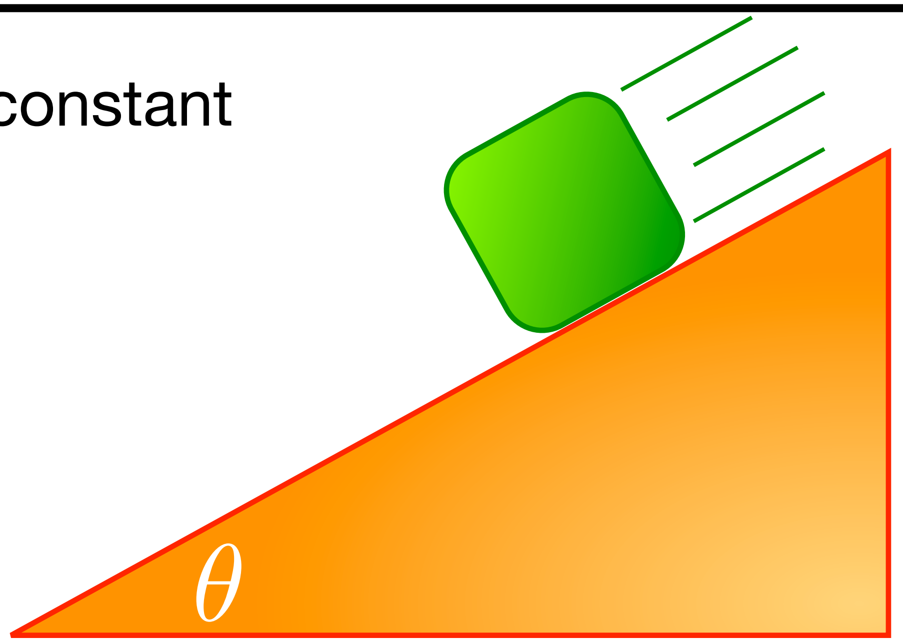
➜ it is often helpful to split the problem up into components

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

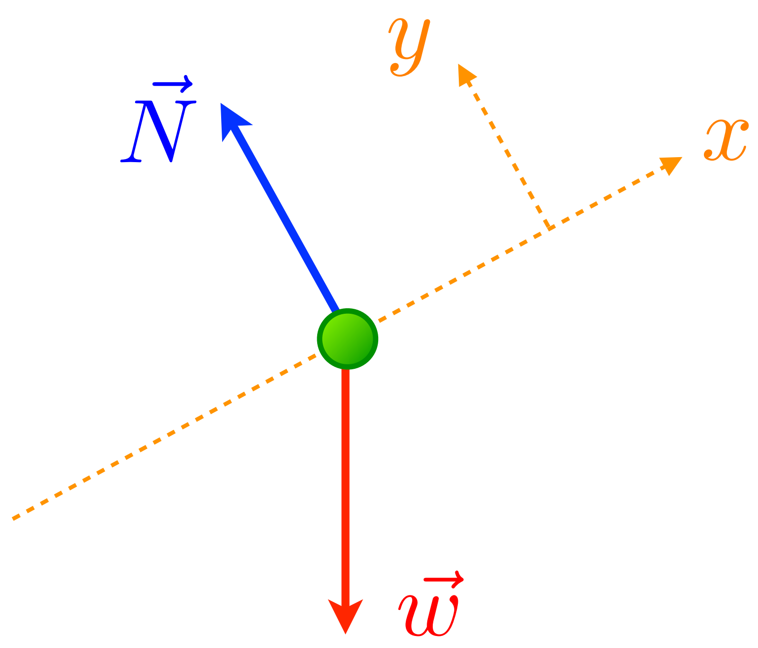
# the sliding box

➜ a box slides down a frictionless incline sloped at a constant angle of θ

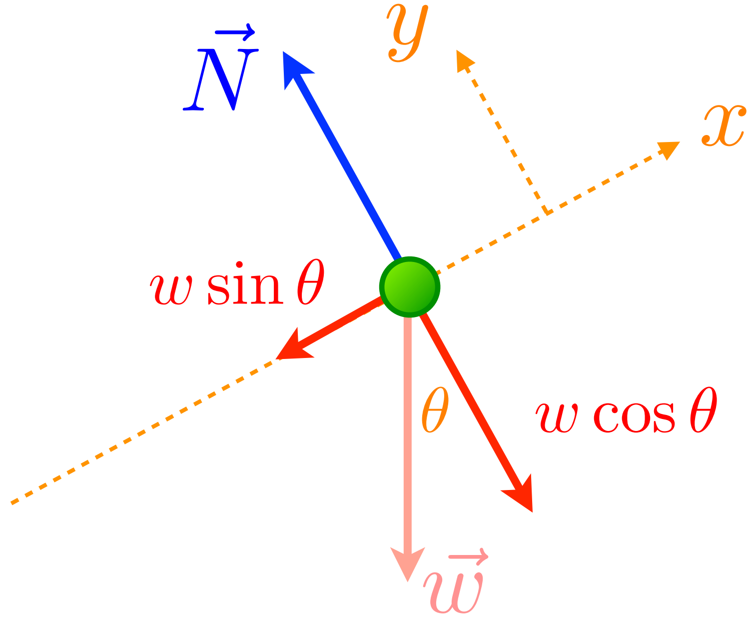



find the acceleration of the box and the normal force exerted by the slope on the box

free-body diagram for the **box**




express in components parallel and perpendicular to the slope



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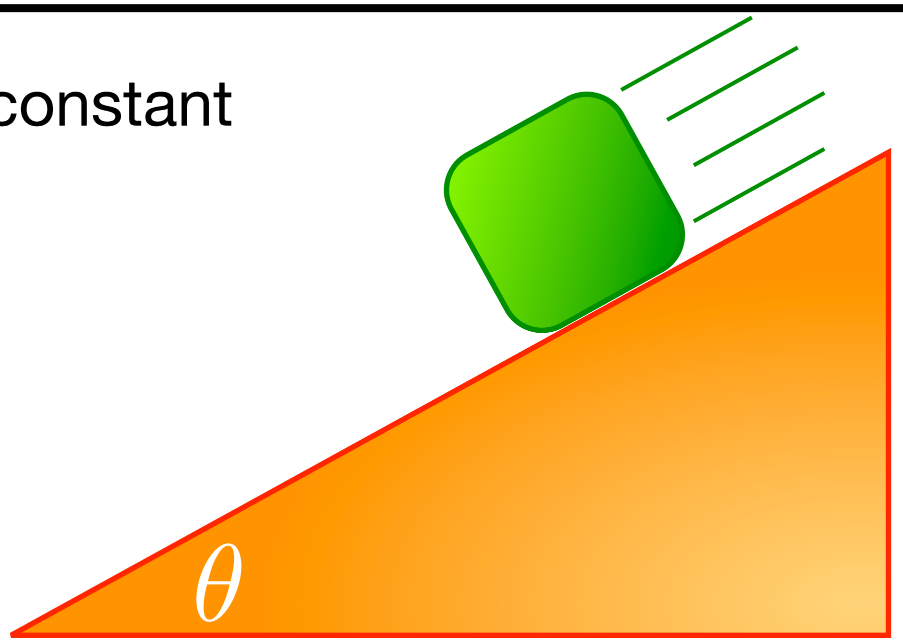
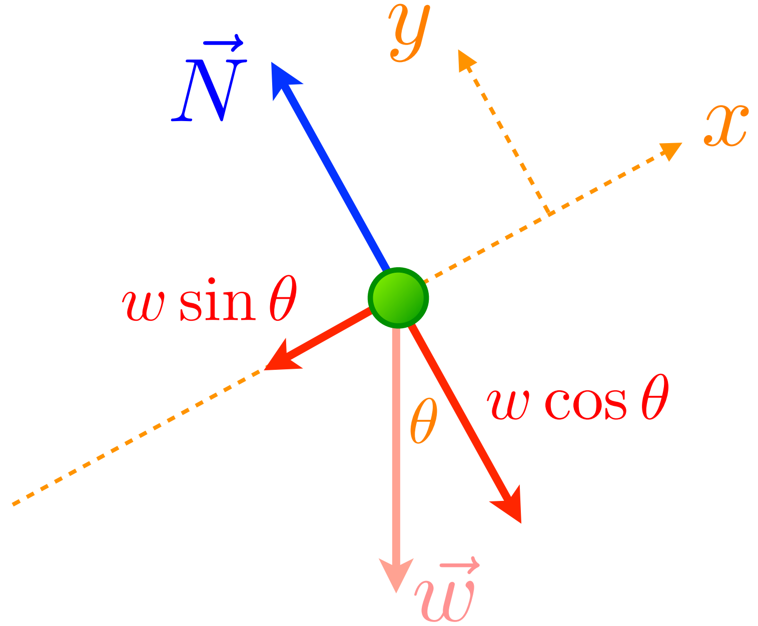



find the acceleration of the box and the normal force exerted by the slope on the box

express in components parallel and perpendicular to the slope




$$\sum F_x = -w\sin\theta$$

box accelerates down the incline

$$-w\sin\theta = ma_x$$

$$a_x = \frac{-mg\sin\theta}{m}$$

$$a_x = -g\sin\theta$$

$$\sum F_y = N - w\cos\theta$$

box doesn't move off the incline $a_y = 0$

$$N = w\cos\theta$$

$$N = mg\cos\theta$$

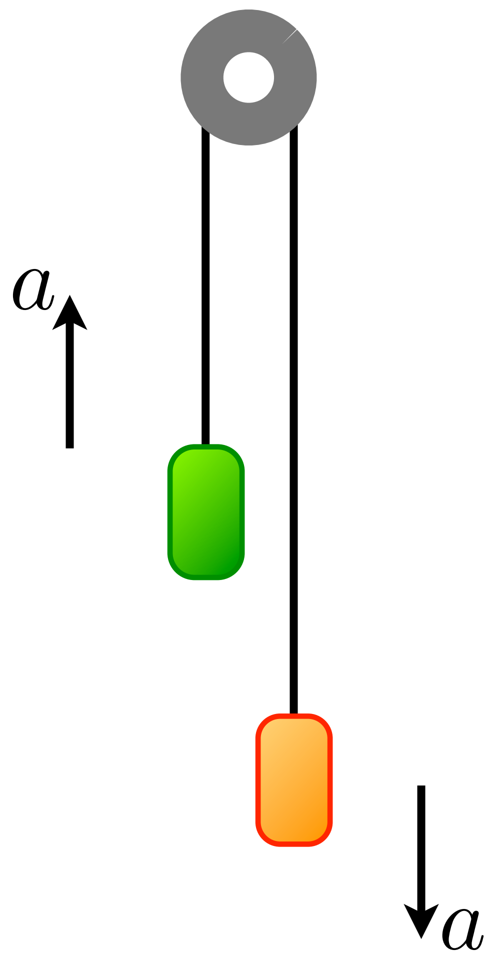
as a cross-check, consider θ→0,90°

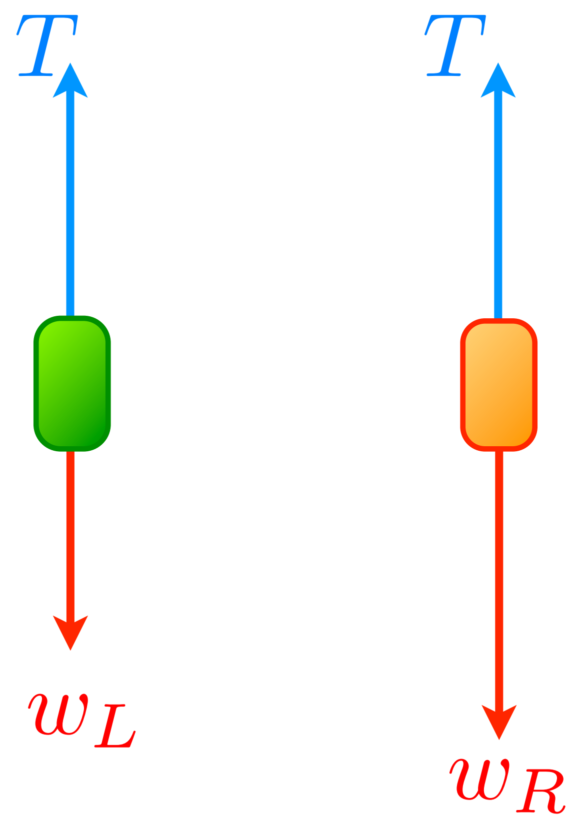
# the Atwood machine

➜ consider this experiment

# the Atwood machine

➜ let's explain the measurement using our theory of forces




$$T - w_L = m_L a$$

$$w_R - T = m_R a$$

$$w_R - w_L = (m_R + m_L)a$$

$$a = \frac{m_R - m_L}{m_R + m_L} g$$

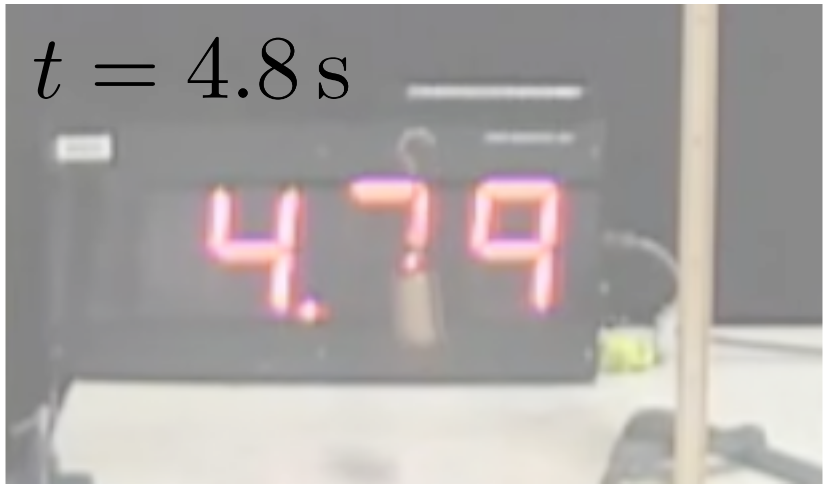
$$y = y_0 - v_0 t + \frac{1}{2} a t^2$$

$$t = \sqrt{\frac{2\Delta y}{a}}$$

$$m_L = 0.550\,\mathrm{kg}$$

$$m_R = 0.560\,\mathrm{kg}$$

$$\Delta y = 1.0\,\mathrm{m}$$



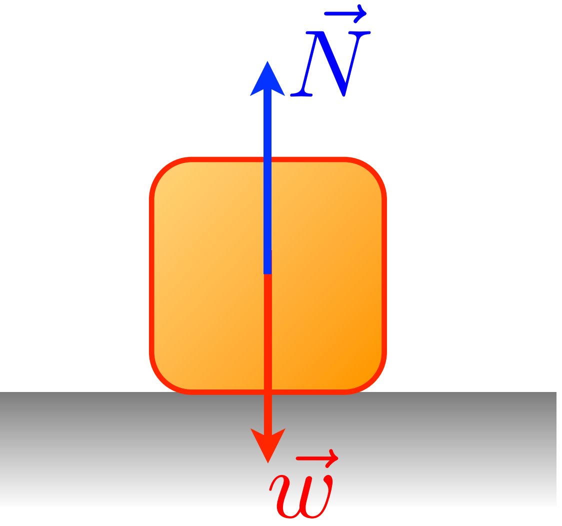
# friction

➜ we already considered one **contact force** present when two surfaces touch, namely the **normal force**, which acts perpendicular to the surfaces

➜ in some cases there can be a **contact force** parallel to the surfaces known as the **friction force**

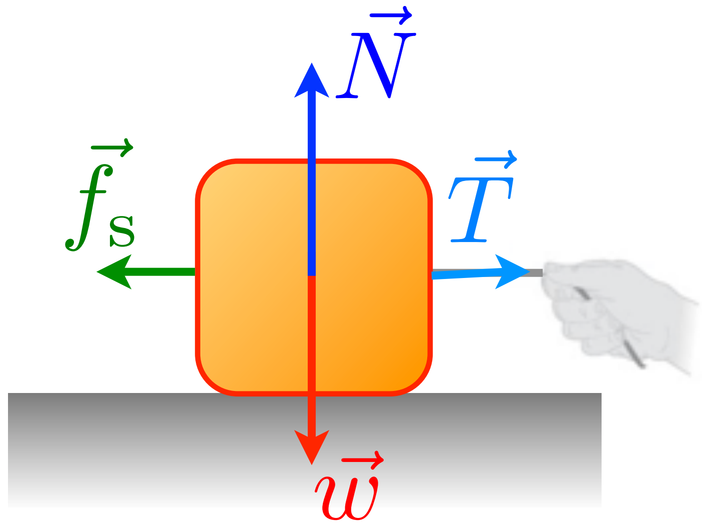
- ➜ **friction** is everywhere ... let's build a simple model to describe it
- ➜ two forms of **friction** - static (not moving) & kinetic (moving)

# friction




no applied force,
box at **rest**
no friction force

# friction

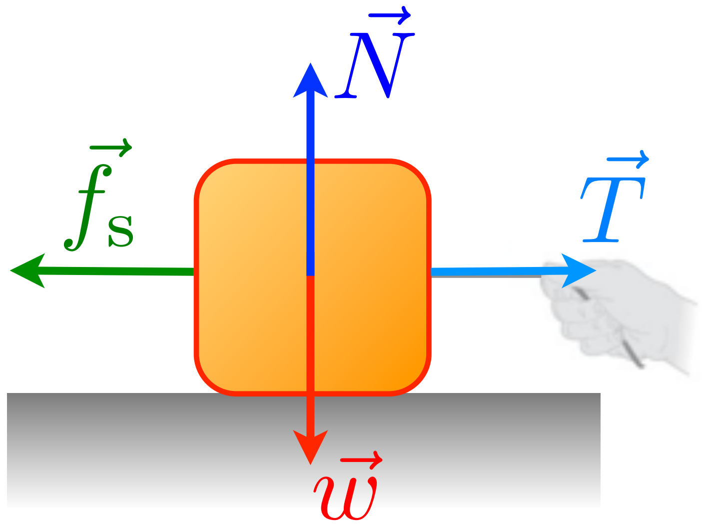



small applied force,
box at **rest**
**static friction force**

$$f_s = T$$

static friction force
matches the pull force &
keeps the box at rest

# friction

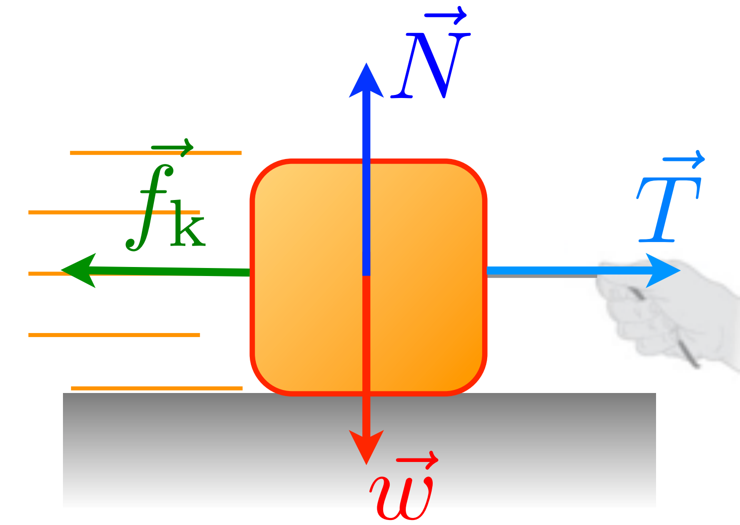



larger applied force,
box at **rest**
**static friction force**

$$f_{\mathrm{s}} = T$$

static friction force increases to match the pull force & keep the box at rest

# friction




even larger applied force, box moving at **constant speed**

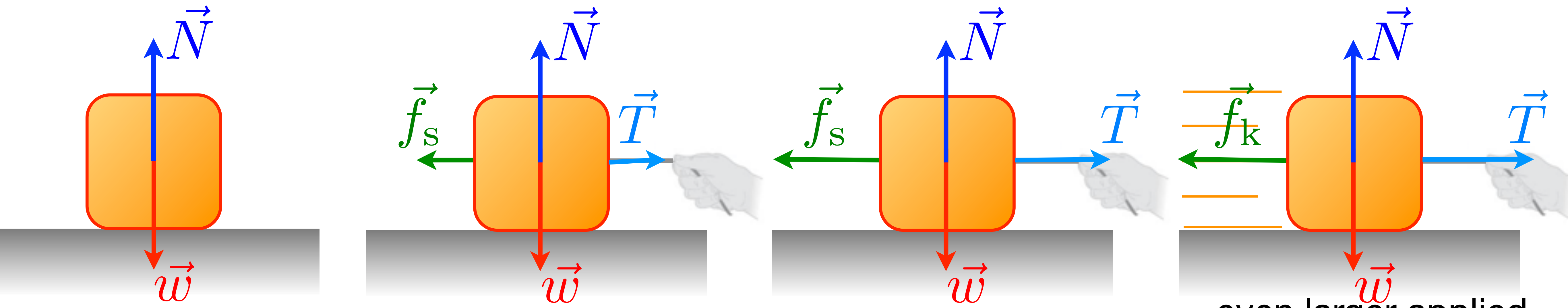
**kinetic friction force**

$$f_\mathrm{k} = T$$

eventually the static friction force cannot get any larger & the box will start to move

once the box is moving there will be a constant kinetic friction force

# friction

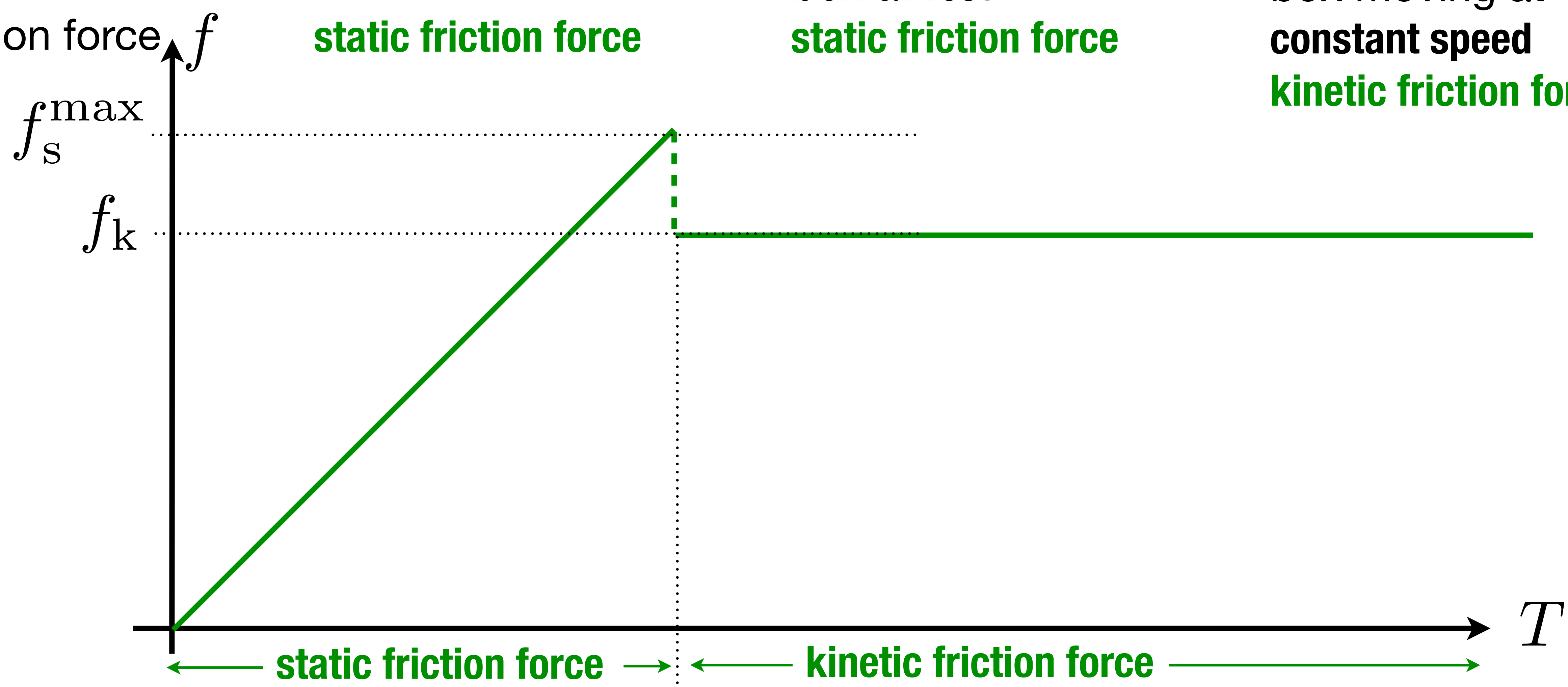



no applied force,
box at **rest**
no friction force

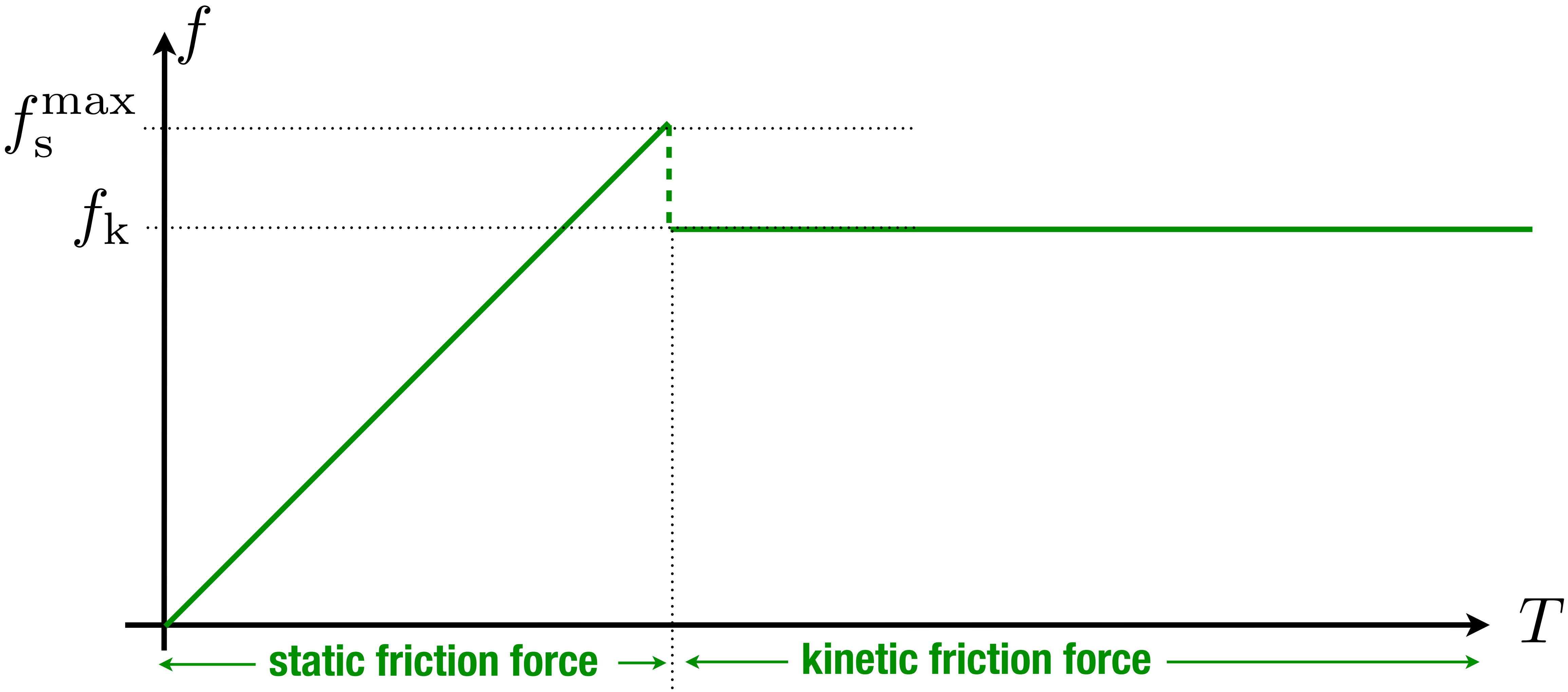
small applied force,
box at **rest**
**static friction force**

larger applied force,
box at **rest**
**static friction force**

even larger applied force,
box moving at
**constant speed**
**kinetic friction force**



# friction




➜ the magnitudes of $f_s^{max}$ and $f_k$ are determined by properties of the two surfaces in contact and can be expressed via **coefficients of friction**

$$f_s^{max} = \mu_s N$$
$$f_k = \mu_k N$$

e.g.

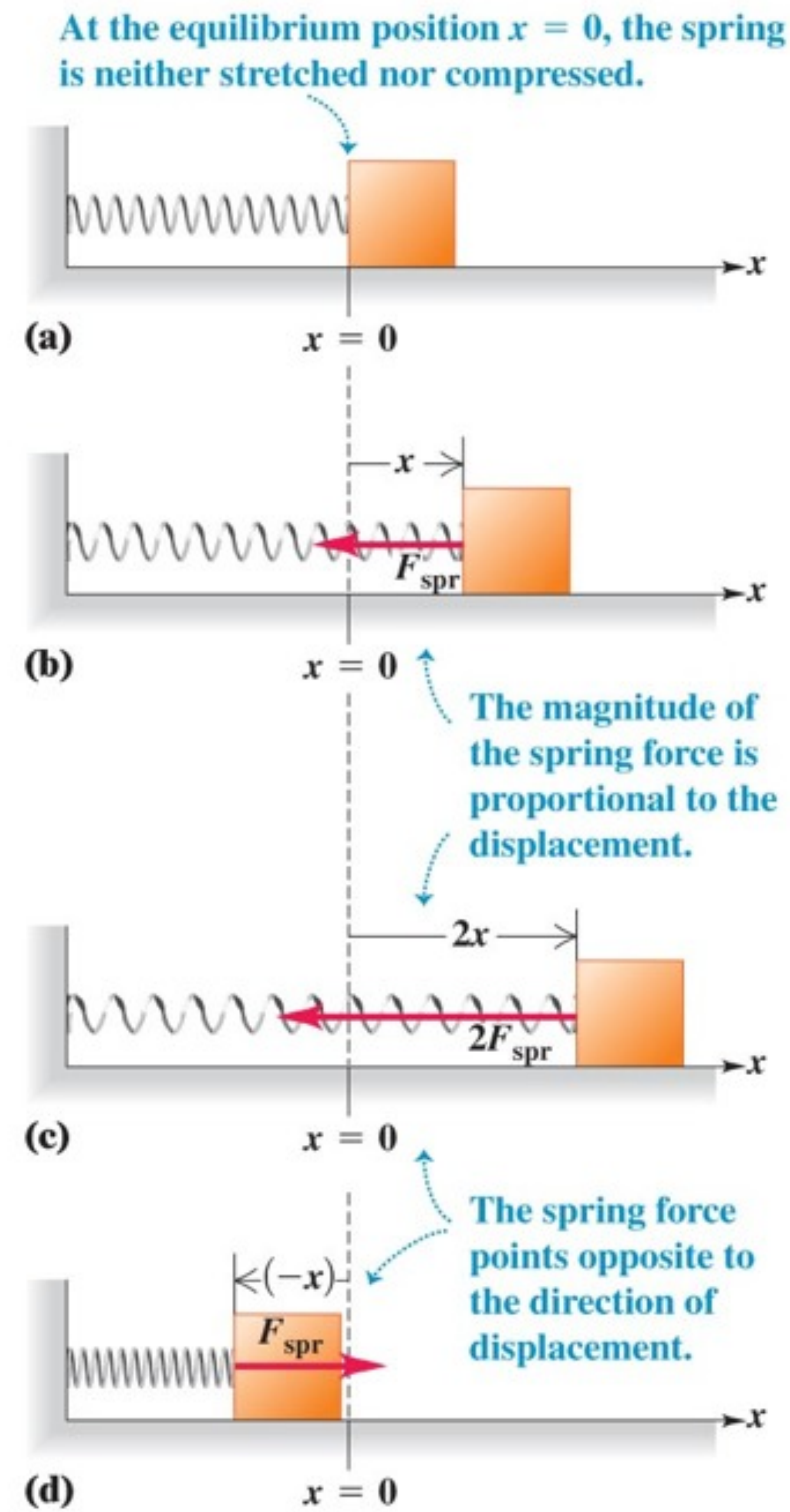
| materials | rubber on concrete | wood on concrete | steel on Teflon |
|---|---|---|---|
| $\mu_s$ | 1.0 | 0.6 | 0.04 |

# elastic forces

➜ springs & Hooke's law

$$F_{\text{spr}} = -kx$$

an empirical, approximate law

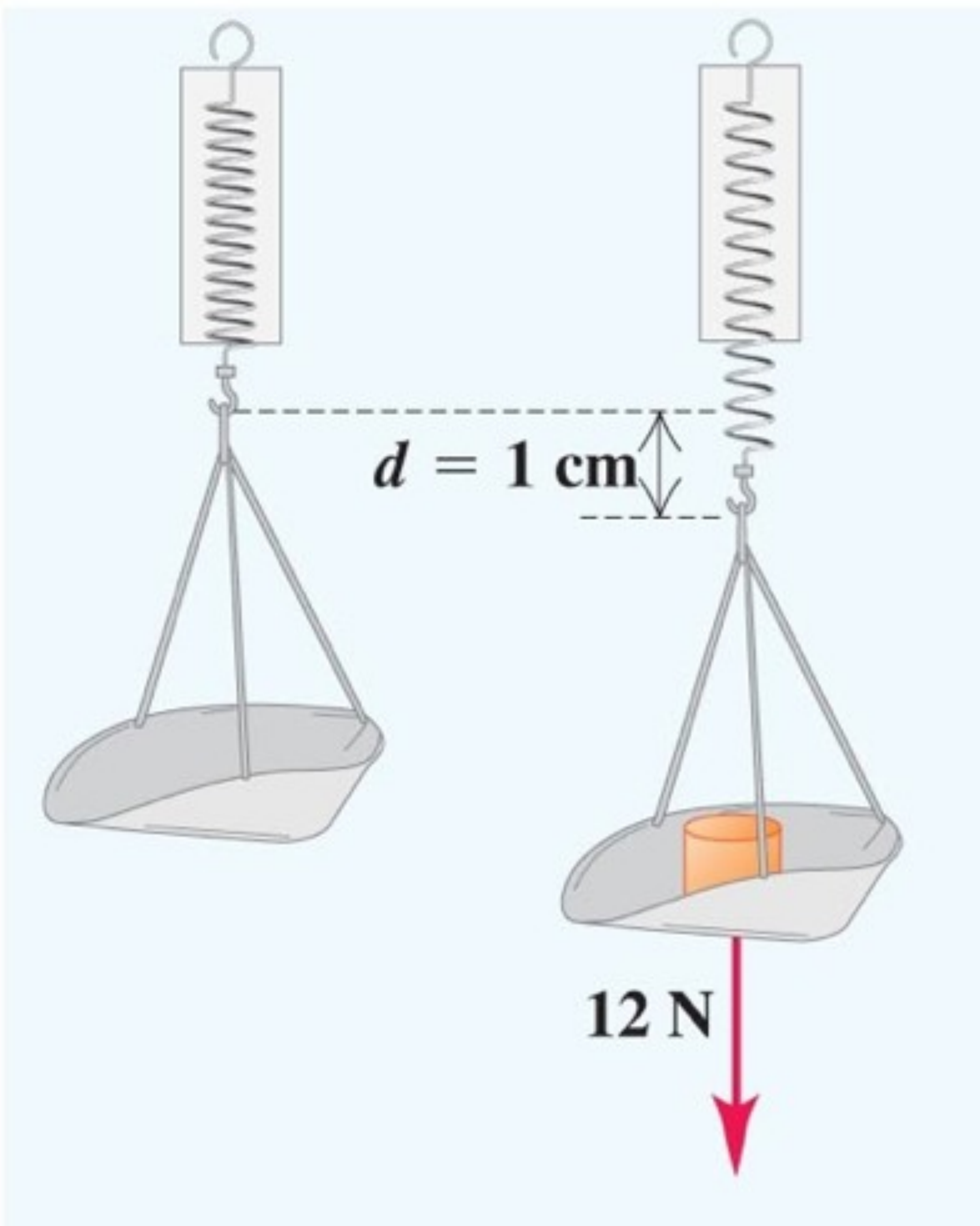


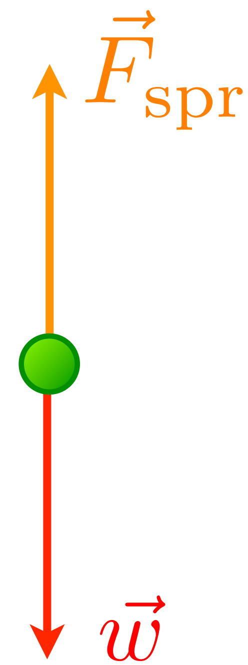
# a spring balance

$F_{\text{spr}} = -kx$

➜ we can use Hooke's law to build a device to measure weight

**calibration**




$$k = \frac{12\,\text{N}}{0.01\,\text{m}} = 1.2 \times 10^3\,\text{N/m}$$

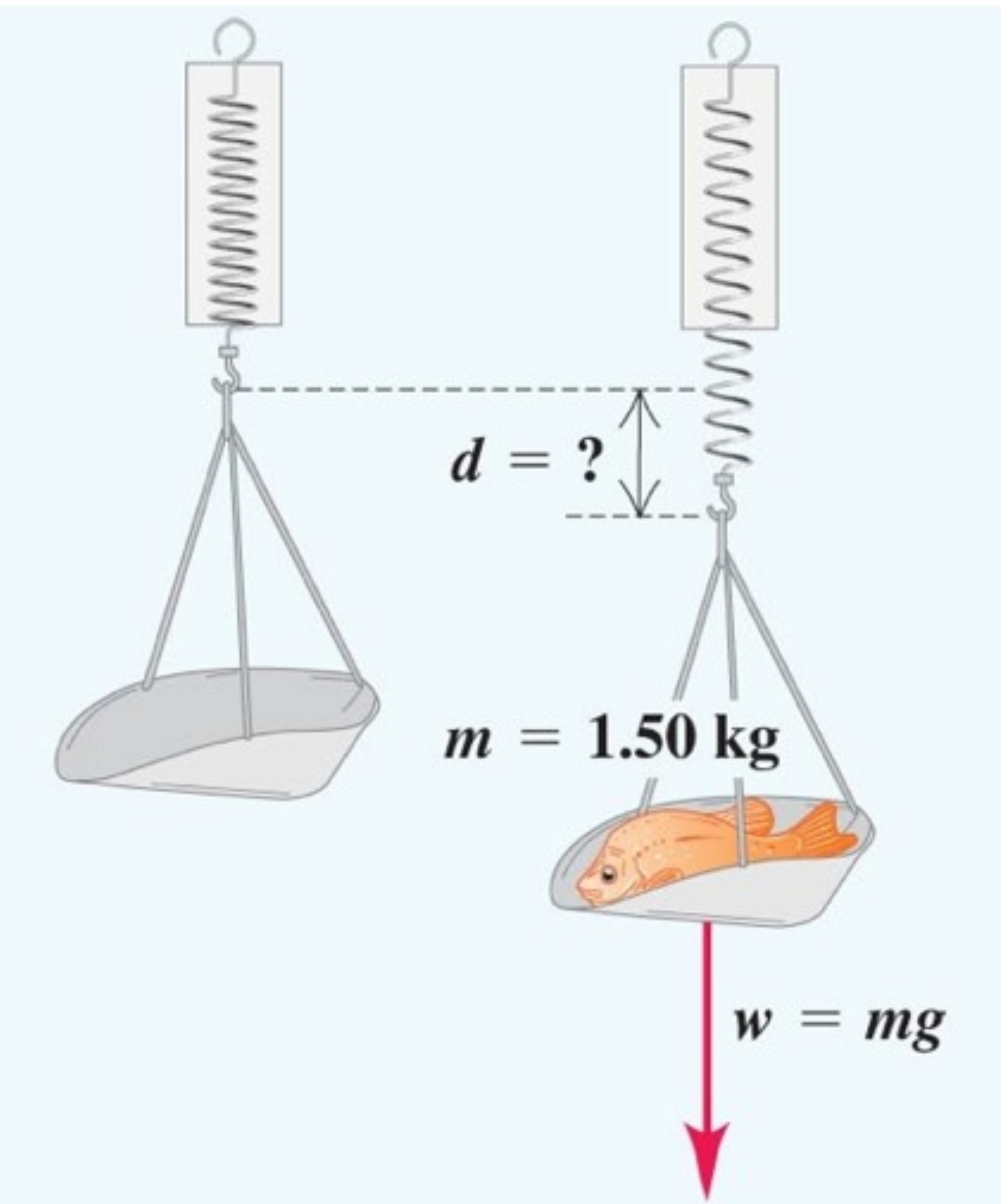
for this spring every centimeter of extension means 12 N of weight

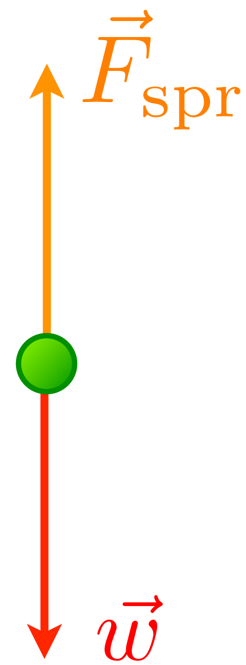
# a spring balance

$$F_{\text{spr}} = -kx$$

➜ we can use Hooke's law to build a device to measure weight

**measurement**




$$|x| = \frac{F_{\text{spr}}}{k} = \frac{mg}{k}$$

$$= \frac{1.50\,\text{kg} \times 9.80\,\text{m/s}^2}{1.2 \times 10^3\,\text{N/m}}$$

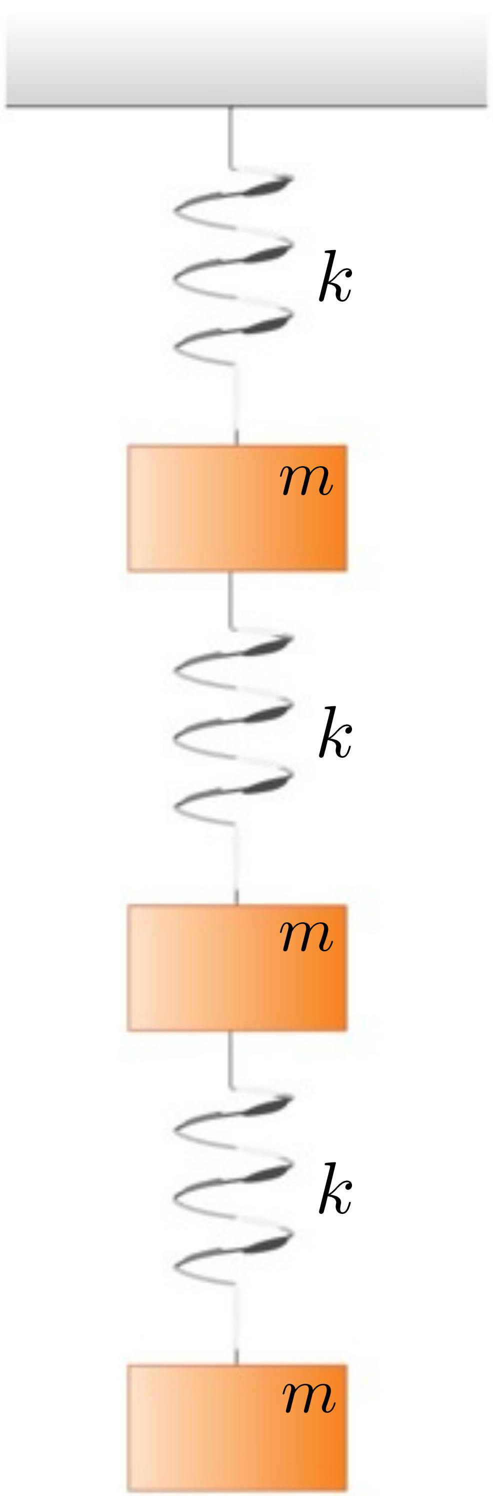
$$= \frac{14.7\,\text{N}}{1.2 \times 10^3\,\text{N/m}}$$

$$= 0.0123\,\text{m} = 1.23\,\text{cm}$$

# springs and weights

➜ three identical masses are hung by three identical massless springs

➜ find the extension of each spring in terms of the mass, *m*, the spring constant, *k* and *g*



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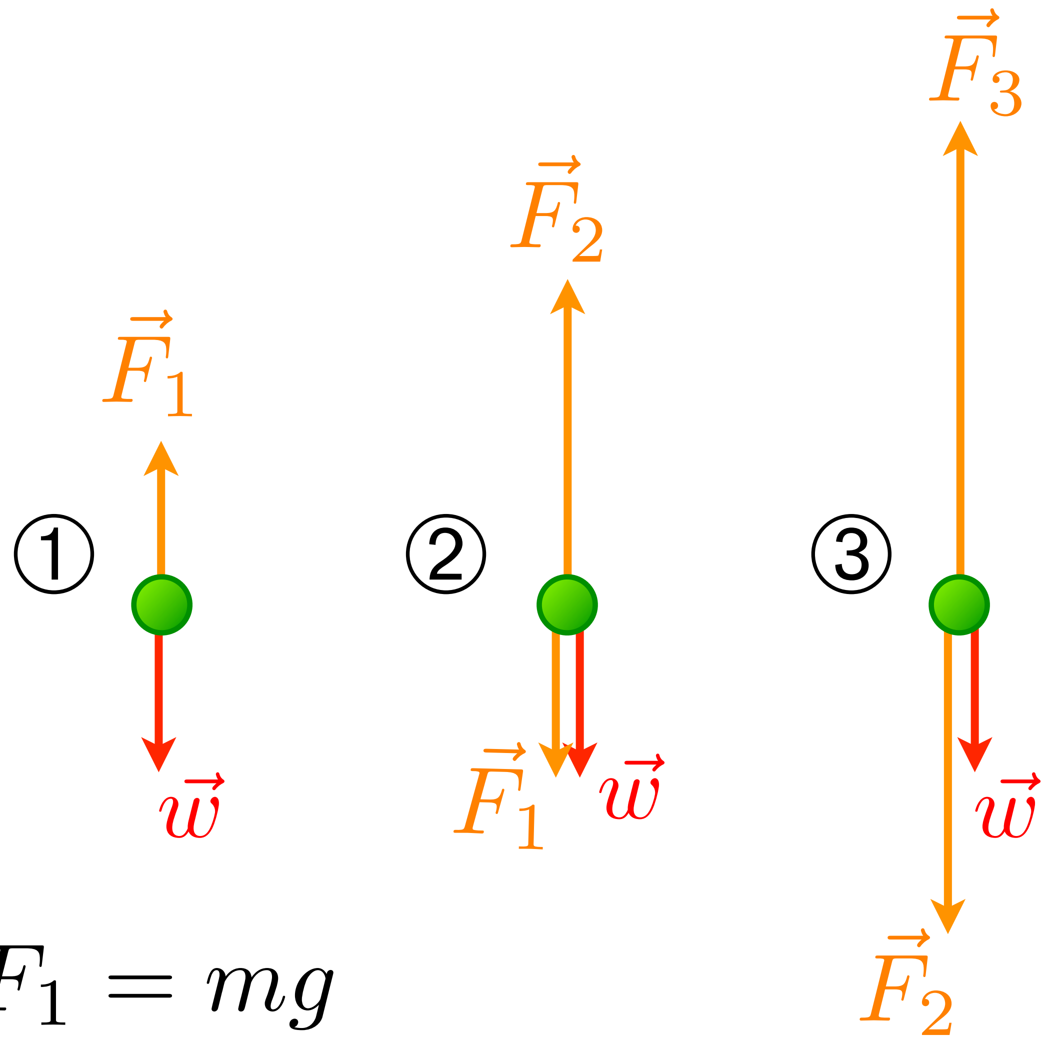



$$F_1 = mg$$

$$F_2 = mg + F_1$$
$$= 2mg$$

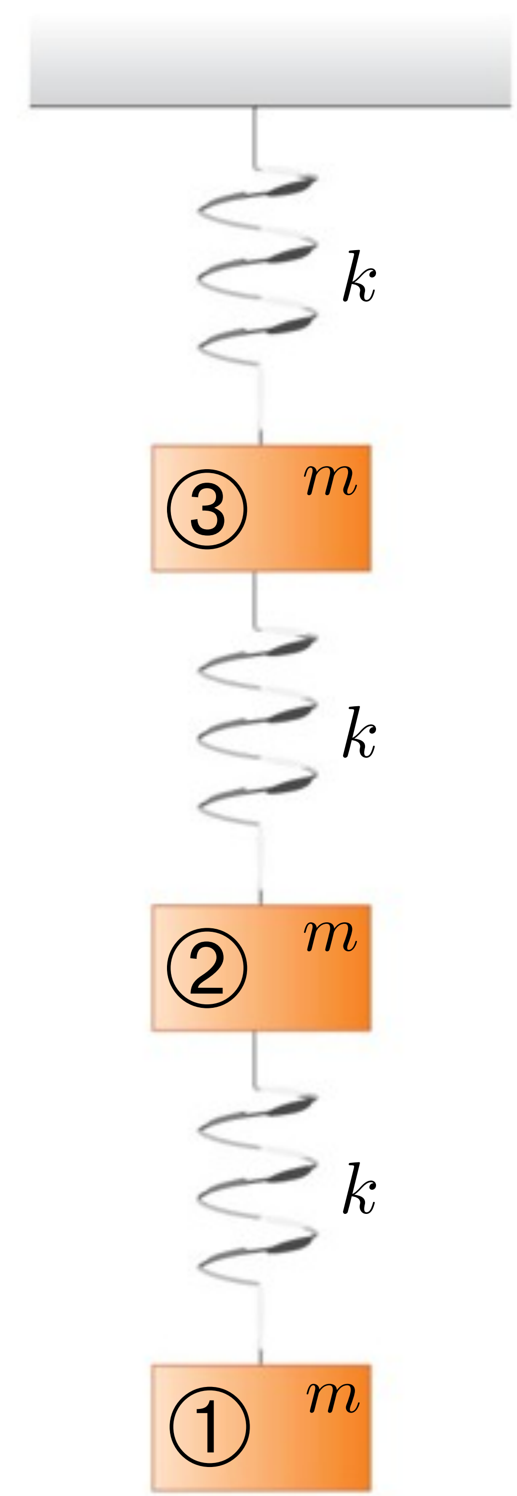
$$F_3 = mg + F_2$$
$$= 3mg$$

---

$$|x_1| = mg/k$$

$$|x_2| = 2mg/k$$

$$|x_3| = 3mg/k$$



# springs and weights

➜ three identical masses are hung by three identical springs

➜ if the unextended length of the springs are 10.0 cm, the spring constant is 8.00 kN/m and the masses are 14.00 kg each, find the lengths of each spring in equilibrium