forces & Newton’s laws of motion
forces (examples)

- **a push** is a force
- **a pull** is a force
- **gravity** exerts a force between all massive objects (without contact) (the force of attraction from the Earth is called the **weight** force)
contact forces

A normal force occurs when an object pushes on a surface, and the force is perpendicular to the surface.

A friction force can occur parallel to the surface of contact.
force vectors

forces have **magnitude** (measured in Newtons, N) and **direction**

e.g. $|\vec{F}| = 10 \text{ N}$
pulling a fridge - resultant force

two guys are moving a fridge by pulling on ropes attached to it
Steve is very strong, Walter is much weaker
two guys are moving a fridge by pulling on ropes attached to it

Steve is very strong, Walter is much weaker

the pair of forces can be replaced by one single force in the direction of the sum of forces with the magnitude of the sum

\[ \vec{F}_{\text{tot}} = \vec{F}_S + \vec{F}_W \]
components of a force
Newton’s first law

Isaac Newton first proposed the following law of nature to attempt to describe objects in motion:

“Every object continues either at rest or in constant motion in a straight line unless it is acted upon by a net force”

a.k.a ‘inertia’

The statement about objects at rest is pretty obvious, but the “constant motion” statement doesn’t seem right according to our everyday observations.
Newton’s first law & friction

“Every object continues either at rest or in constant motion in a straight line unless it is acted upon by a net force.”

The problem is we can’t easily test the law, because we can’t set up a situation where there is no force.

Friction is ubiquitous. We can try to minimise friction though.

(a) Table: puck stops short
(b) Ice: puck slides farther
(c) Air-hockey table: puck slides even farther
curling & inertia
Newton’s second law

→ The first law describes what happens when no force acts on an object
→ The second law describes the response of the object to a force being applied

we know that different objects respond differently to the same magnitude of force

→ push a shopping cart
→ push a freight train → very different responses

“The acceleration of an object is directly proportional to the resultant force acting on it and inversely proportional to its mass. The direction of the acceleration is the direction of the resultant force.”

or, in a much more compact notation, \[ \sum \vec{F} = m \vec{a} \]
Newton’s second law

“The acceleration of an object is directly proportional to the resultant force acting on it and inversely proportional to its mass. The direction of the acceleration is the direction of the resultant force.”

$$\sum \vec{F} = m\vec{a}$$

→ OK, so to move an object at rest we need to accelerate it

$$\vec{a} \neq 0$$

means there must be a net force acting on the object

→ to change the velocity of an object, we need to accelerate it

$$\sum \vec{F} = 0$$

so

$$\vec{a} = 0$$

→ the mass of an object determines how much acceleration you get for a given force

$$|\vec{a}| = \frac{|\vec{F}|}{m}$$

→ push a shopping cart ($m$ is small)

→ push a freight train ($m$ is big)

→ big acceleration

→ small acceleration
"the" unit of force

→ the Newton, N, is defined to be the force required to impart an acceleration of 1 m/s² to a 1 kg mass

\[ F = ma \]

\[ 1 \, \text{N} = 1 \, \text{kg m/s}^2 \]

there are other units, e.g. pounds (lb) we could use, but let’s stick to SI (“metric”) for now
a worker with spikes on his shoes pulls with a constant horizontal force of magnitude 20 N on a box of mass 40 kg resting on the flat, frictionless surface of a frozen lake. What is the acceleration of the box?

\[ \sum F_x = +20 \text{ N} \]

\[ m a_x = F_x \]
\[ a_x = \frac{F_x}{m} = \frac{20 \text{ N}}{40 \text{ kg}} \]
\[ = 0.50 \frac{\text{kg m/s}^2}{\text{kg}} \]
\[ a_x = 0.50 \text{ m/s}^2 \]
mass & weight

→ we need to be careful to distinguish these terms

→ **mass** is related to the amount of matter ("stuff") in an object

→ **weight** is specifically the **force** on an object from the gravitational attraction of the Earth

mass, $m$

are mass and weight related?

experimentally we found that all objects in free-fall accelerate toward the Earth with acceleration of magnitude $g = 9.80 \text{ m/s}^2$

\[
F_y = ma_y
\]

\[
-w \quad m \times (-g)
\]

\[
w = mg \quad \text{true for all objects}
\]

but the value of $g$ is a property of the Earth, it wouldn’t be the same value on the Moon
normal force (at equilibrium)

→ consider the forces on a box sitting at rest on the floor

- the weight force points down
  but this can’t be the only force on the box
  - if it was it would accelerate downwards!

- the normal force points upward
  the box compresses the surface of the floor at the microscopic level and the floor pushes back

\[
\sum F_y = N + (-w) = N - w
\]

\[
m a_y = 0 \quad \begin{cases} N = w \end{cases}
\]
Newton’s third law

“ To every action there is always opposed an equal reaction ”

If an object \( A \) exerts a force on object \( B \), object \( B \) exerts a force of equal magnitude and opposite direction on \( A \)

\[
\vec{F}_A \text{ on } B = -\vec{F}_B \text{ on } A
\]
normal forces & Newton’s third law

→ box A sits on top of box B at rest
all the forces

- box A sits on top of box B at rest

  what forces act on box A?

  - normal force on A from B
  - weight of A

  equal & opposite because the box is at rest - the first law
all the forces

→ box A sits on top of box B at rest

what is the third law reaction force to the normal force on A from B?

equal & opposite because of the **third** law - N.B. forces act on **different** objects
all the forces

box A sits on top of box B at rest

what is the third law reaction force to weight of A?

so just as the Earth attracts you toward it, you attract the Earth toward you
all the forces

→ box A sits on top of box B at rest

is the normal force on A the reaction force to the weight of A?

NO!

they are equal & opposite, but they aren’t an action-reaction pair

remove box B & the normal force goes away
bathroom scales

→ your bathroom scales measure the force you exert on the scales

the reaction force of this is the normal force of the scales on you

you also have your weight force

normal force on you from scales

normal force on you from scales

normal force on scales from you

but if you’re at rest & remain at rest, your acceleration is zero

& thus the scales measure your weight
now suppose you stand on bathroom scales while riding an elevator

★ elevator accelerating down

![Diagram showing forces on an elevator accelerating down]

smaller reading on the scales
you “lost weight”

★ elevator accelerating up

![Diagram showing forces on an elevator accelerating up]

larger reading on the scales
you “gained weight”
→ consider a uniform rope whose ends are being pulled on

→ look at a small section with mass $m$

Newton’s 2nd law applied to this section of rope

$$T_R - T_L = ma$$

→ rope in equilibrium (not accelerating), $T_R = T_L$ & tension same throughout

→ rope is massless, $T_R = T_L$ & tension same throughout

we will usually assume ropes to be effectively massless or in equilibrium such that the tension is the same throughout the rope
systems in equilibrium in more than one dimension

→ here by ‘in equilibrium’, we mean at rest or moving with constant velocity

→ in that case $\vec{a} = \vec{0}$ and by Newton’s second law $\sum \vec{F} = \vec{0}$

→ or “all the forces on an object must balance”

really just Newton’s first law

$\sum F_x = 0$

$\sum F_y = 0$

→ it is often helpful to split the problem up into components
two-dimensional equilibrium

A car engine of mass 500 kg hangs at rest from a set of chains as shown. Find the tension in each chain, assuming their masses are negligible.
two-dimensional equilibrium

\( \rightarrow \) a car engine of mass 500 kg hangs at rest from a set of chains as shown. Find the tension in each chain, assuming their masses are negligible.

\[
\begin{align*}
\sum F_y &= T_1 - w = 0 \\
\implies T_1 &= w \\
\sum F_y &= T_3 \sin \theta - T_1 = 0 \\
\implies T_3 \sin \theta &= w \\
\implies T_3 &= \frac{w}{\sin \theta} \\
\sum F_x &= T_3 \cos \theta - T_2 = 0 \\
\implies T_2 &= T_3 \cos \theta
\end{align*}
\]

\( \theta = 60^\circ \)
systems out of equilibrium

→ here by ‘in equilibrium’, we mean at rest or moving with constant velocity

→ so systems out of equilibrium have an acceleration

→ and we must use Newton’s second law in full

\[ \sum \vec{F} = m \vec{a} \]

→ it is often helpful to split the problem up into components

\[ \sum F_x = ma_x \]
\[ \sum F_y = ma_y \]
the sliding box

→ a box slides down a frictionless incline sloped at a constant angle of $\theta$

find the acceleration of the box and the normal force exerted by the slope on the box

free-body diagram for the box

express in components parallel and perpendicular to the slope

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the sliding box

→ a box slides down a frictionless incline sloped at a constant angle of $\theta$

find the acceleration of the box and the normal force exerted by the slope on the box

express in components parallel and perpendicular to the slope

$$\sum F_x = -w \sin \theta$$

box accelerates down the incline

$$-w \sin \theta = ma_x$$

$$a_x = \frac{-mg \sin \theta}{m}$$

$$a_x = -g \sin \theta$$

$$\sum F_y = N - w \cos \theta$$

box doesn’t move off the incline $a_y = 0$

$$N = w \cos \theta$$

$$N = mg \cos \theta$$

as a cross-check, consider $\theta \to 0, 90^\circ$
the Atwood machine

→ consider this experiment
the Atwood machine

→ let’s explain the measurement using our theory of forces

\[ T - w_L = m_L a \]
\[ w_R - T = m_R a \]
\[ w_R - w_L = (m_R + m_L) a \]
\[ a = \frac{m_R - m_L}{m_R + m_L} g \]

\[ y = y_0 - v_0 t + \frac{1}{2} a t^2 \]
\[ t = \sqrt{\frac{2 \Delta y}{a}} \]

\[ m_L = 0.550 \text{ kg} \]
\[ m_R = 0.560 \text{ kg} \]
\[ \Delta y = 1.0 \text{ m} \]

\[ t = 4.8 \text{ s} \]
friction

→ we already considered one contact force present when two surfaces touch, namely the normal force, which acts perpendicular to the surfaces

→ in some cases there can be a contact force parallel to the surfaces known as the friction force

→ friction is everywhere ... let’s build a simple model to describe it

→ two forms of friction - static (not moving) & kinetic (moving)
no applied force, box at **rest**
no friction force
friction

small applied force, box at rest

**static friction force**

\[ f_s = T \]

static friction force matches the pull force & keeps the box at rest
friction

larger applied force, box at rest

**static friction force**

\[ f_s = T \]

static friction force increases to match the pull force & keep the box at rest
friction

Eventually, the static friction force cannot get any larger, and the box will start to move once the box is moving. There will be a constant kinetic friction force:

$$f_k = T$$

Even larger applied force, box moving at constant speed.

**Kinetic friction force**
**friction**

- no applied force, box at **rest**
  - no friction force

- small applied force, box at **rest**
  - **static friction force**

- larger applied force, box at **rest**
  - **static friction force**

- even larger applied force, box moving at **constant speed**
  - **kinetic friction force**

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**Graph:**
- **Static friction force ($f_s$)** increases linearly with applied force ($T$).
- **Kinetic friction force ($f_k$)** remains constant.

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**Equations:**
- $f_s = \mu_s N$
- $f_k = \mu_k N$

friction

The magnitudes of $f_s^{\text{max}}$ and $f_k$ are determined by properties of the two surfaces in contact and can be expressed via coefficients of friction.

The static friction force is given by $f_s^{\text{max}} = \mu_s N$, and the kinetic friction force is given by $f_k = \mu_k N$.

<table>
<thead>
<tr>
<th>materials</th>
<th>rubber on concrete</th>
<th>wood on concrete</th>
<th>steel on Teflon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_s$</td>
<td>1.0</td>
<td>0.6</td>
<td>0.04</td>
</tr>
</tbody>
</table>
**elastic forces**

springs & Hooke’s law

\[ F_{spr} = -kx \]

an empirical, approximate law

At the equilibrium position \( x = 0 \), the spring is neither stretched nor compressed.

The magnitude of the spring force is proportional to the displacement.

The spring force points opposite to the direction of displacement.
A spring balance

we can use Hooke’s law to build a device to measure weight

\[ F_{spr} = -kx \]

Calibration

For this spring every centimeter of extension means 12 N of weight

\[ k = \frac{12 \text{ N}}{0.01 \text{ m}} = 1.2 \times 10^3 \text{ N/m} \]
a spring balance

we can use Hooke’s law to build a device to measure weight

\[ F_{spr} = -kx \]

\[ |x| = \frac{F_{spr}}{k} = \frac{mg}{k} \]

\[ = \frac{1.50 \text{ kg} \times 9.80 \text{ m/s}^2}{1.2 \times 10^3 \text{ N/m}} \]

\[ = \frac{14.7 \text{ N}}{1.2 \times 10^3 \text{ N/m}} \]

\[ = 0.0123 \text{ m} = 1.23 \text{ cm} \]
springs and weights

→ three identical masses are hung by three identical massless springs
→ find the extension of each spring in terms of the mass, \( m \), the spring constant, \( k \) and \( g \)
springs and weights

- three identical masses are hung by three identical massless springs
- find the extension of each spring in terms of the mass, $m$, the spring constant, $k$, and $g$

\[ F_1 = mg \]

\[ F_2 = mg + F_1 = 2mg \]

\[ F_3 = mg + F_2 = 3mg \]

\[ |x_1| = \frac{mg}{k} \]

\[ |x_2| = \frac{2mg}{k} \]

\[ |x_3| = \frac{3mg}{k} \]
springs and weights

- three identical masses are hung by three identical springs
- if the unextended length of the springs are 10.0 cm, the spring constant is 8.00 kN/m and the masses are 14.00 kg each, find the lengths of each spring in equilibrium