work & energy

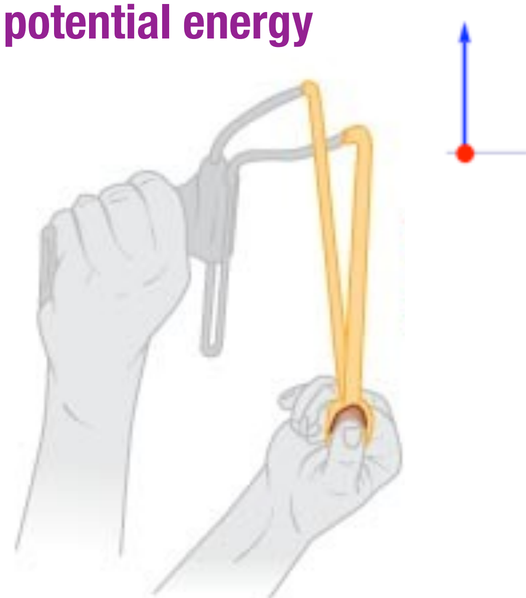
conservation of energy

entirely
gravitational potential
energy

kinetic energy
turning into
gravitational potential
energy

gravitational potential
energy
turning into
kinetic energy

stretched rubber stores
elastic potential energy



elastic potential energy
converted into
kinetic energy

conservation of energy

➜ the total energy in an isolated system is a constant quantity

➜ we'll largely limit ourselves to **mechanical energy**, examples being

➜ **kinetic energy** - energy due to motion

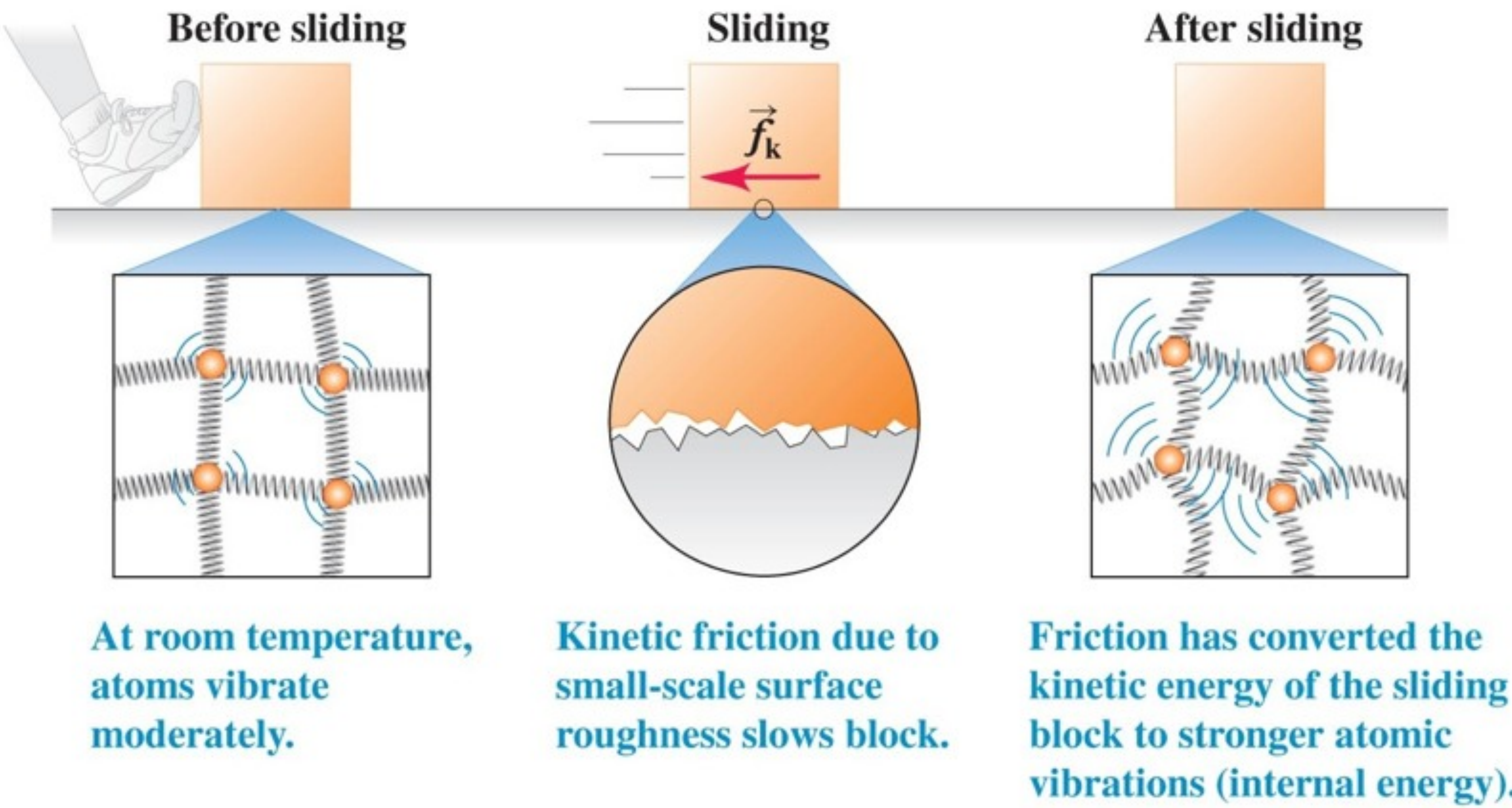
➜ **gravitational potential energy** - energy stored by being elevated above the Earth

➜ **elastic potential energy** - energy stored in an elastic deformation

dissipative forces & internal energy

➜ we've already mentioned that practically all systems experience friction of some sort

➜ friction is a dissipative force - it transforms **mechanical energy** into **internal energy**

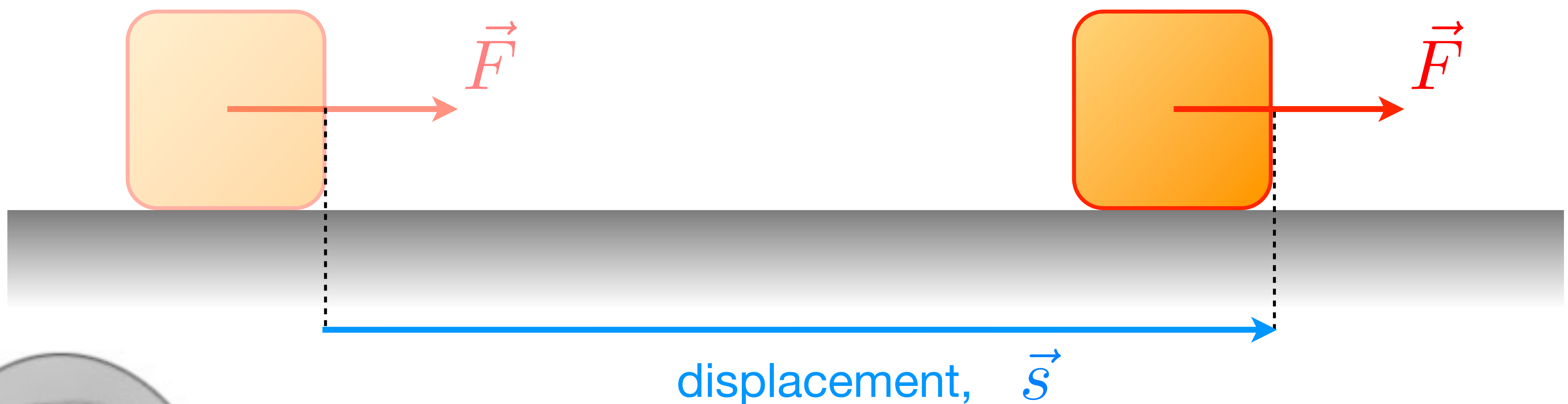


➜ in most cases this energy is effectively lost to us **(thermodynamics)**

work - the relation between force & energy

➜ we use **work** to mean the change in the kinetic energy of an object due to external forces

e.g. consider pushing on a block with a constant force



work, $W = F\,s$

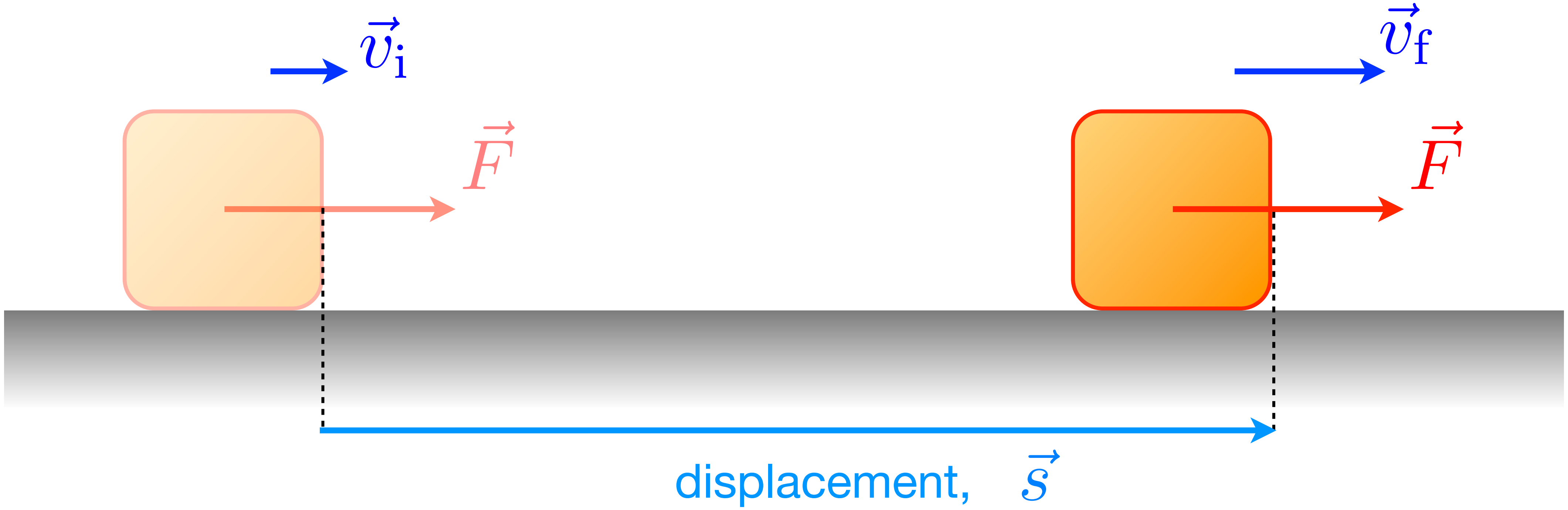
James Joule
(1818-1889)

units of work are N m,
also known as Joules, J

work & kinetic energy

➜ can we define kinetic energy in a quantitative way ?

e.g. consider pushing on a block, initially moving with speed v_i with a constant force over a distance s, ending with speed v_f



work, $W = F\,s$

constant force ⇒ constant acceleration

$$v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2as \qquad \& \qquad F = ma$$

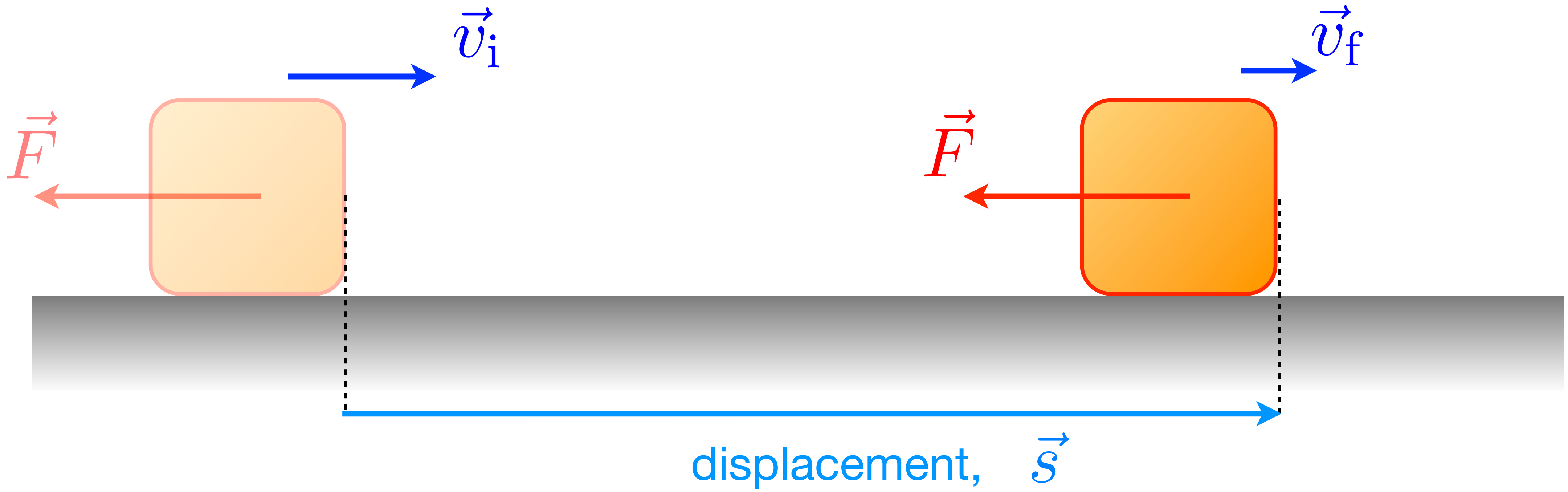
$$\tfrac{1}{2}mv_\mathrm{f}^2 = \tfrac{1}{2}mv_\mathrm{i}^2 + F\,s$$

$$K_\mathrm{f} = K_\mathrm{i} + W$$

“work-energy theorem”

can work reduce the kinetic energy of an object ?

➜ sure, just have the force acting against the motion :



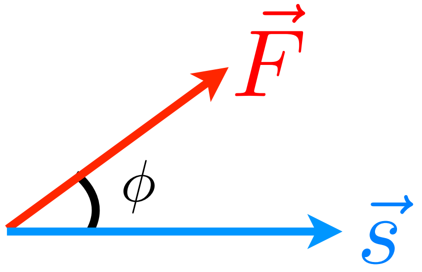
"work-energy theorem"

$$K_\mathrm{f} = K_\mathrm{i} + W$$

but K_f can only be smaller than K_i if W is negative, $W = -F\,s$?

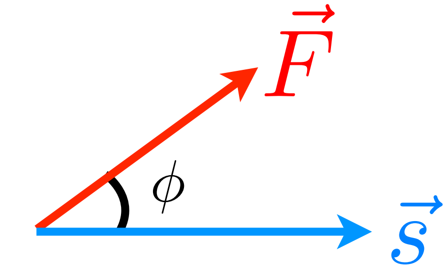
the relative direction between the force & the displacement is important !

$$W = Fs\cos\phi$$

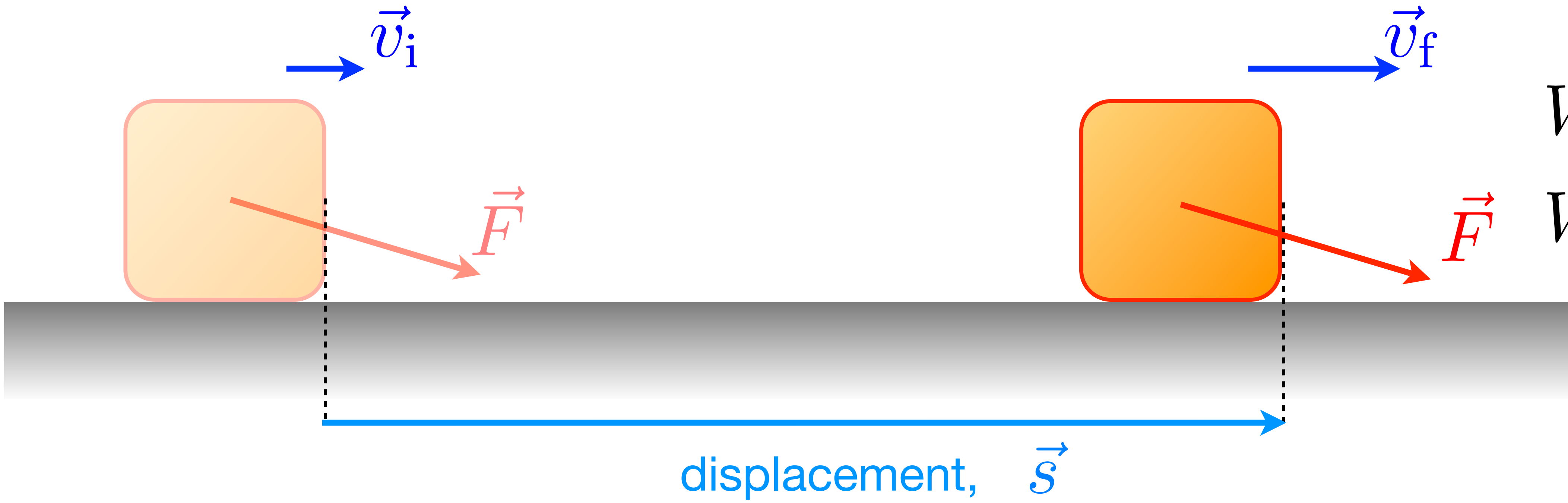


some force does no work

$$W = Fs\cos\phi$$



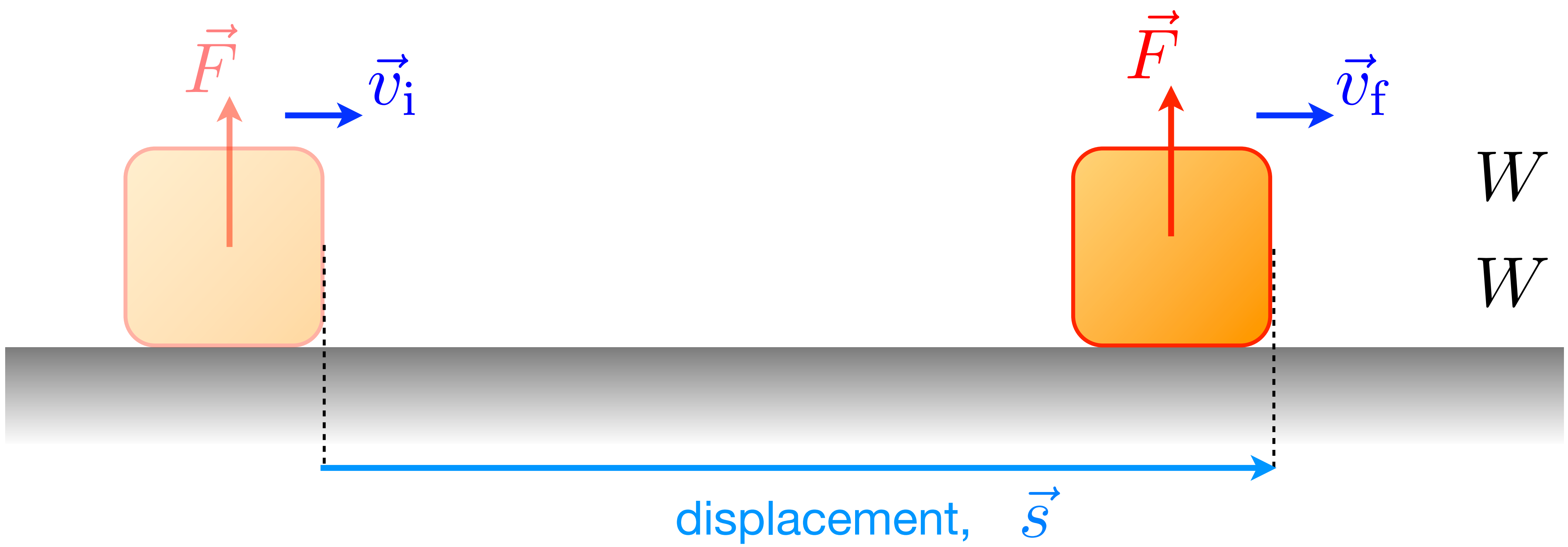
➜ only the component of force parallel to the displacement does work



$$W = Fs\cos\phi$$

$$W < Fs$$

➜ so a force perpendicular to the displacement will do no work

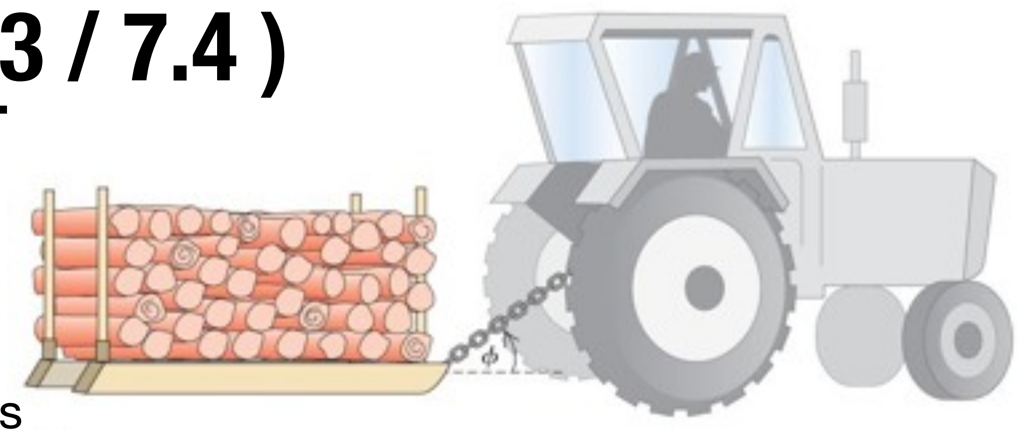


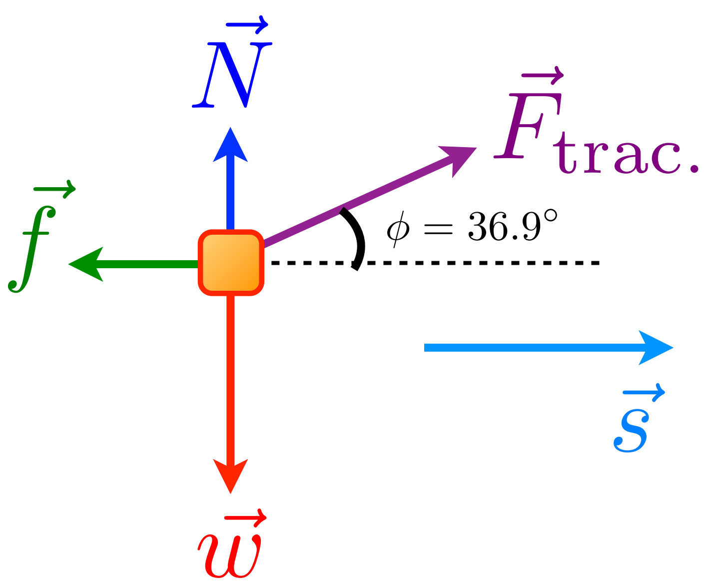
$$W = Fs\cos 90^\circ$$

$$W = 0$$

work done by several forces (examples 7.3 / 7.4)

The sled is pulled a distance of 20.0 m by a tractor along level frozen ground. The weight of the sled & its load is 14,700 N. The tractor exerts a constant force of 5000 N at an angle of 36.9° above the horizontal. A constant 3500 N friction force opposes the motion. Find the work done on the sled by each force and the total work done by all the forces. If the sled's initial speed is 2.00 m/s, what is its final speed?





the **normal** and **weight** forces do no work on the sled
- they are perpendicular to the displacement

$$\phi = 90^\circ \qquad \cos 90^\circ = 0$$

the **tractor force** does work

$$W_{\text{trac.}} = F_{\text{trac.}} s \cos\phi = (5000\,\text{N})(20.0\,\text{m})\cos 36.9^\circ$$
$$W_{\text{trac.}} = 80.0\,\text{kJ}$$

the **friction force** does work

$$W_{\text{fric.}} = f s \cos\phi = (3500\,\text{N})(20.0\,\text{m})\cos 180^\circ$$
$$W_{\text{fric.}} = -70.0\,\text{kJ}$$

$$W_{\text{tot.}} = W_{\text{trac.}} + W_{\text{fric.}} = 10.0\,\text{kJ}$$

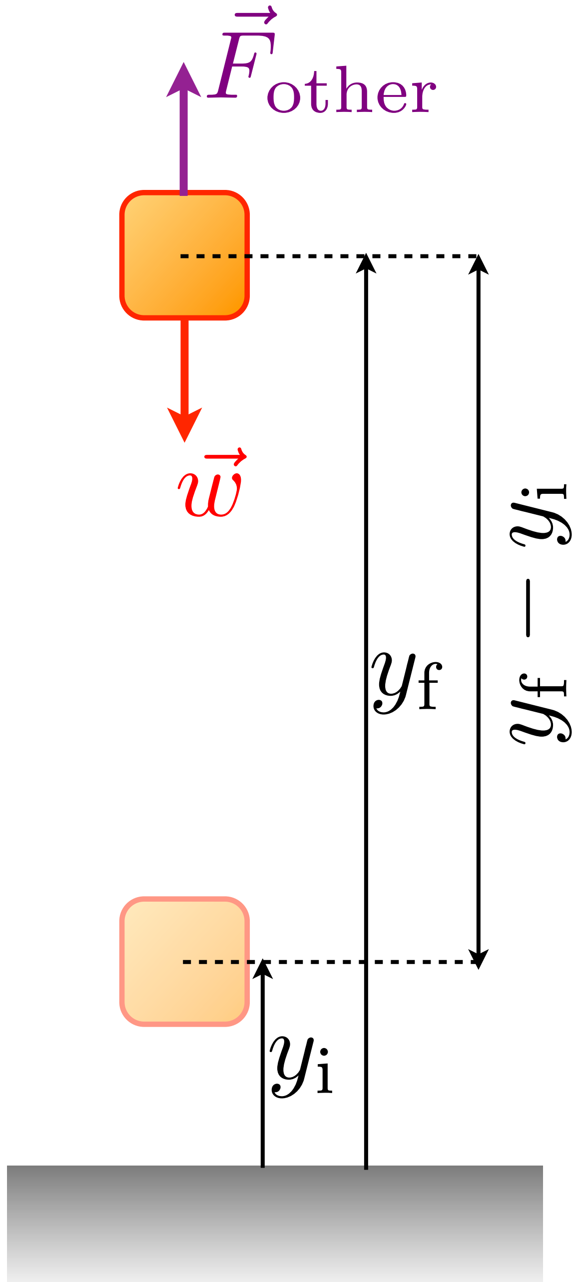
work-energy theorem

$$\tfrac{1}{2}mv_{\text{f}}^2 = \tfrac{1}{2}mv_{\text{i}}^2 + W_{\text{tot.}} \qquad v_{\text{f}} = \sqrt{v_{\text{i}}^2 + \frac{2W_{\text{tot.}}}{m}} \qquad v_{\text{f}} = 4.16\,\text{m/s}$$

potential energy

➜ for forces that are **conservative**, we can define a **potential energy** that accounts for energy "stored" by doing work

➜ e.g. **gravitational potential energy**



suppose we lift a block of mass *m* up from a height of y_i to a height of y_f

the work done by gravity on the block is

$$W_{\text{grav.}} = -mg\left(y_f - y_i\right)$$

& the work-energy theorem is

$$K_{\text{f}} = K_{\text{i}} + W_{\text{grav.}} + W_{\text{other}}$$

& thus

$$K_{\text{f}} = K_{\text{i}} - mgy_{\text{f}} + mgy_{\text{i}} + W_{\text{other}}$$

or

$$K_{\text{f}} + mgy_{\text{f}} = K_{\text{i}} + mgy_{\text{i}} + W_{\text{other}}$$

$$K_{\text{f}} + U_{\text{f}} = K_{\text{i}} + U_{\text{i}} + W_{\text{other}}$$

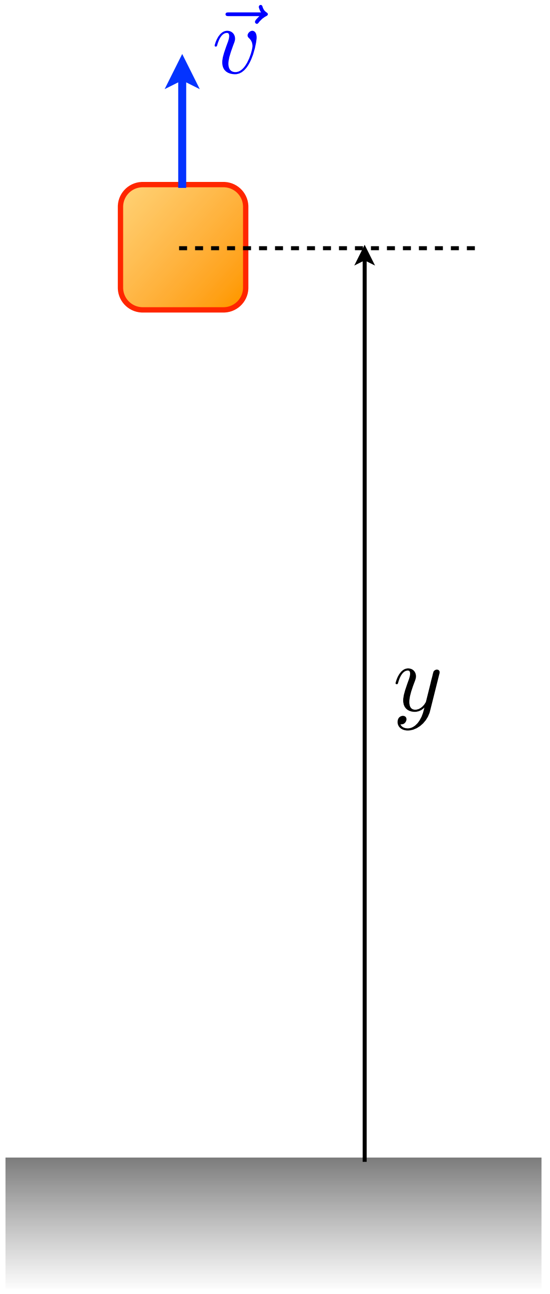
gravitational potential energy

➜ if the weight is the only force acting on a body, conservation of energy can be expressed

$$K_{\mathrm{f}} + U_{\mathrm{f}} = K_{\mathrm{i}} + U_{\mathrm{i}}$$

where $K = \frac{1}{2}mv^2$

$U = mgy$



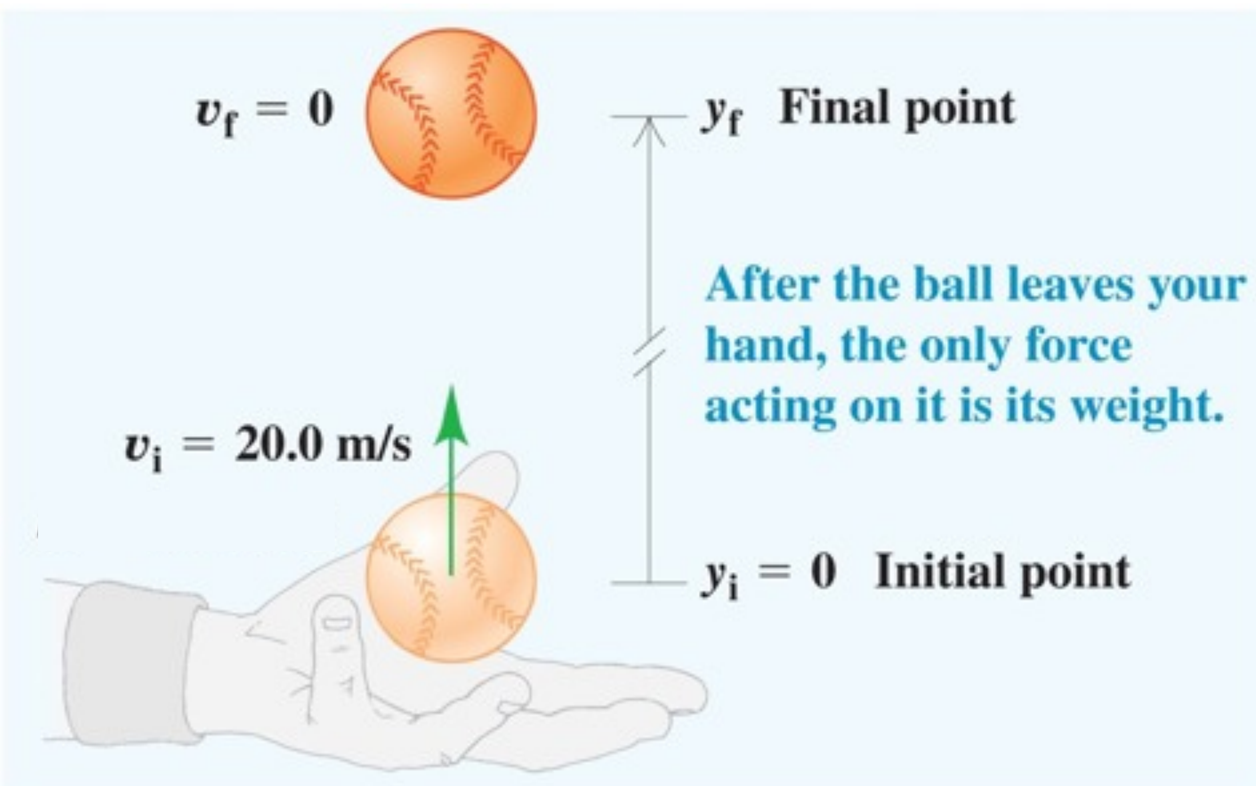
conservation of energy

$$K_f + U_f = K_i + U_i$$

$$K = \frac{1}{2}mv^2$$

$$U = mgy$$

You throw a ball of unknown mass straight up in the air, giving it an initial speed of 20.0 m/s. How high does it go, neglecting air resistance? Use conservation of energy.



$$K_f = 0$$

$$U_i = 0$$

$$K_i = U_f$$

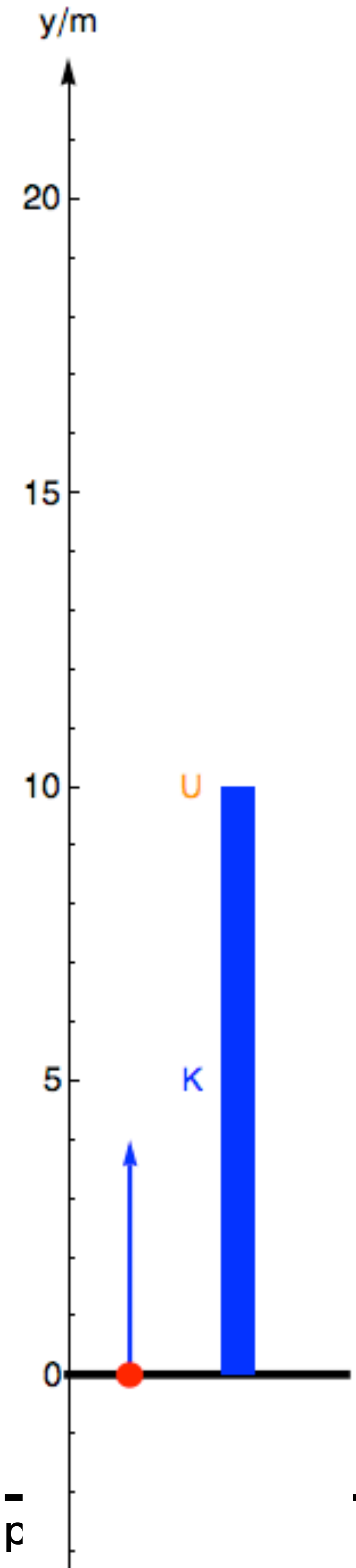
$$\frac{1}{2}mv_i^2 = mgy_f$$

$$y_f = \frac{v_i^2}{2g}$$

$$y_f = \frac{(20.0\,\mathrm{m/s})^2}{2 \times 9.80\,\mathrm{m/s^2}} = 20.4\,\mathrm{m}$$

conservation of energy

$$K_\mathrm{f} + U_\mathrm{f} = K_\mathrm{i} + U_\mathrm{i}$$

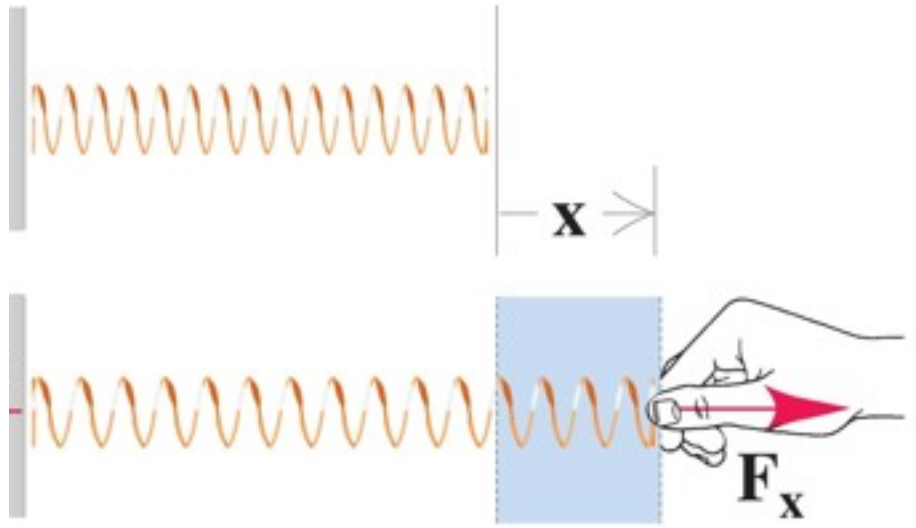


elastic potential energy

➜ compressing/extending a spring obeying Hooke's law will also 'store' energy

but in this case the force is not constant with displacement

$$F = kx$$

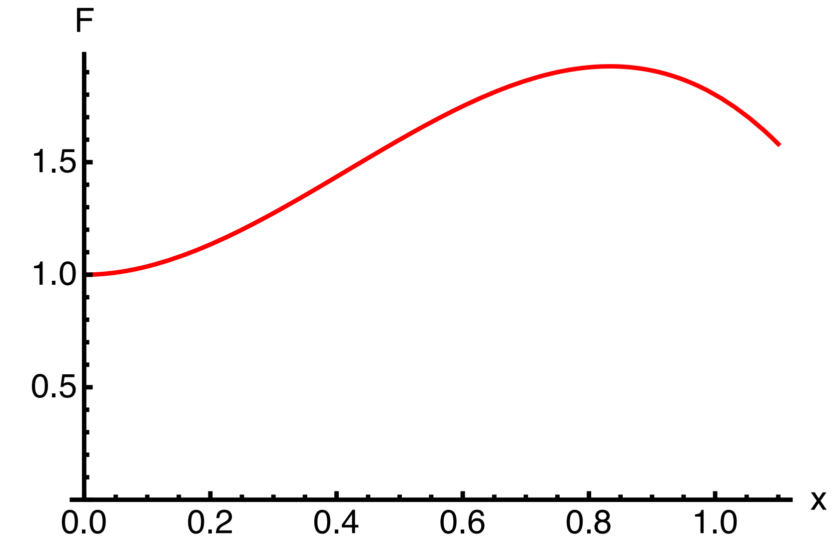


this is the force on the spring from the pulling hand

➜ we need to find a way to compute work when the force varies

work from a varying force

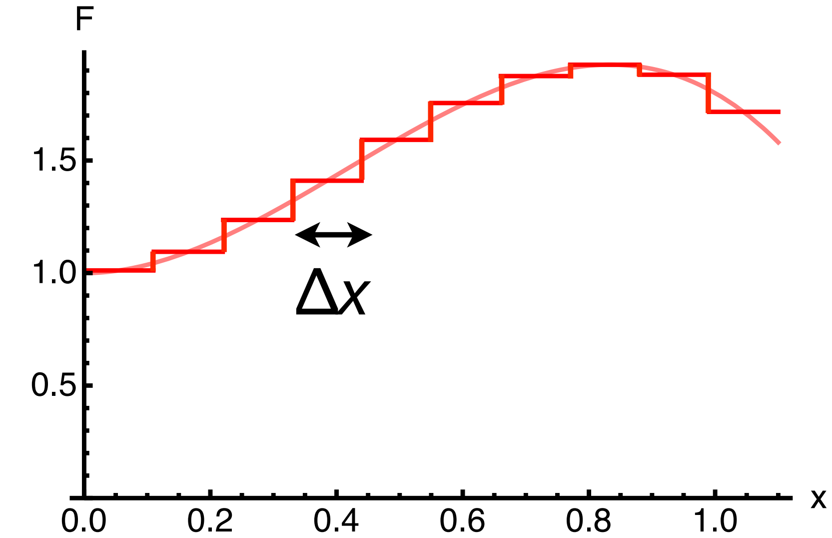
e.g. consider the case shown, when the force behaves in a complicated way with displacement



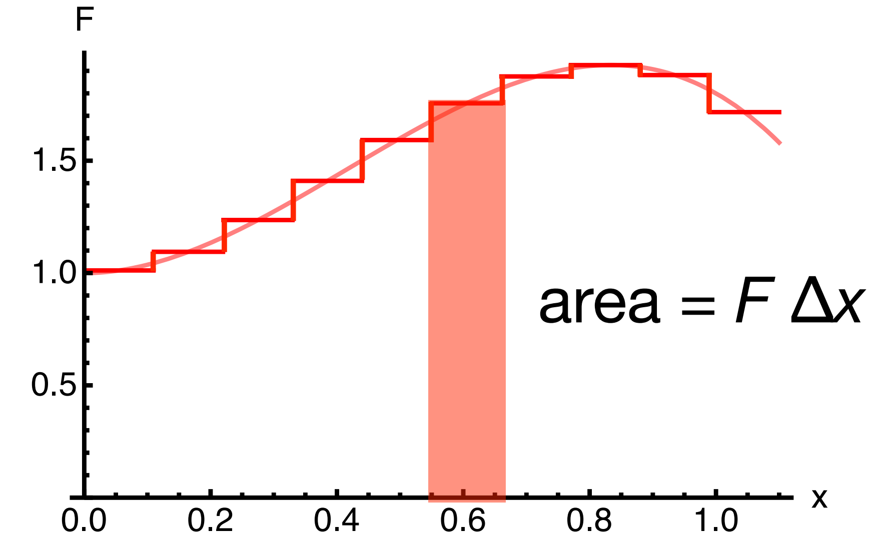
if the force was constant for a displacement Δx, we'd use

$$\text{work done, } \Delta W = F\,\Delta x$$

we can try considering the complicated F(x) to be a series of small constant changes



so the work done is **approximated** by the sum of the areas of these rectangles



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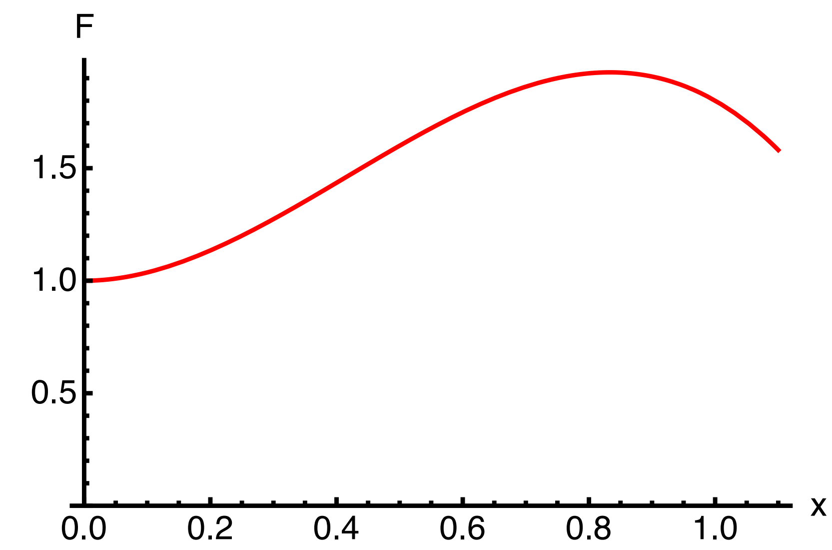

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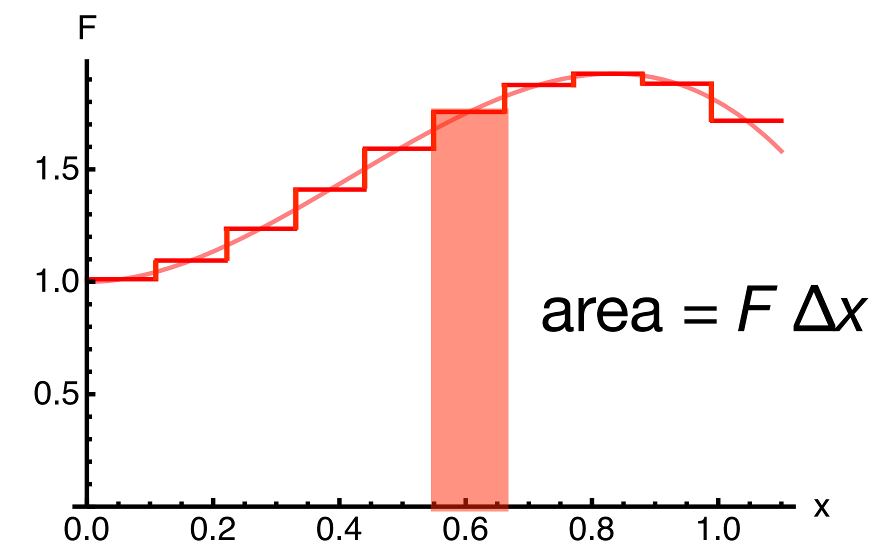
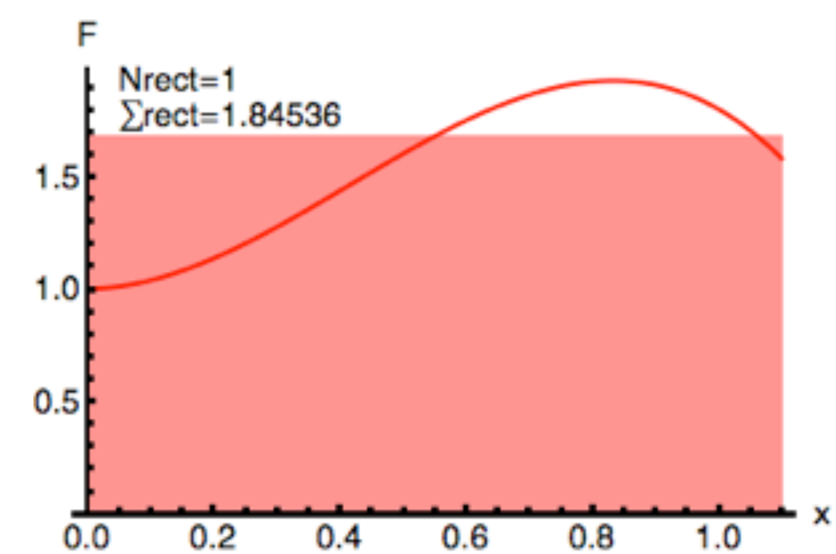


as we reduce the width Δx we'll get close to the correct answer, which is seen to be the **area under the curve**



elastic potential energy

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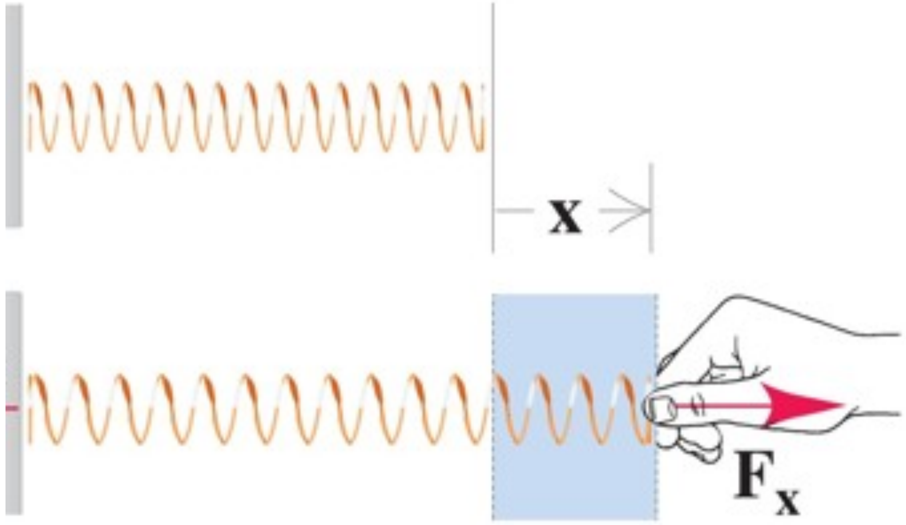
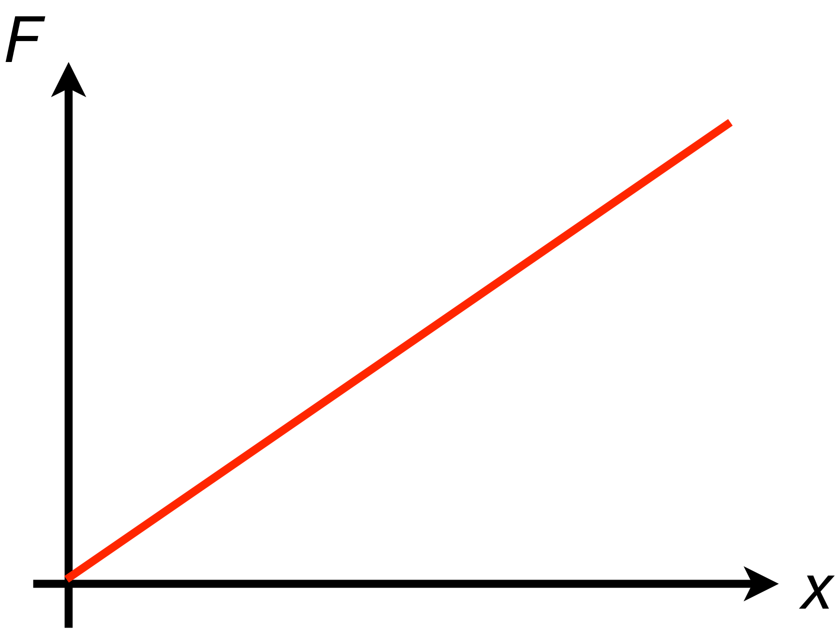
but in this case the force is not constant with displacement

$$F = kx$$



➜ so the work done corresponds to the area under the *F*,*x* curve



➜ but this is easy, it's just a triangle

area = ½×base×height

$$W = \frac{1}{2}xF$$
$$= \frac{1}{2}x(kx)$$

$$W = \frac{1}{2}kx^2$$

elastic potential energy

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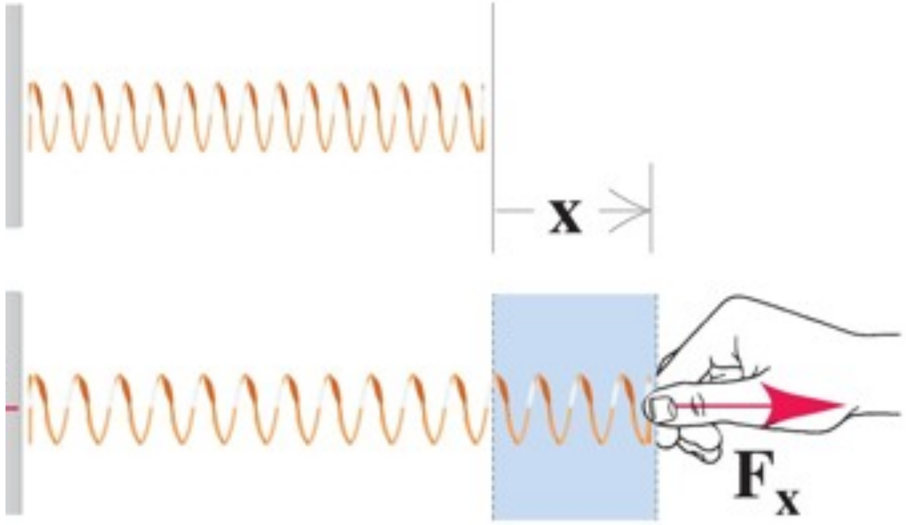

but in this case the force is not constant with displacement

$$F = kx$$

$$W = \frac{1}{2}kx^2$$

➜ considering a block attached to the end of a spring, we can treat the conservative Hooke's law force as giving us a potential energy

$$K_\mathrm{i} + U_\mathrm{i} = K_\mathrm{f} + U_\mathrm{f}$$

$$U_\mathrm{i} = \frac{1}{2}kx_\mathrm{i}^2$$