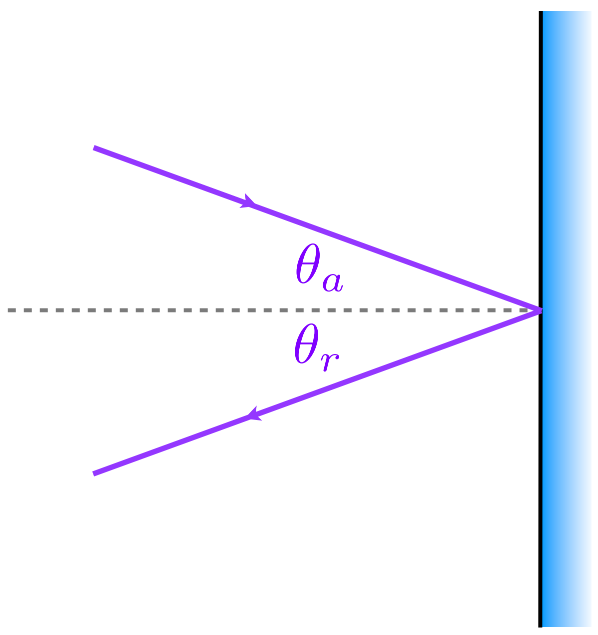
geometric optics

geometric optics of a plane (flat) mirror

➜ we've already learnt a rule that applies when light reflects from a smooth surface

angle of incidence = angle of reflection



geometric optics of a plane (flat) mirror

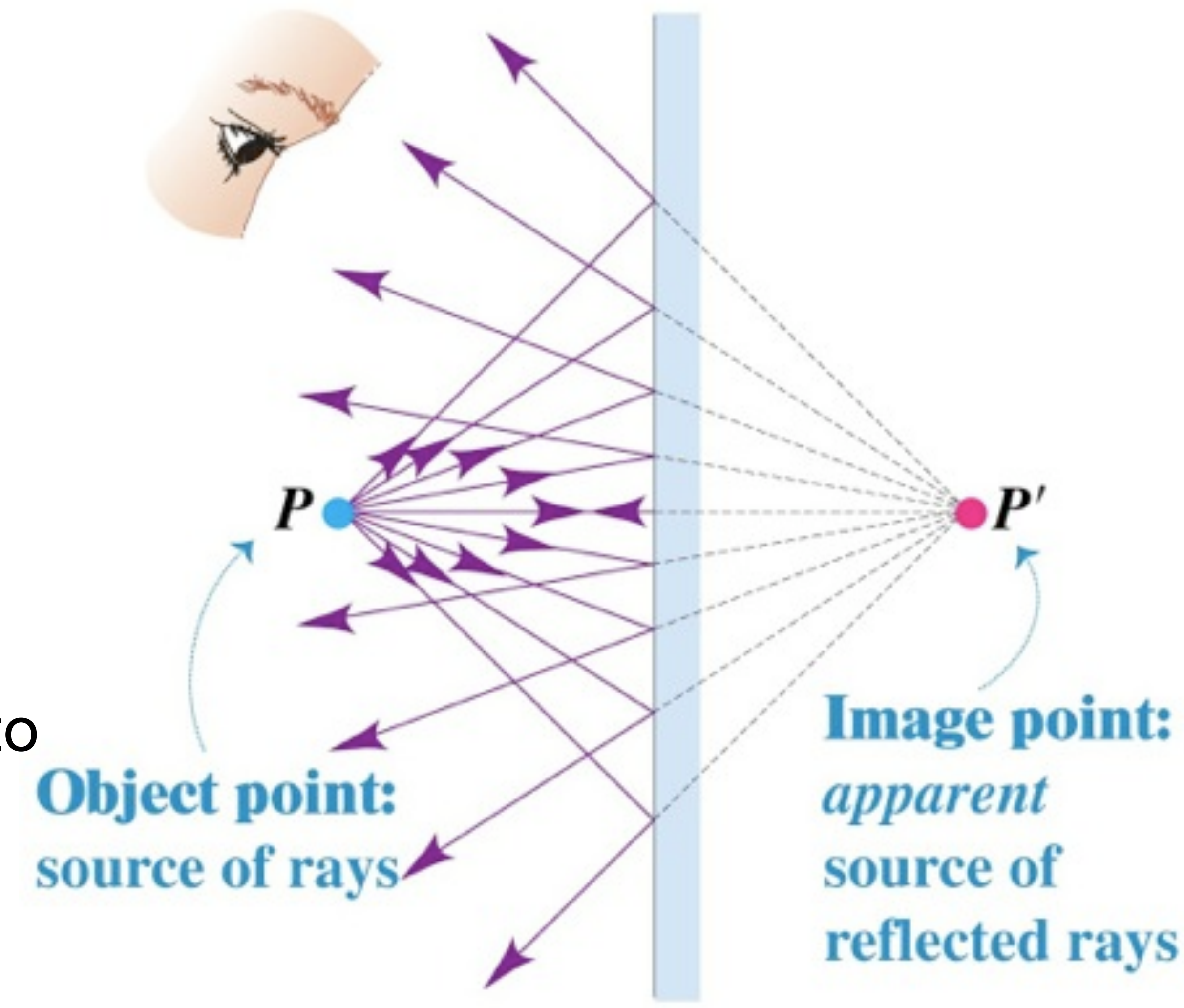
➜ consider an object which emits spherical wavefronts, or in other words, rays in every direction (this is basically anything illuminated)

➜ place the object in front of a plane mirror

➜ we can draw rays diverging from the object - they are reflected at the mirror

➜ tracking the reflected rays back they appear to be diverging from a point behind the mirror

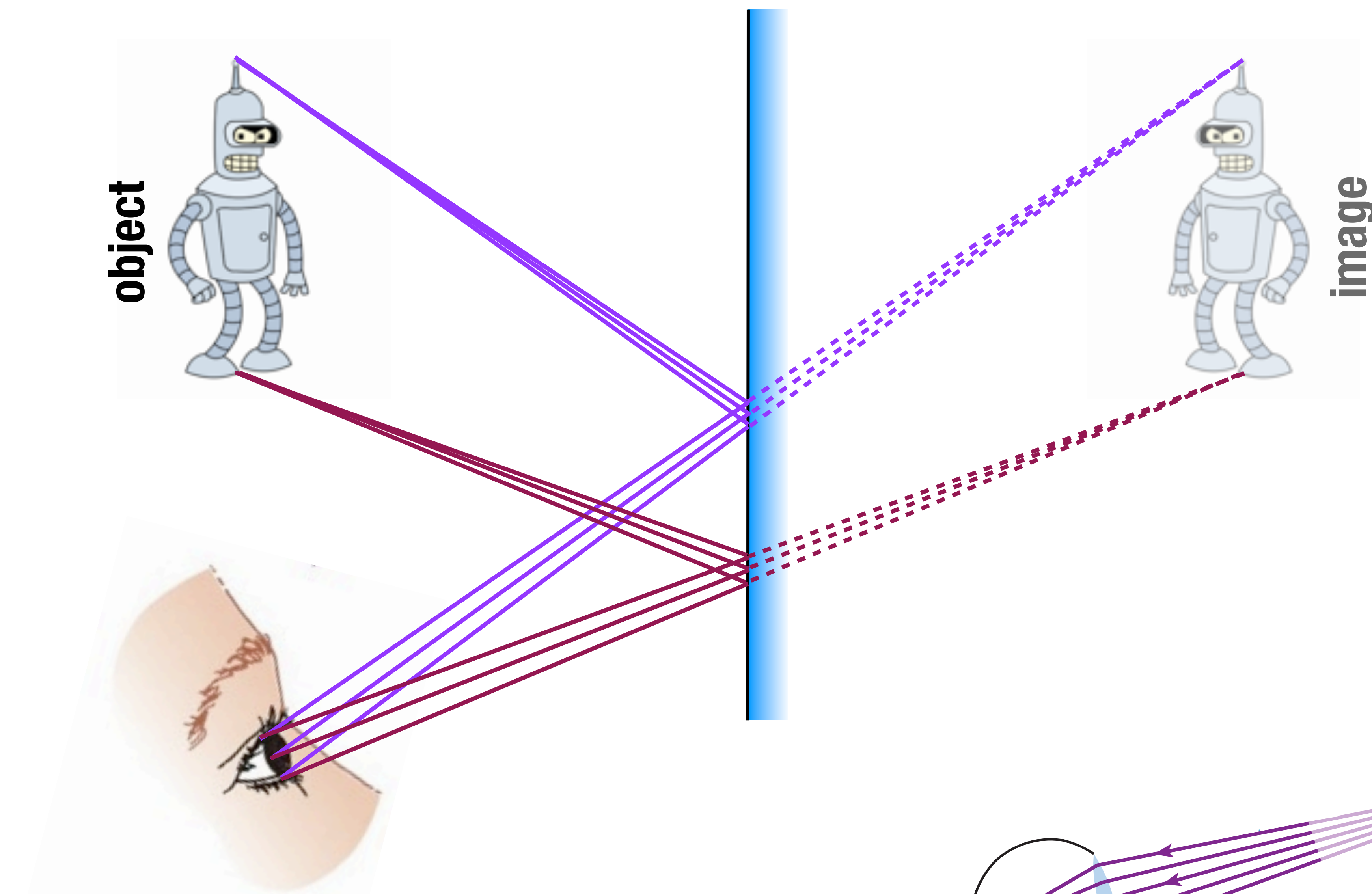
we call this the **image**

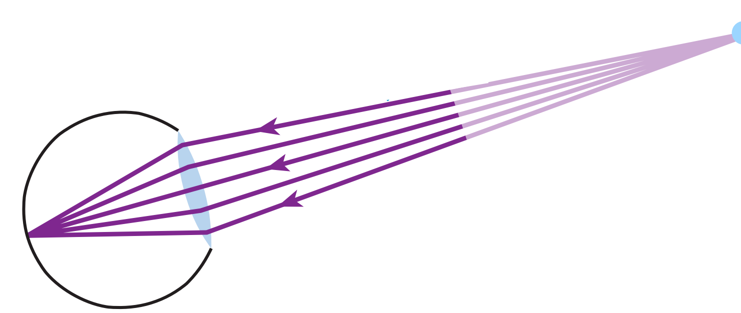


geometric optics of a plane (flat) mirror

➜ image formation for an extended object

➜ consider rays from different parts of the object



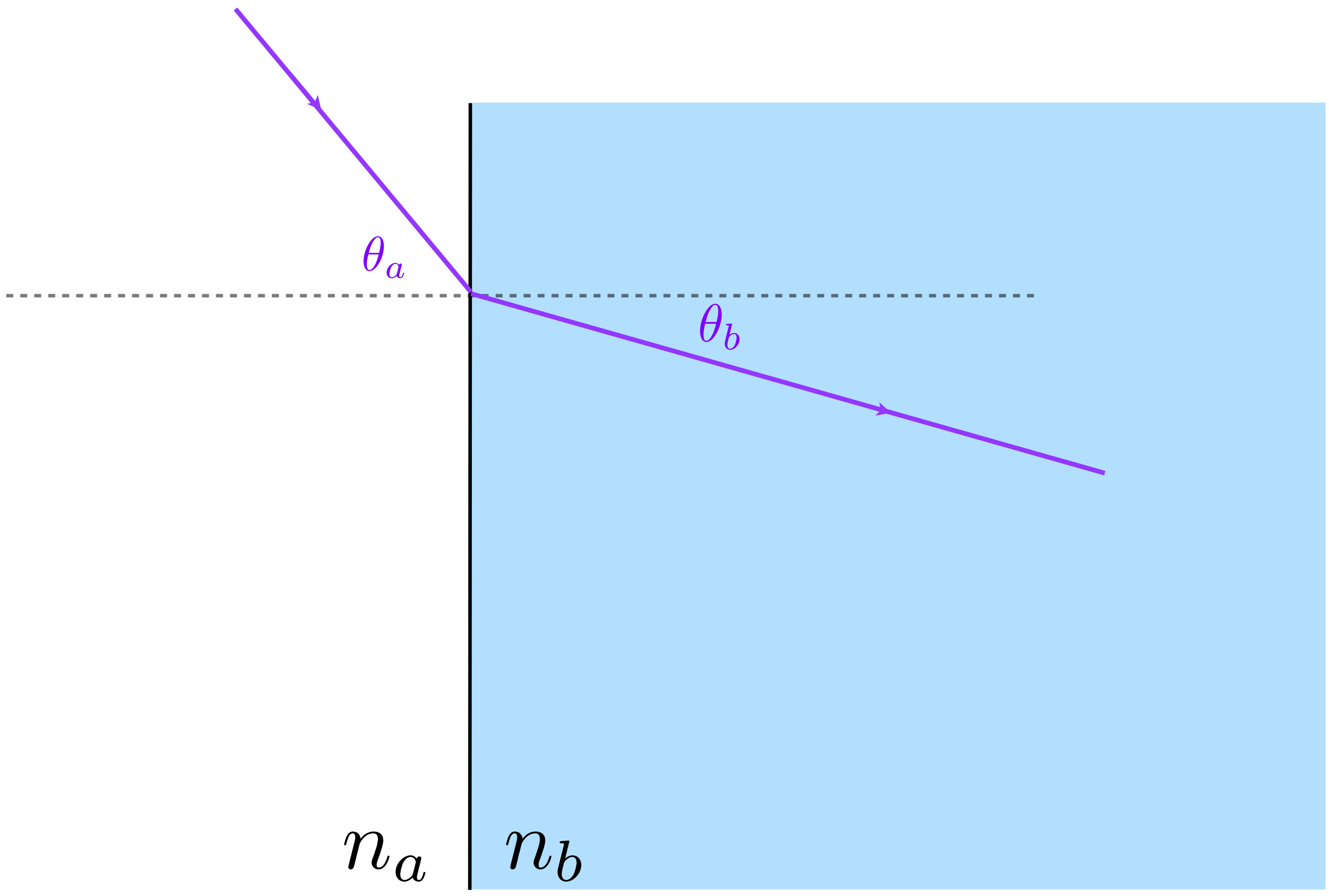


we'll consider how the eye focusses the image later when we look at lenses

geometric optics of a plane refracting surface

➜ we've already learnt a rule that applies when light refracts at the interface between two media

Snell's law : $n_a \sin\theta_a = n_b \sin\theta_b$



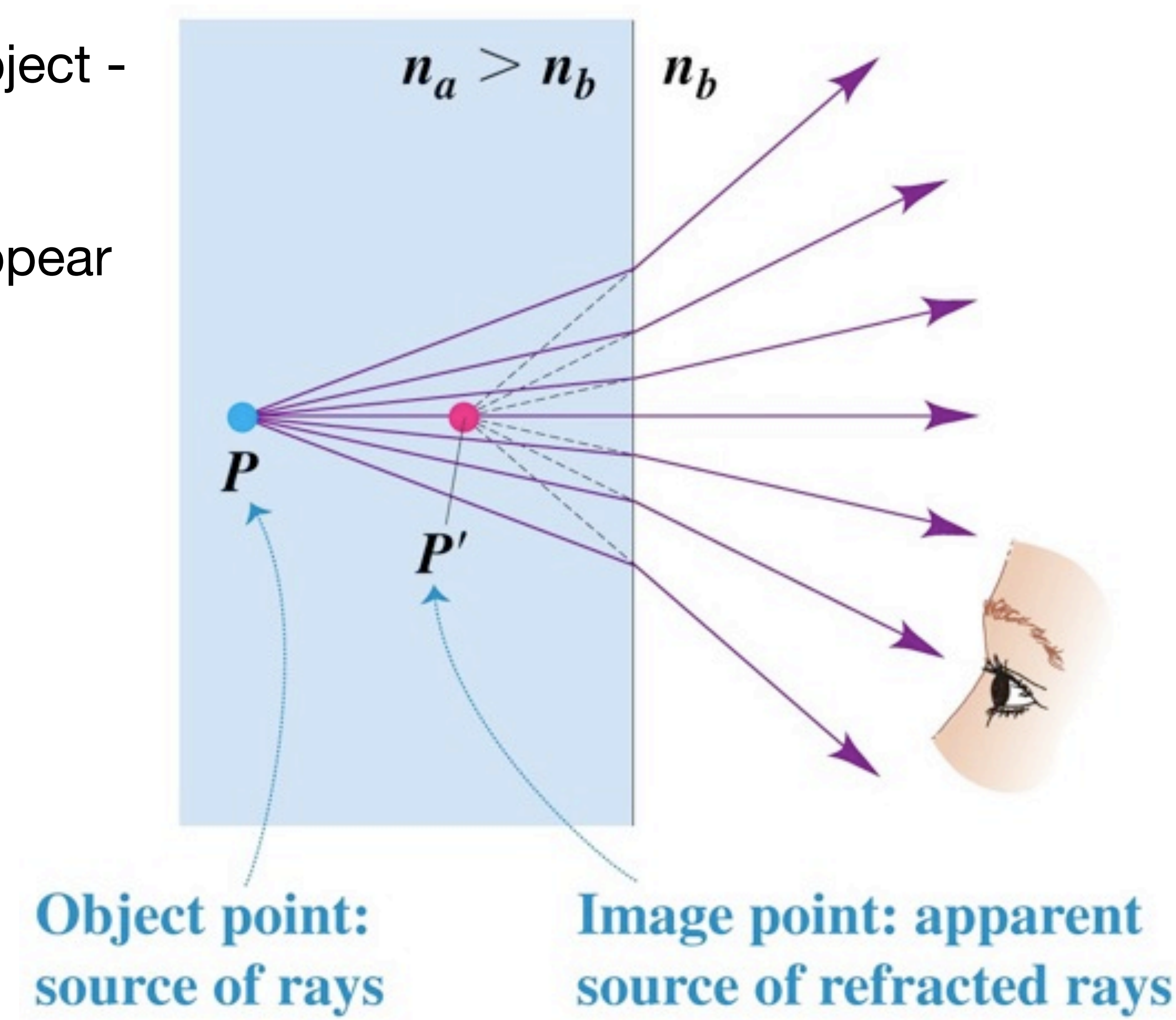
geometric optics of a plane refracting surface

➜ consider an object which emits spherical wavefronts, or in other words, rays in every direction

➜ place the object in medium ***a***

➜ we can draw rays diverging from the object - they are refracted at the interface

➜ tracking the reflected rays back they appear to be diverging from a different point in medium ***a***
we call this the **image**

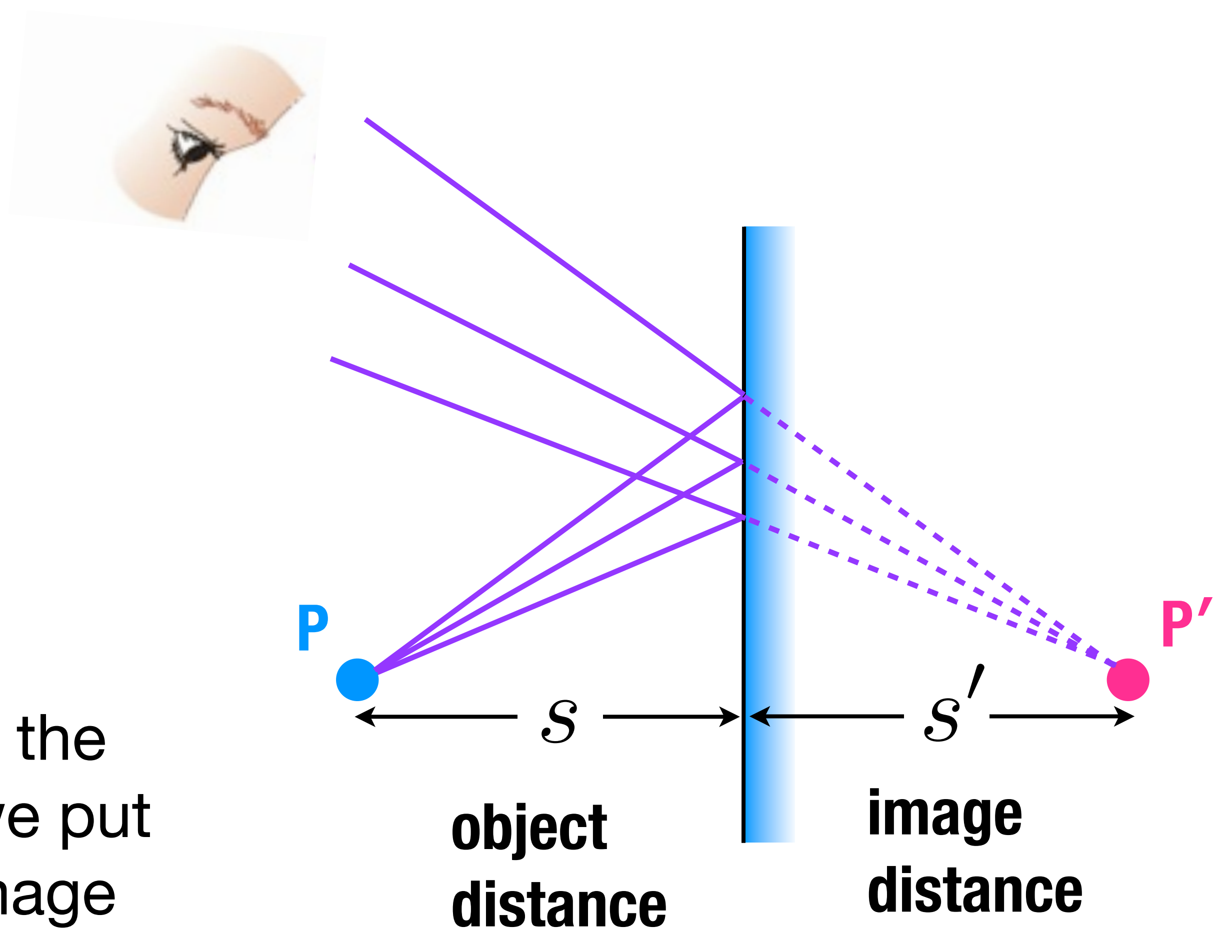


quantitative geometric optics

➜ we can do better than just these qualitative statements and diagrams

➜ we can derive formulae that tell us exactly where the image will be and how magnified it is relative to the object

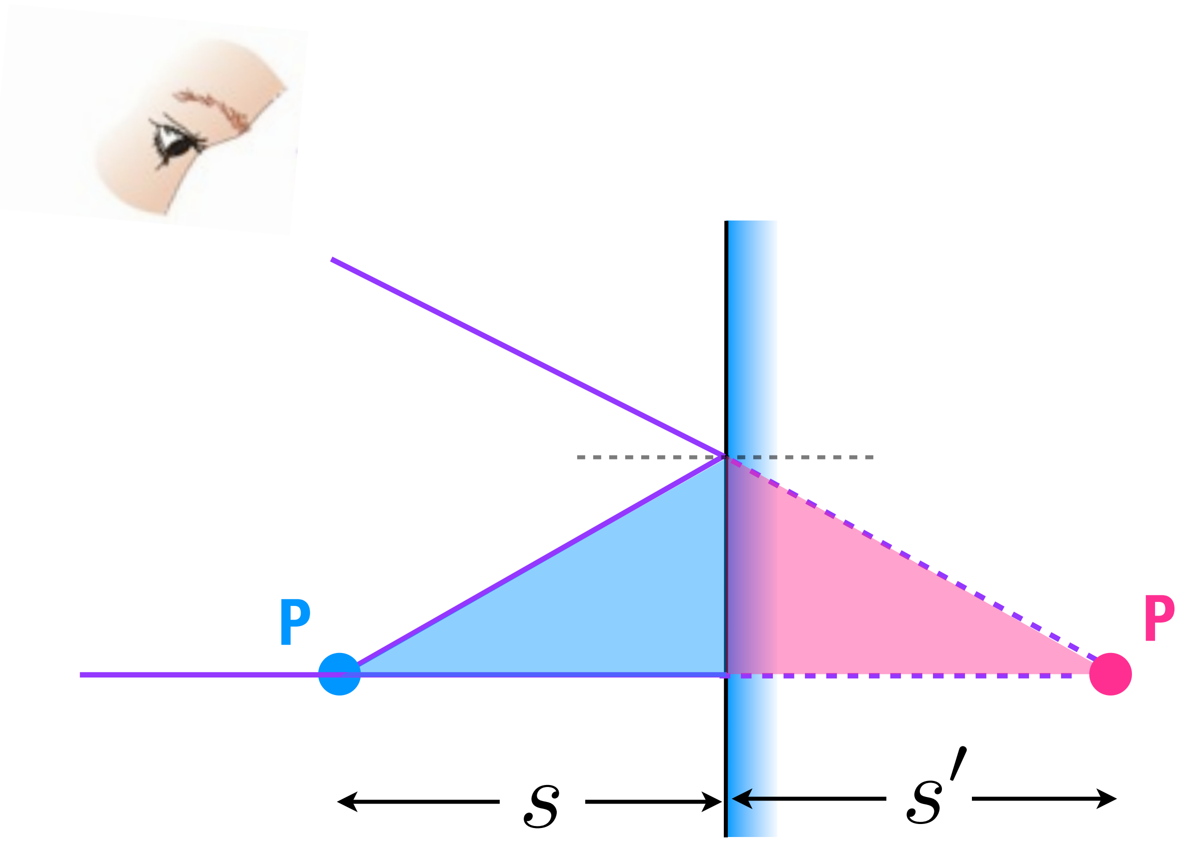
➜ first we should define some terms
object distance
image distance



➜ we call this image a **virtual image** - the rays appear to come from **P′** but if we put a screen at **P′** we wouldn't see an image

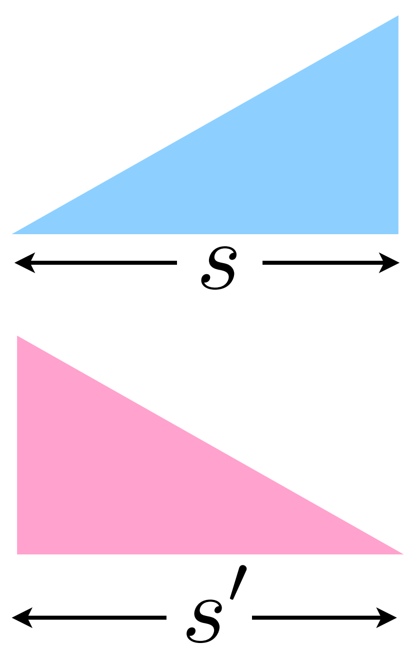
quantitative geometric optics

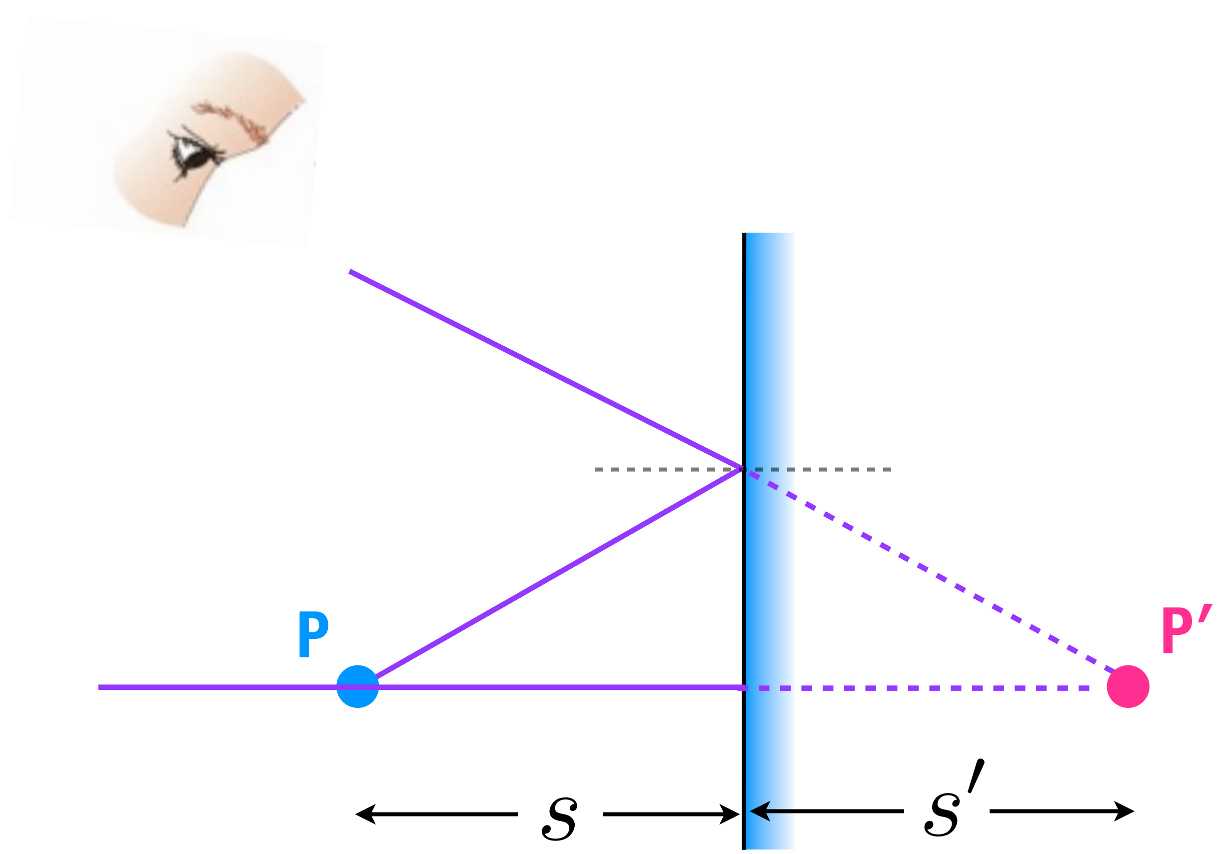
➜ using the fact that angle of incidence = angle of reflection
and doing some simple geometry we can find a relationship between *s* and *s′*



quantitative geometric optics

➜ using the fact that angle of incidence = angle of reflection
and doing some simple geometry we can find a relationship between *s* and *s′*





$$|s| = |s'|$$

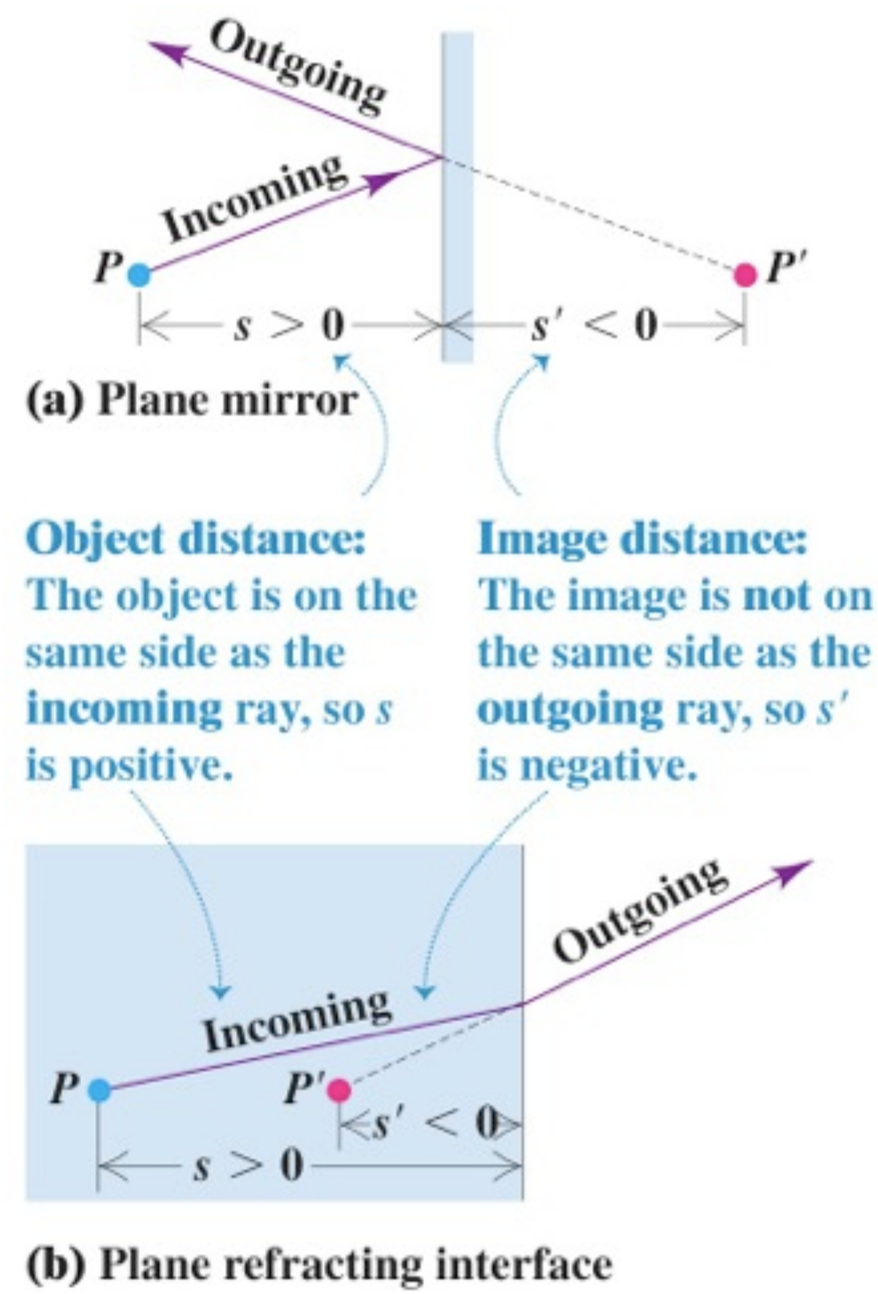
sign rules in optics

➜ looks like another annoying thing to remember - but they will turn out to be very useful when we deal with more complicated optical systems

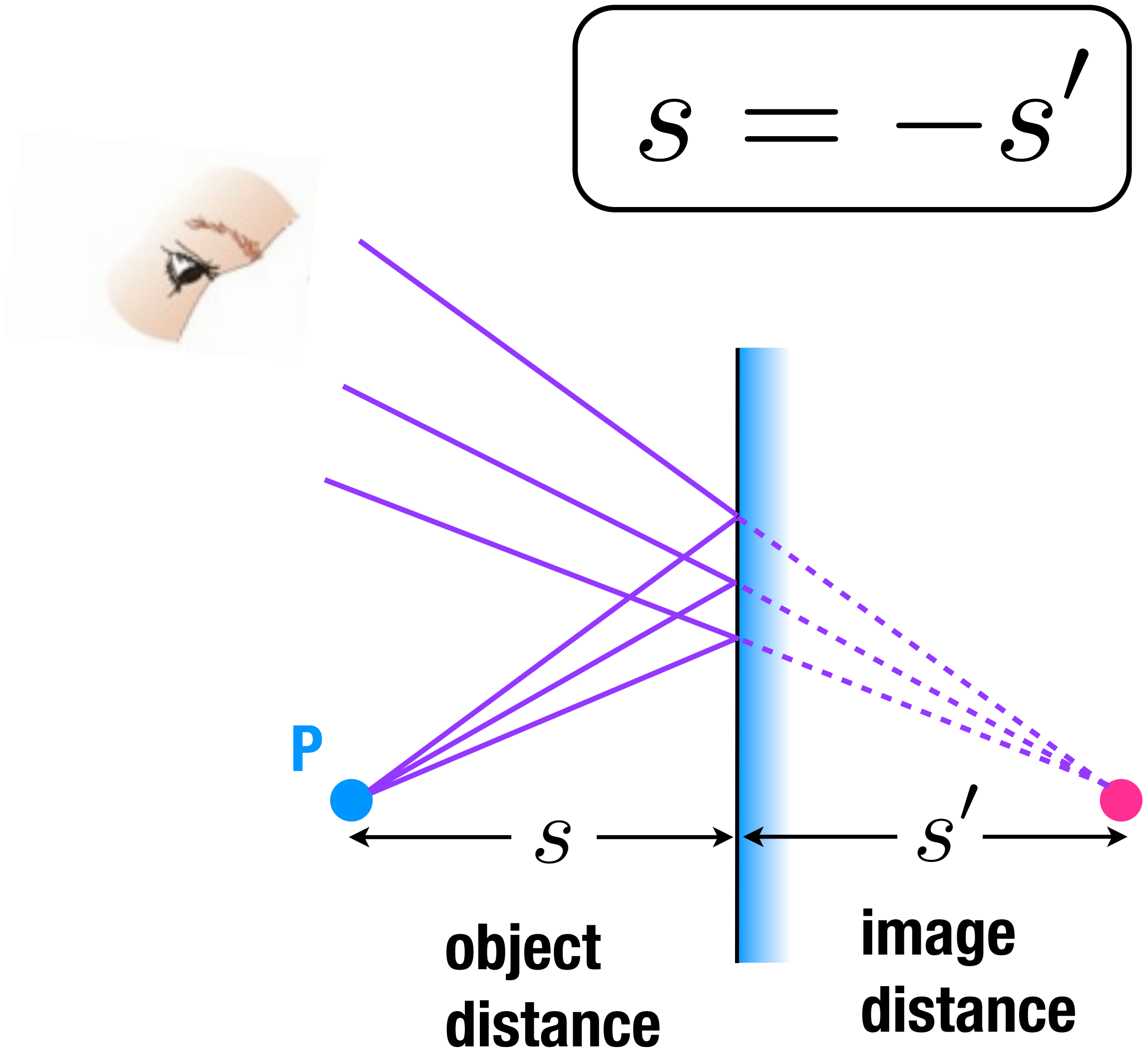
➜ **object distance**: positive when the object is on the same side of the reflecting or refracting surface as the incoming rays (otherwise negative)

➜ **image distance**: positive when the image is on the same side of the reflecting or refracting surface as the outgoing rays (otherwise negative)

negative image distance means a virtual image



plane mirror

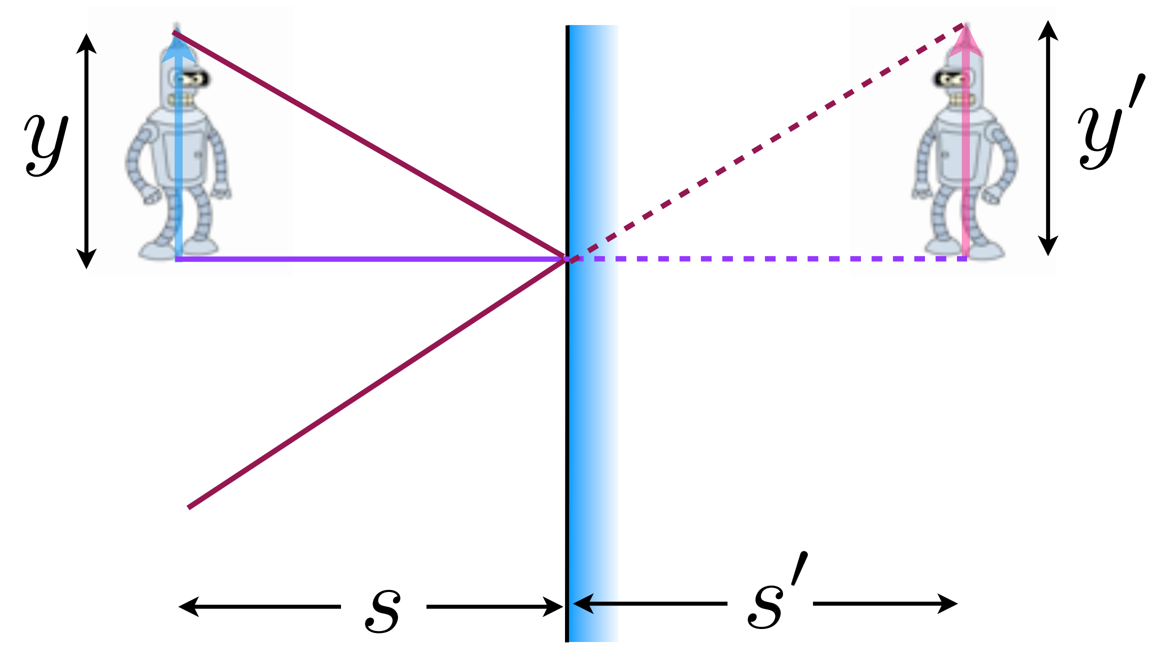


quantitative geometric optics - size of objects

➜ what about objects that have a size?
➜ potentially, reflection or refraction could magnify or diminish the image relative to the object

➜ the same geometric construction shows that

$$y' = y$$



➜ **magnification** is defined by

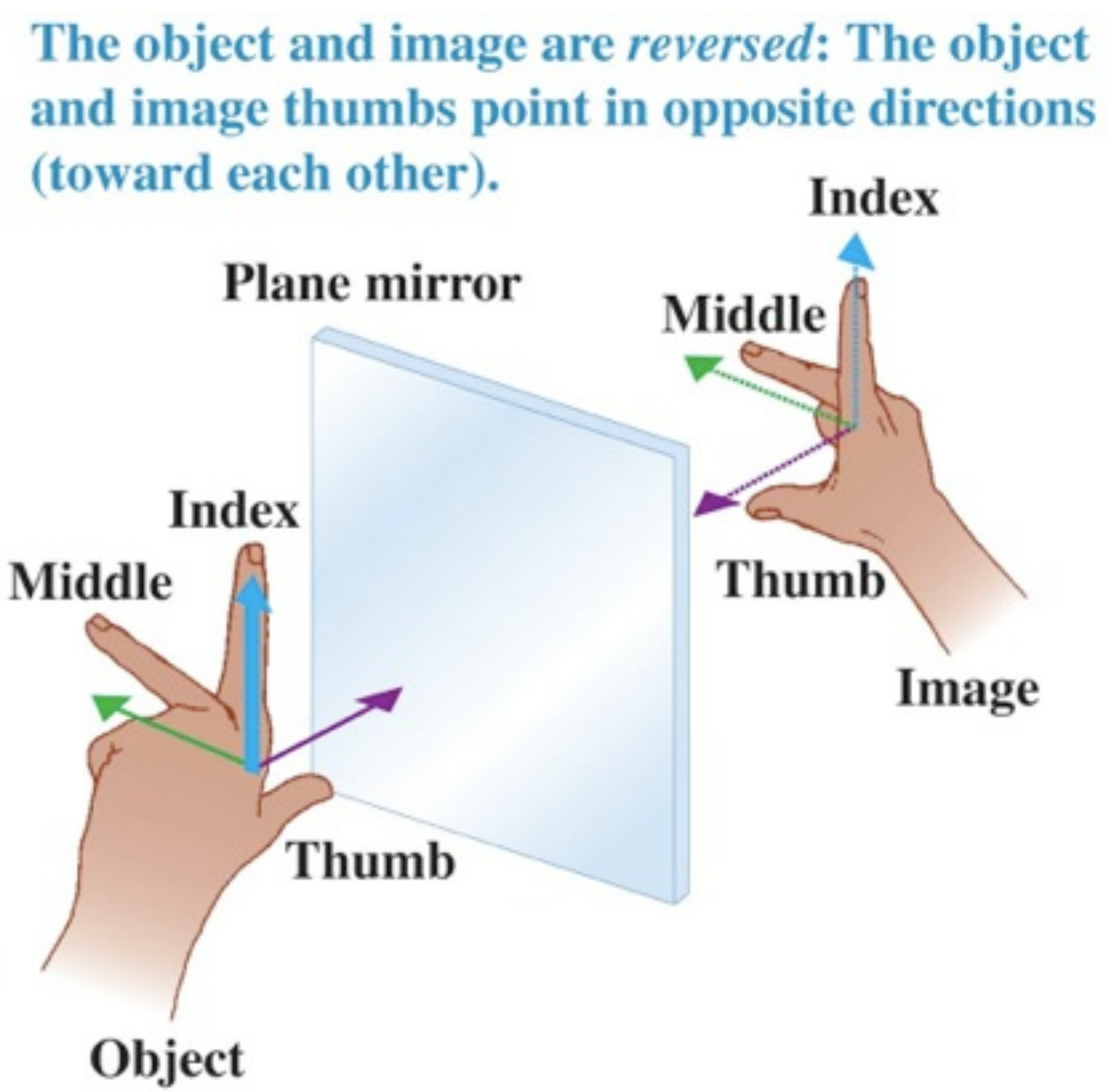
$$m = \frac{y'}{y}$$

a plane mirror has m=1

reversal of images

➜ although the image in a plane mirror is the same size as the object it is not identical to it

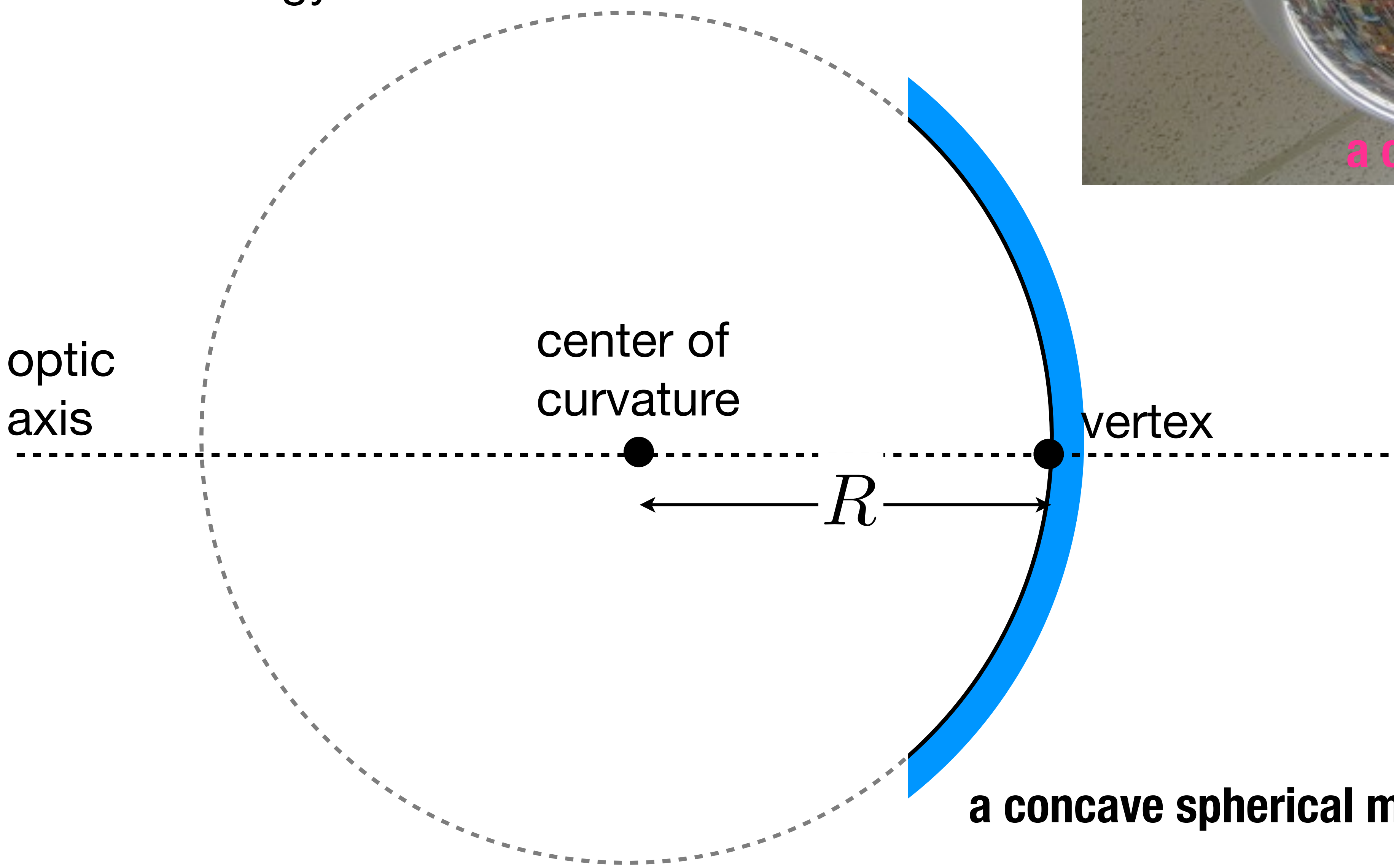
➜ it is **reversed**



reflection at a spherical surface

➜ mirrors can be made which are not flat but rather shaped like part of a sphere

➜ some terminology

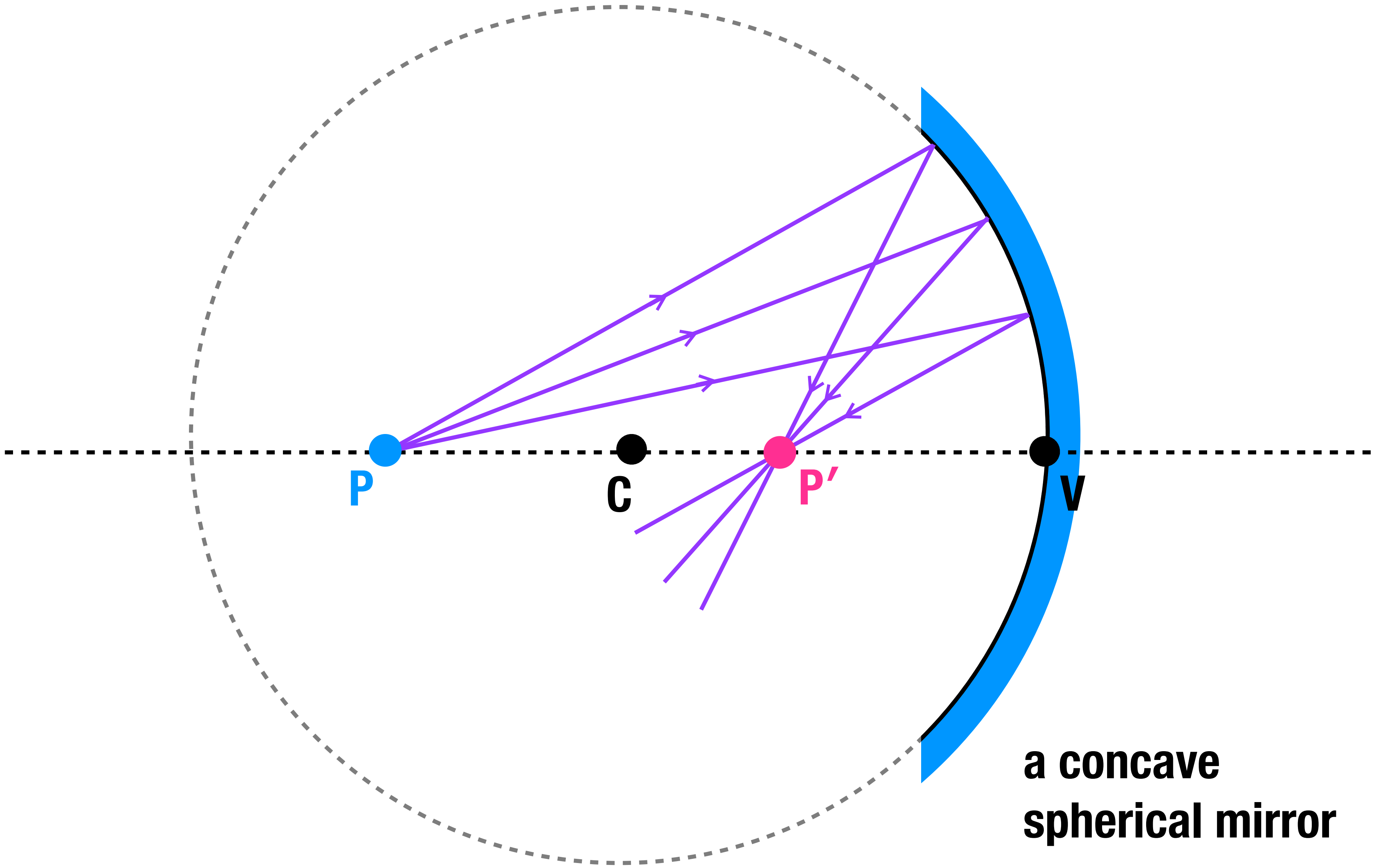


R = "radius of curvature"

reflection at a spherical surface

➜ image formation in a spherical mirror

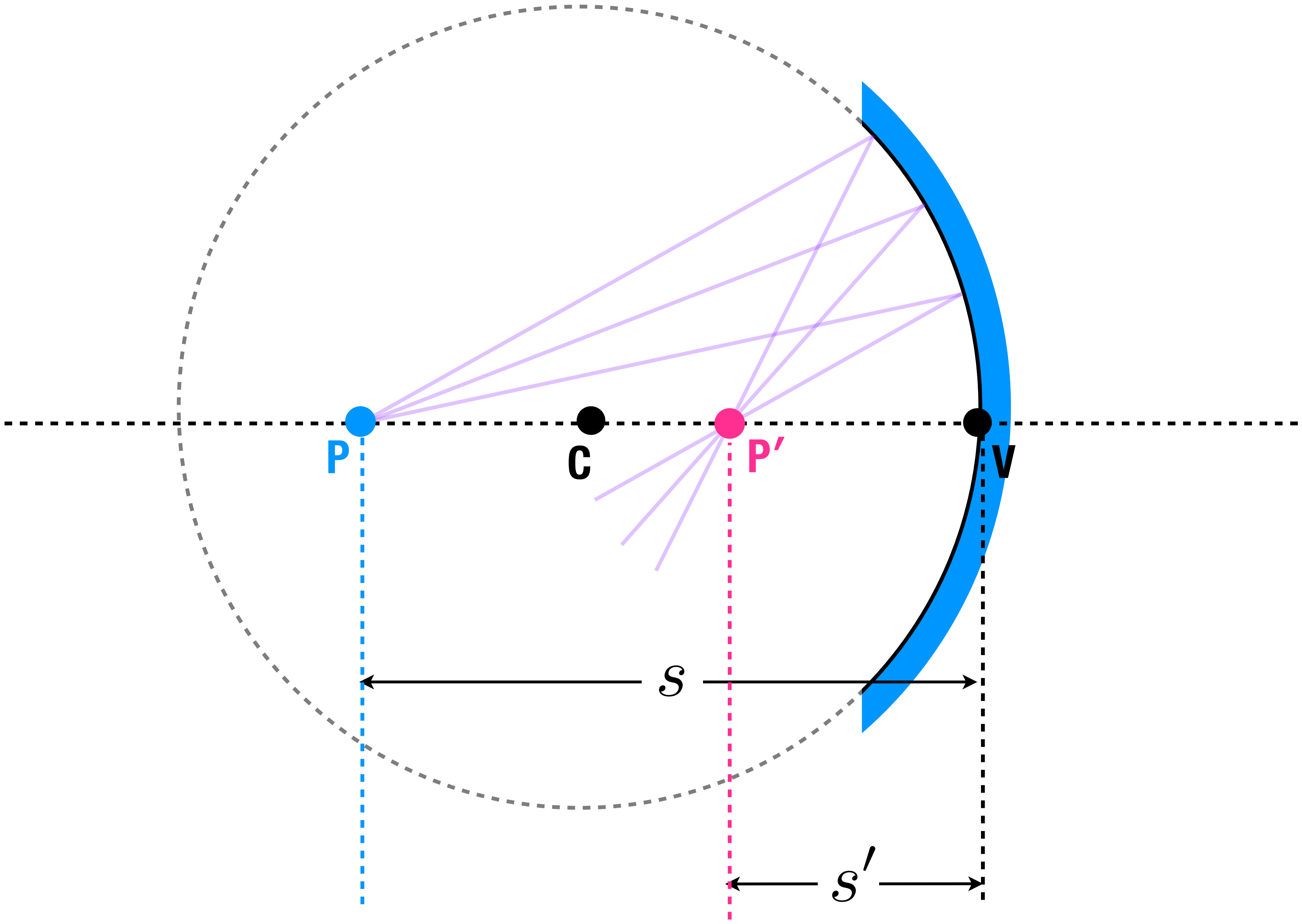
➜ angle of incidence = angle of reflection still holds



reflection at a spherical surface

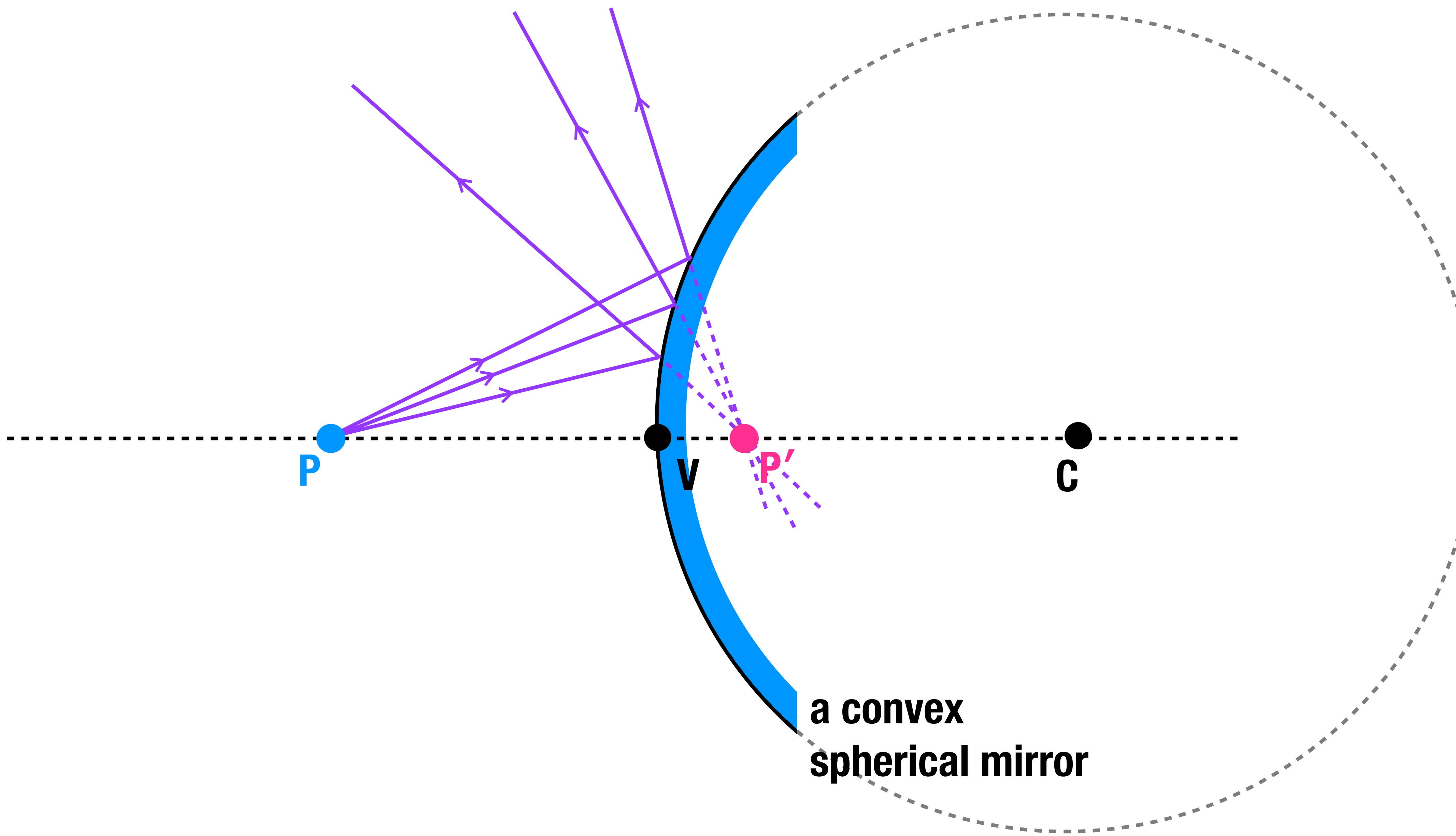
➜ image formation in a spherical mirror

➜ defining the object/image distances

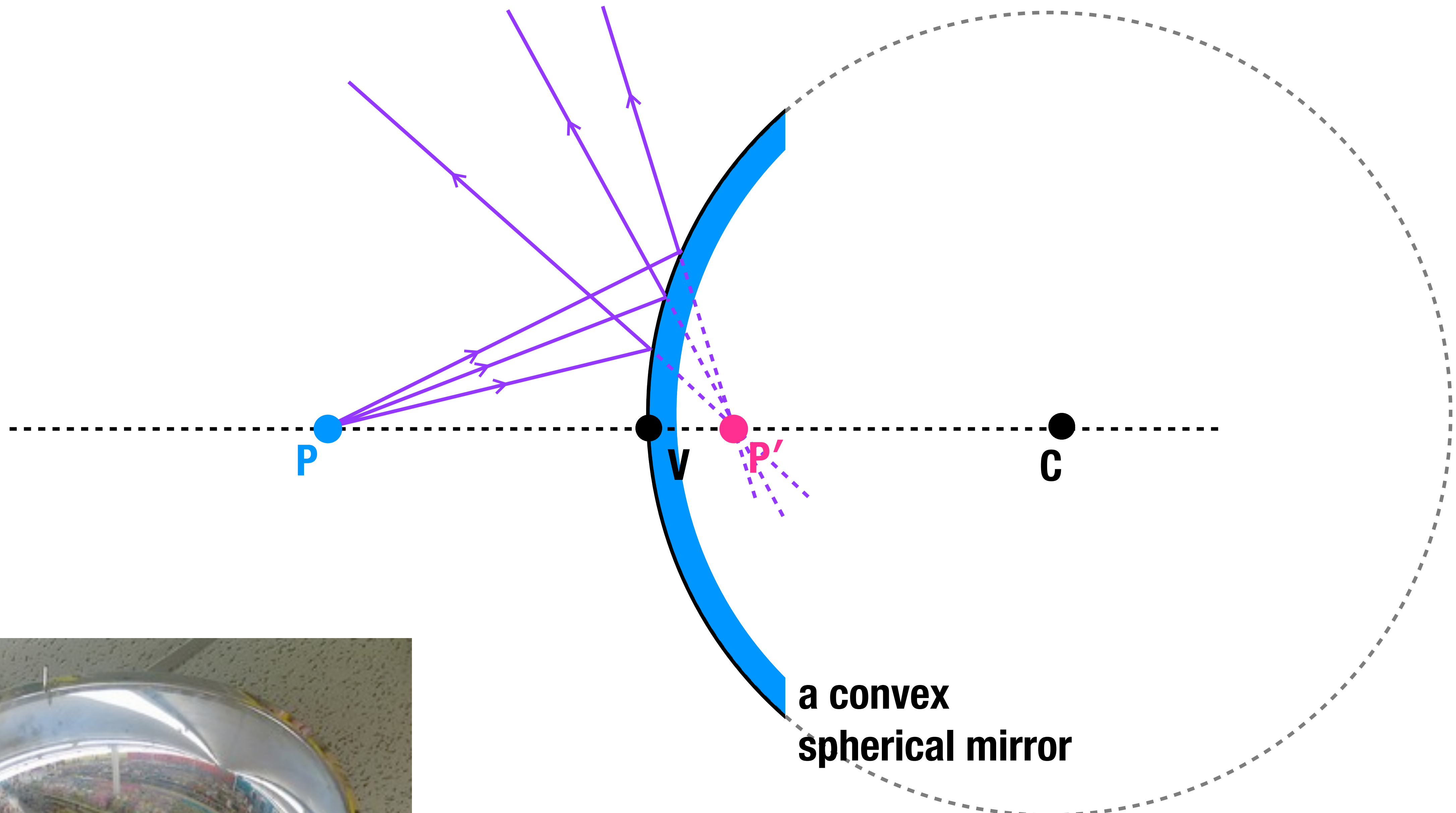


reflection at a spherical surface

➜ image formation in a **convex** spherical mirror



reflection at a spherical surface

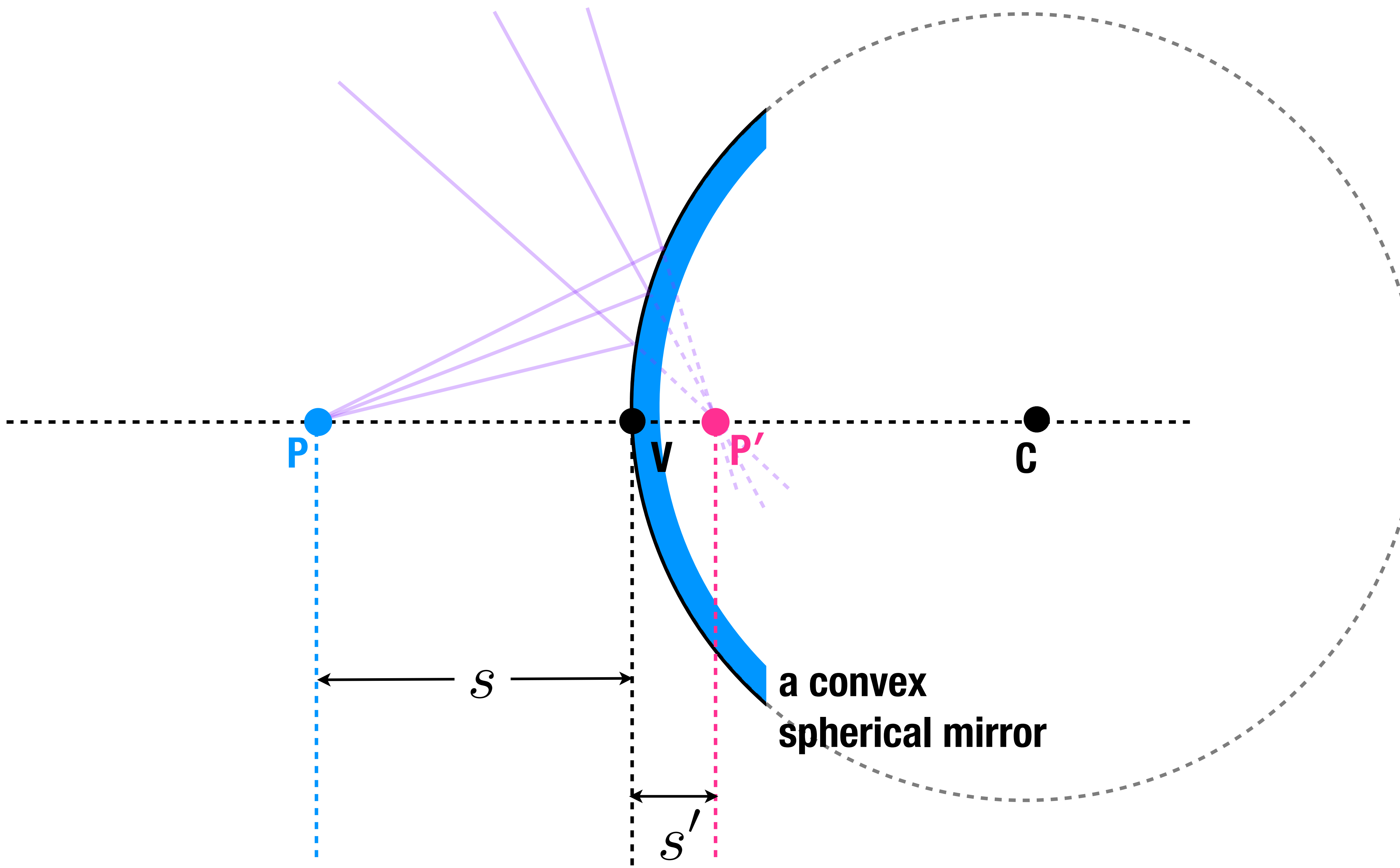


see an image on a screen placed at P′ ?

reflection at a spherical surface

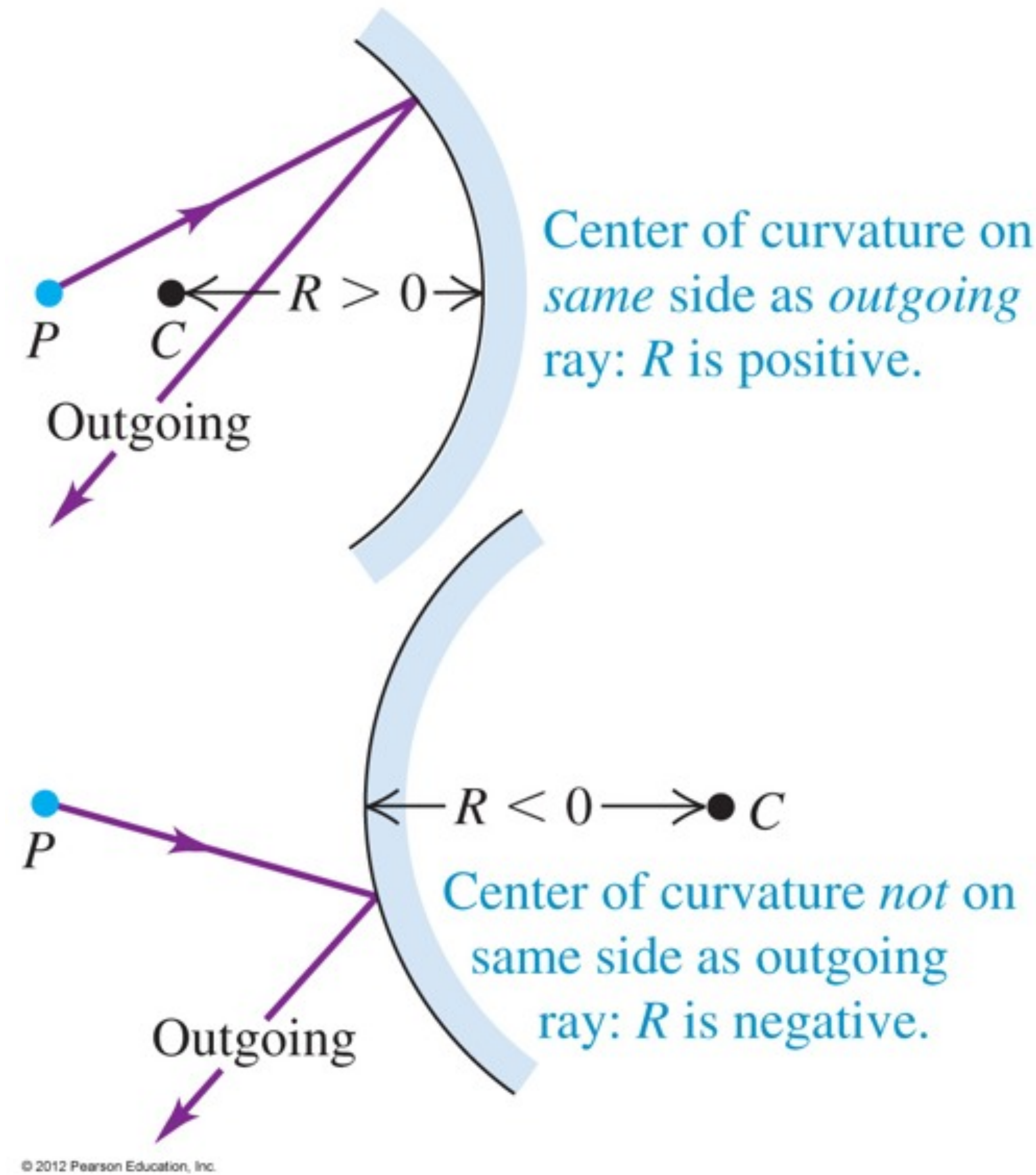
➜ image formation in a spherical mirror

➜ defining the object/image distances



reflection at a spherical surface

➜ a sign convention for the radius of curvature

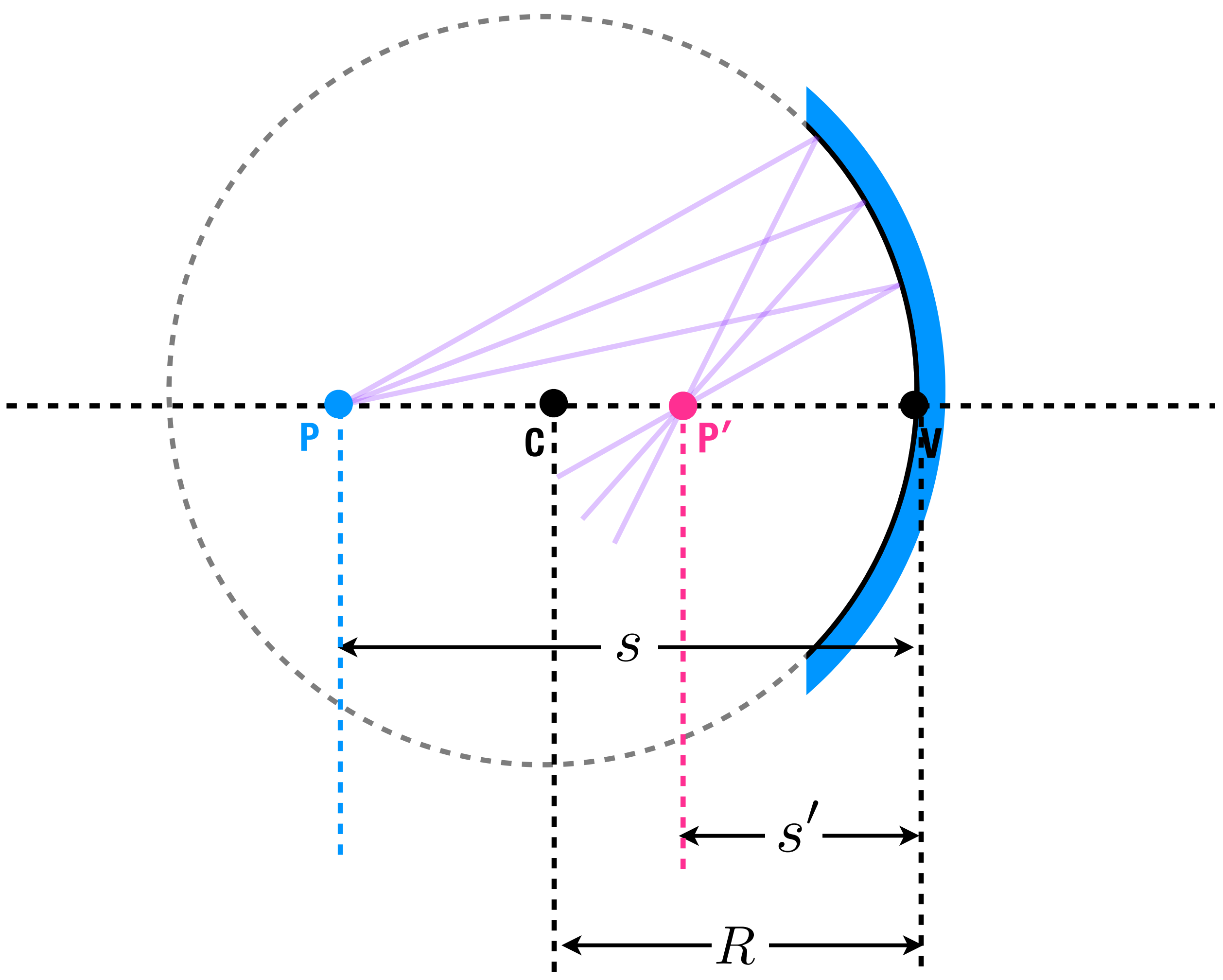


reflection at a spherical surface

➜ for rays close to the optic axis

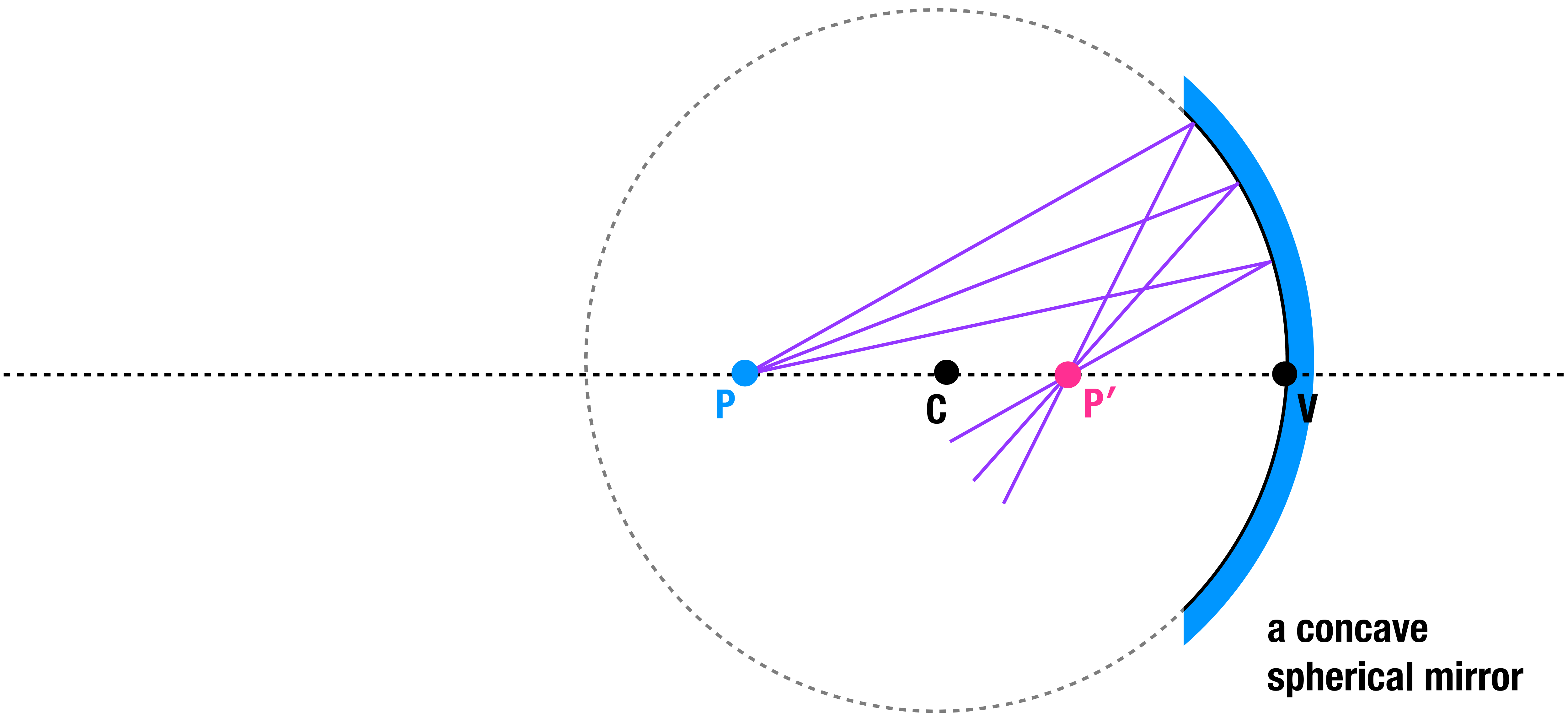
$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

a geometric derivation is in the textbook



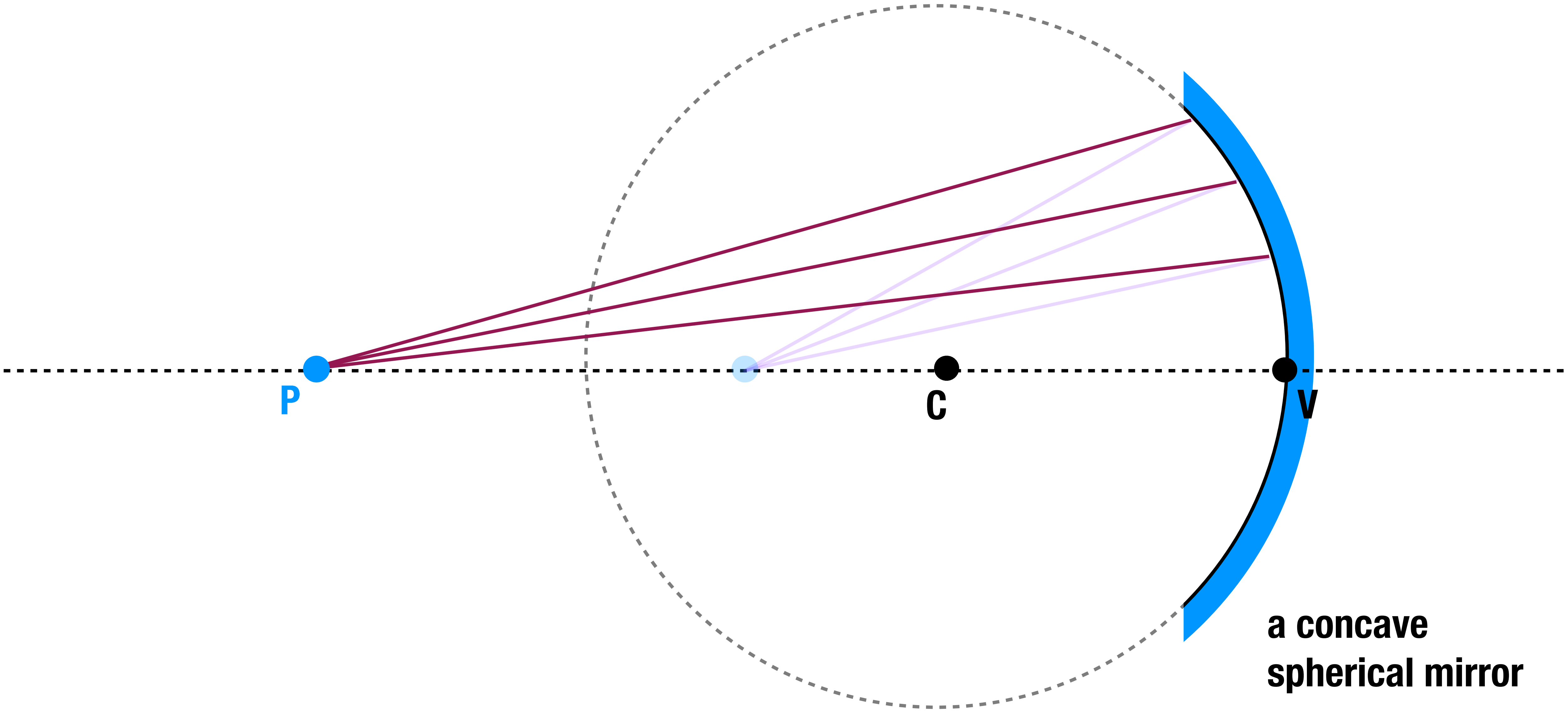
the focal point

➜ suppose the object is very far away, then the rays from it are almost parallel



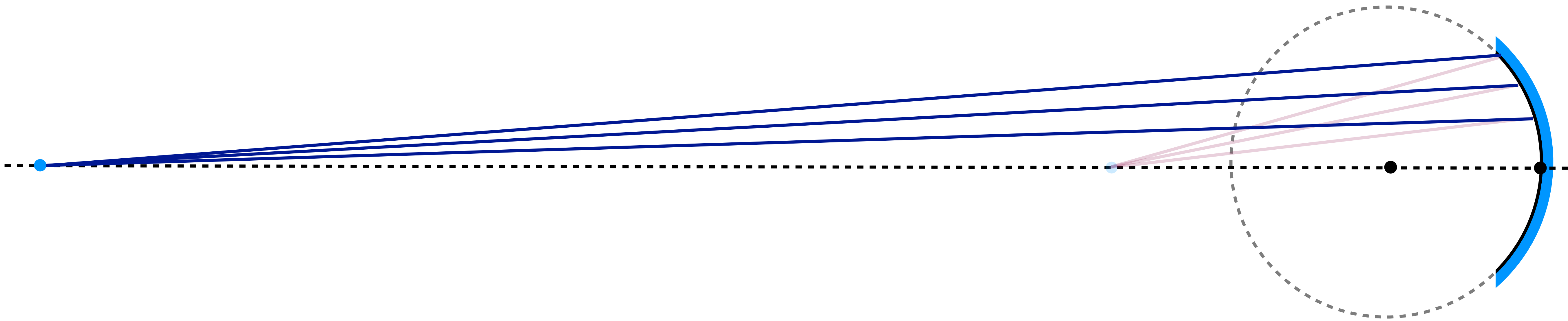
the focal point

➜ suppose the object is very far away, then the rays from it are almost parallel



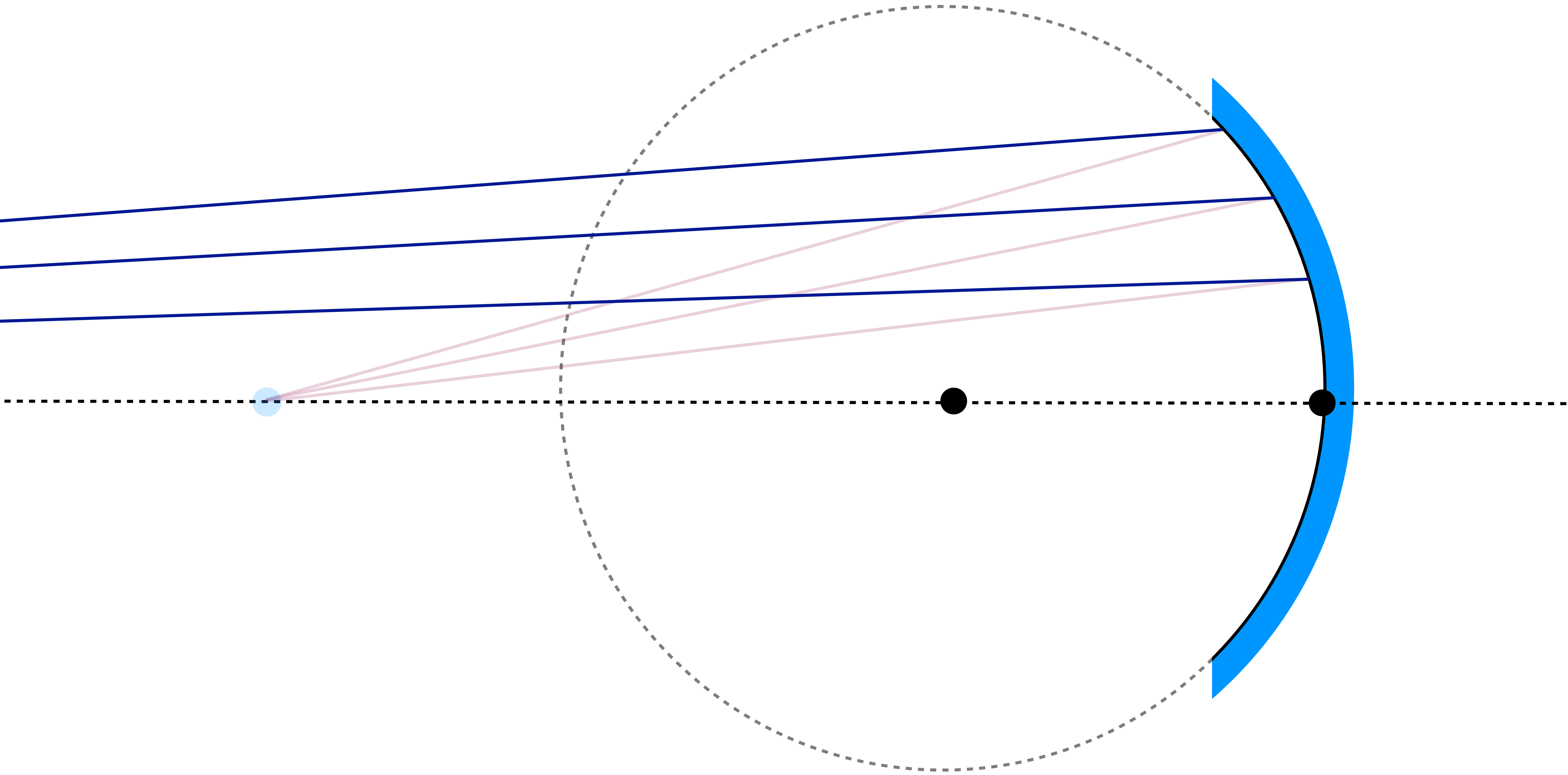
the focal point

➜ suppose the object is very far away, then the rays from it are almost parallel



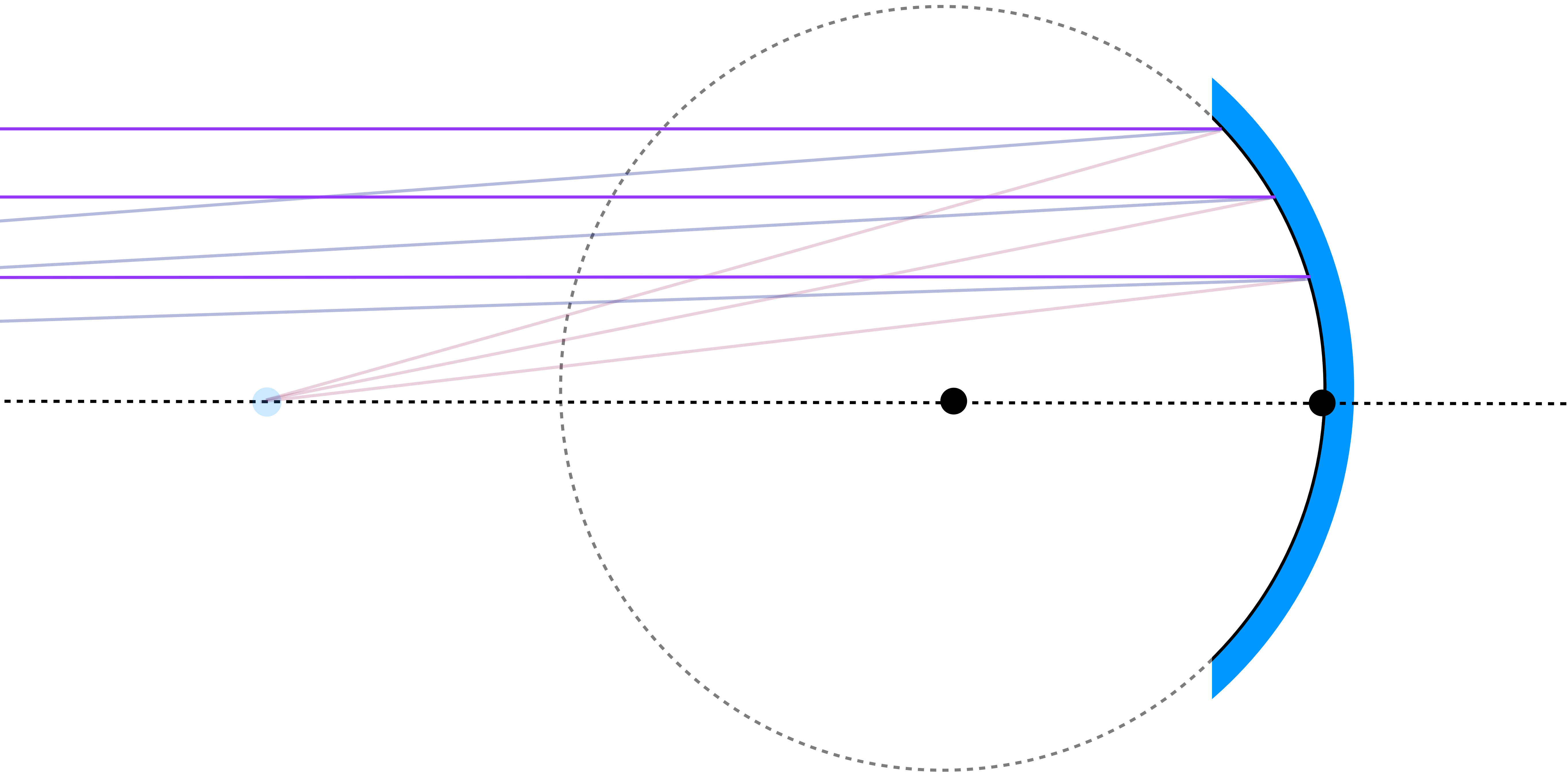
the focal point

➜ suppose the object is very far away, then the rays from it are almost parallel



the focal point

➜ suppose the object is very far away, then the rays from it are almost parallel

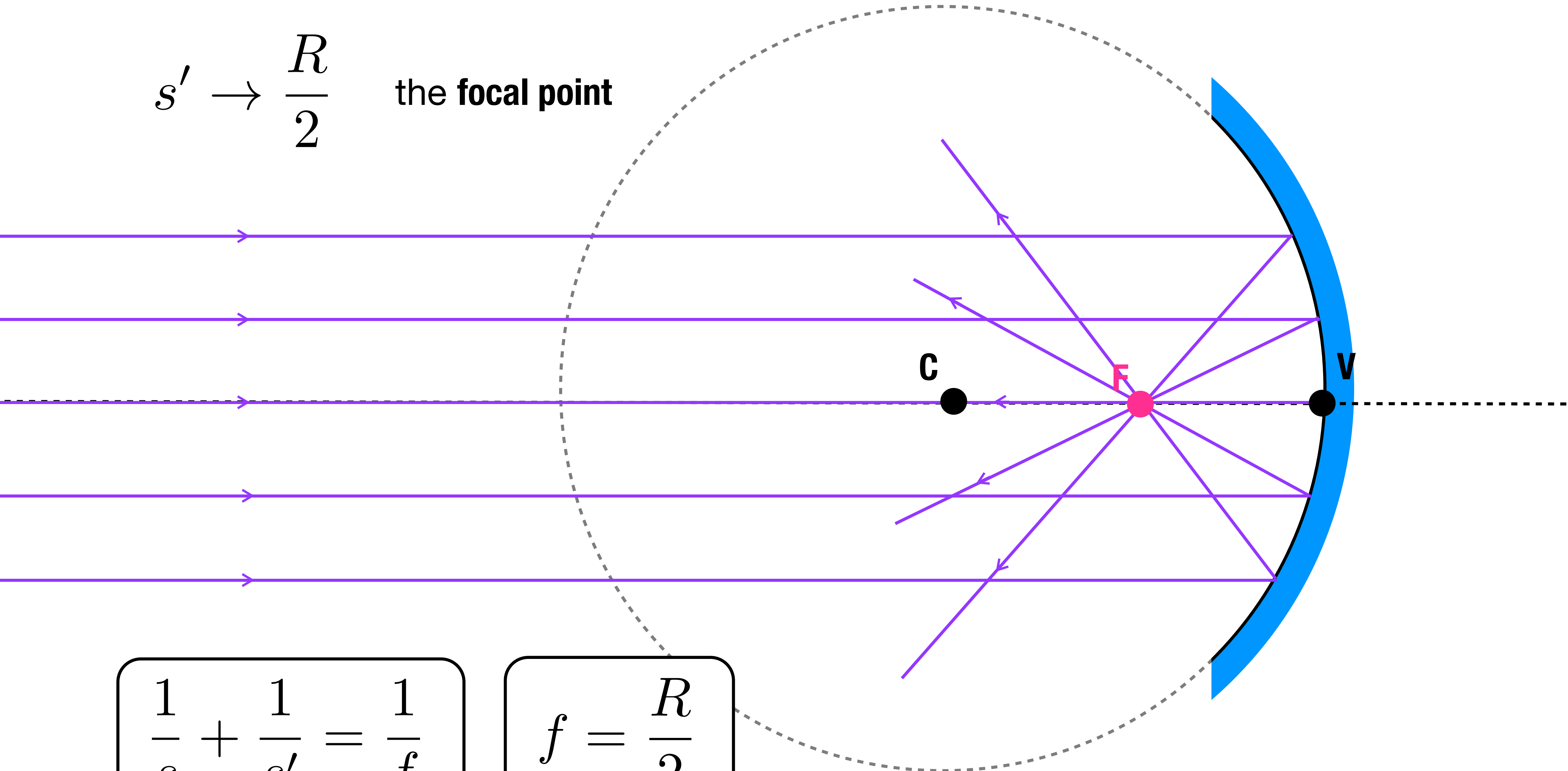


the focal point

➜ suppose the object is very far away, then the rays from it are almost parallel

$$s \to \infty$$

$$s' \to \frac{R}{2}$$ the **focal point**

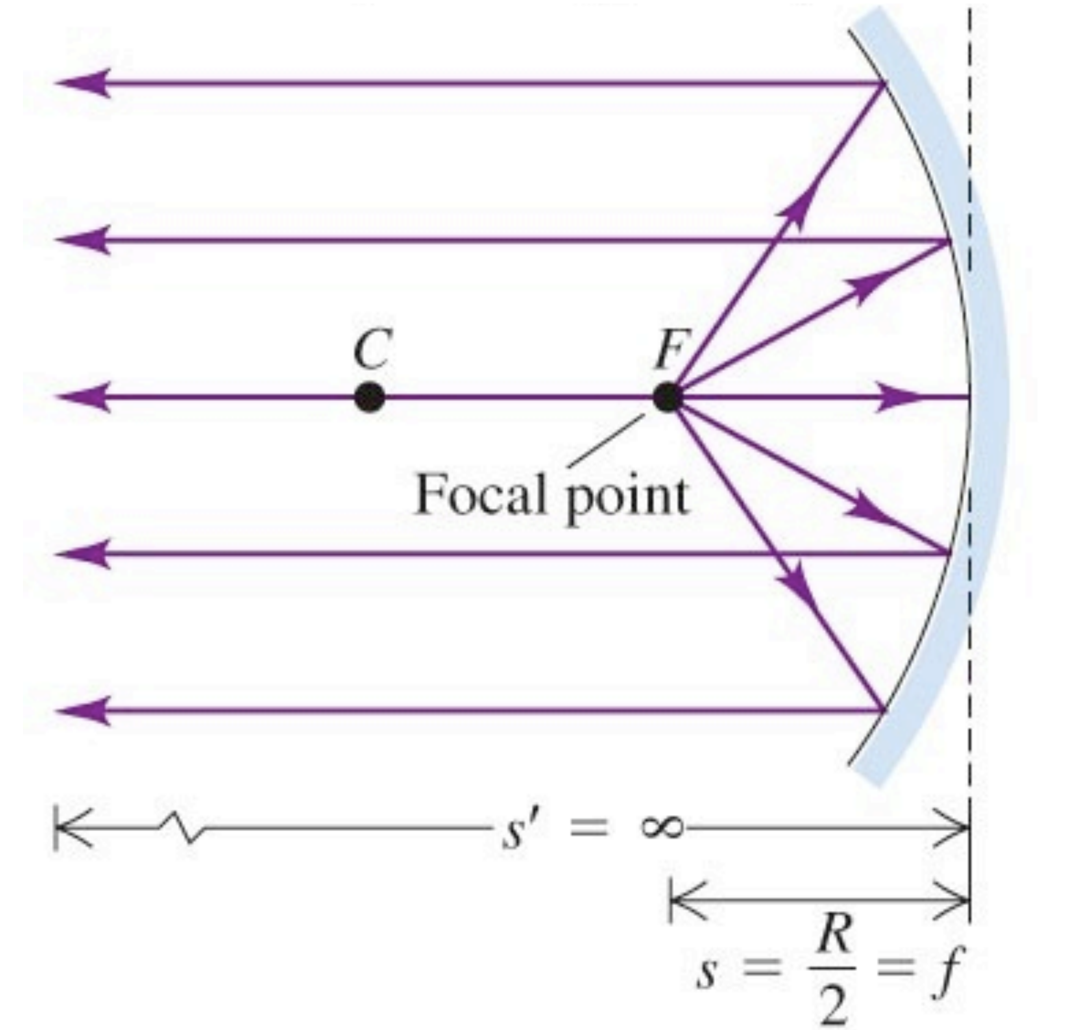


$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

$$\boxed{f = \frac{R}{2}}$$

the focal point

➜ imagine running this in reverse - put the object at the focal point



(b) Rays diverging from the focal point re
to form parallel outgoing rays.

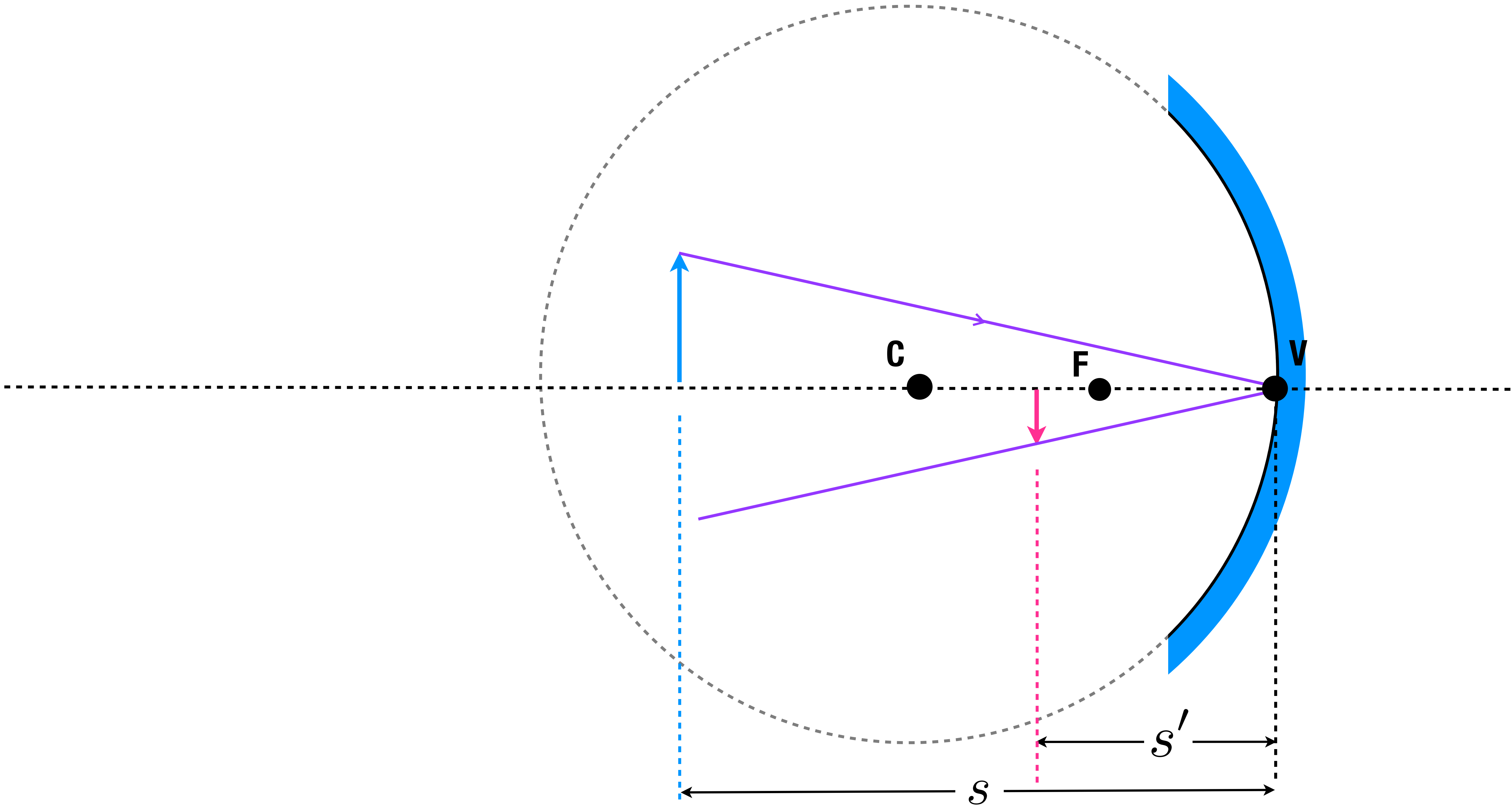
properties of the focal point:

➜ any incoming ray parallel to the optic axis is reflected through the focal point

➜ any ray passing through the focal point is reflected parallel to the optic axis

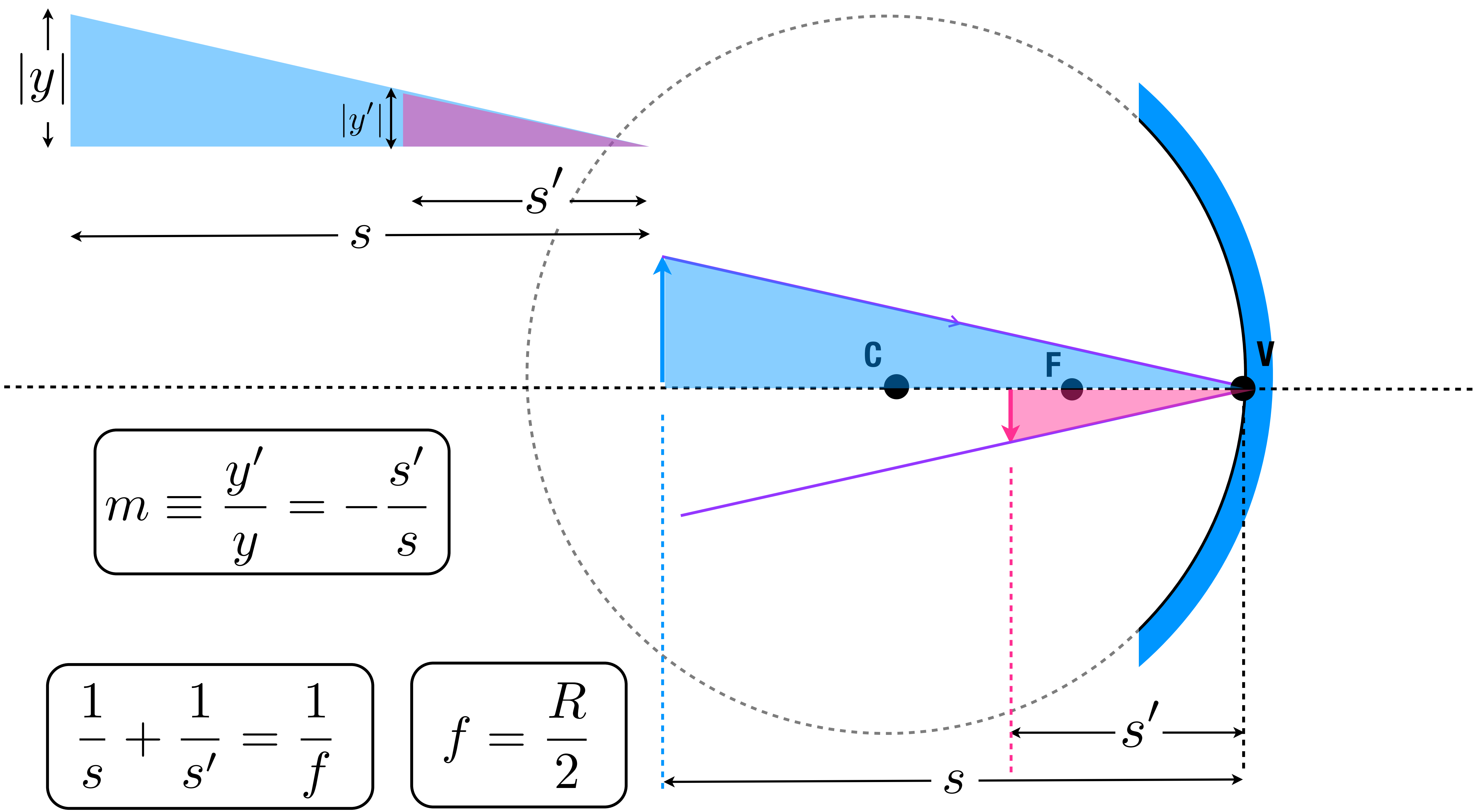
magnification in a spherical mirror

➜ do objects get magnified viewed in a spherical mirror ?



magnification in a spherical mirror

➜ do objects get magnified viewed in a spherical mirror ?



$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$f = \frac{R}{2}$$

graphical method - principal ray tracing

➜ we'll use the formulas soon, but first let's explore a graphical technique

➜ we'll single out some special rays whose path is easy to work out

➜ we call them **principal rays**

➜ a ray parallel to the optic axis is reflected through the focal point of the mirror

➜ a ray through the focal point is reflected parallel to the optic axis

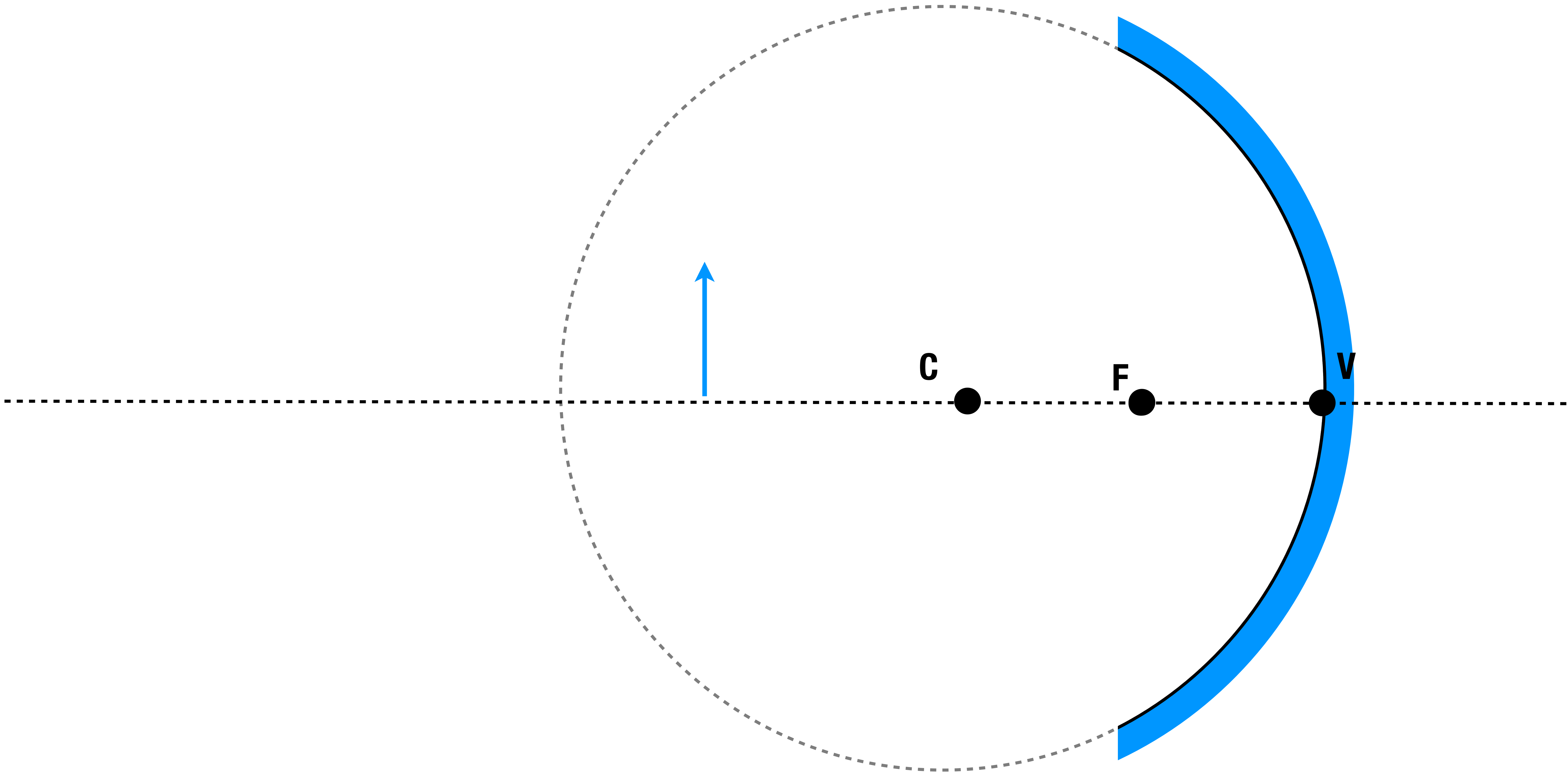
➜ a ray along the radius passing through the center of curvature is reflected back along the same line

➜ a ray reflecting at the vertex is reflected forming an equal angle to its original direction

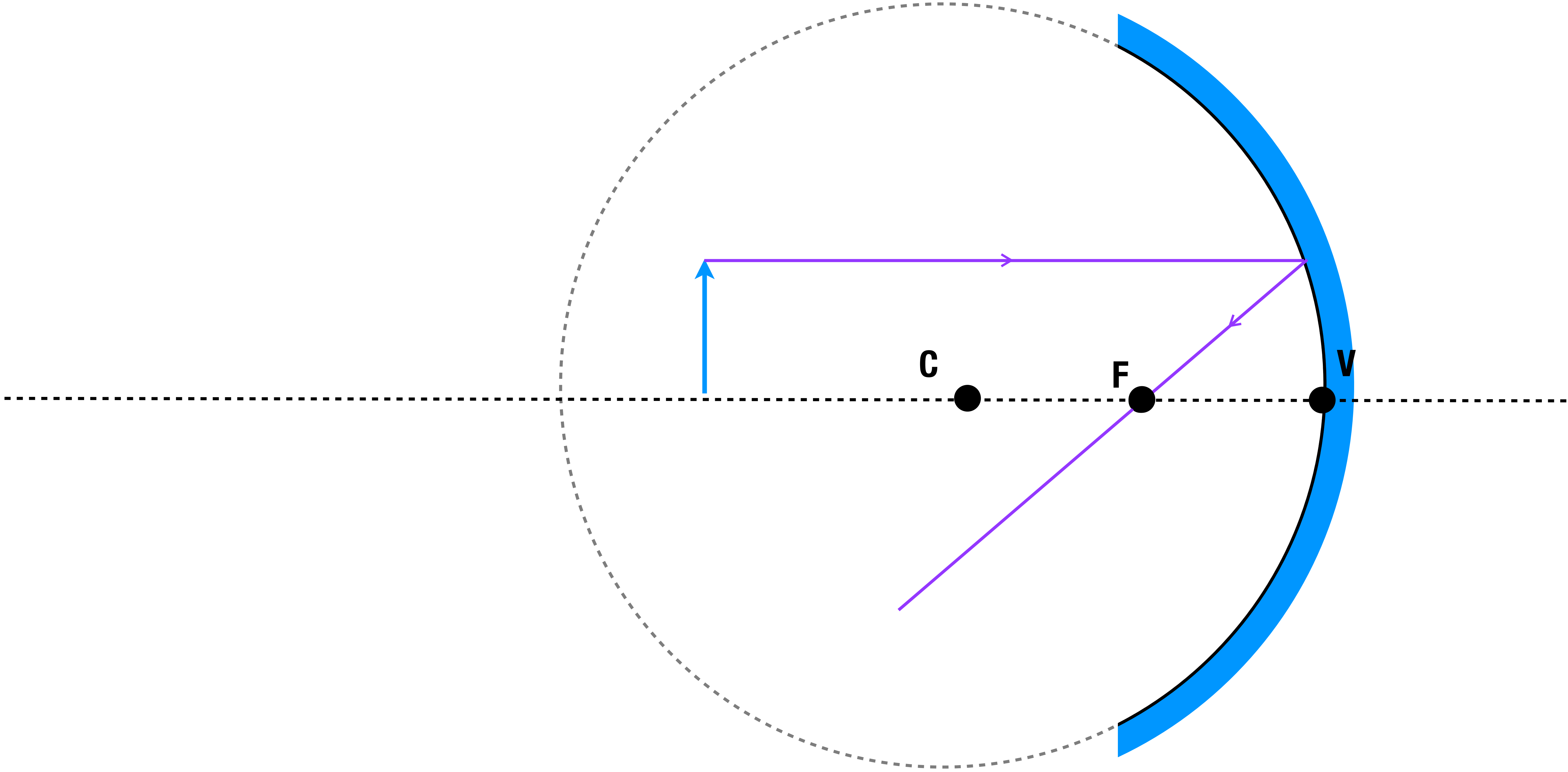
➜ usually drawing any two of these rays describes the image position and size - drawing more checks our answer

➜ no tricky sign conventions here

graphical method - principal ray tracing

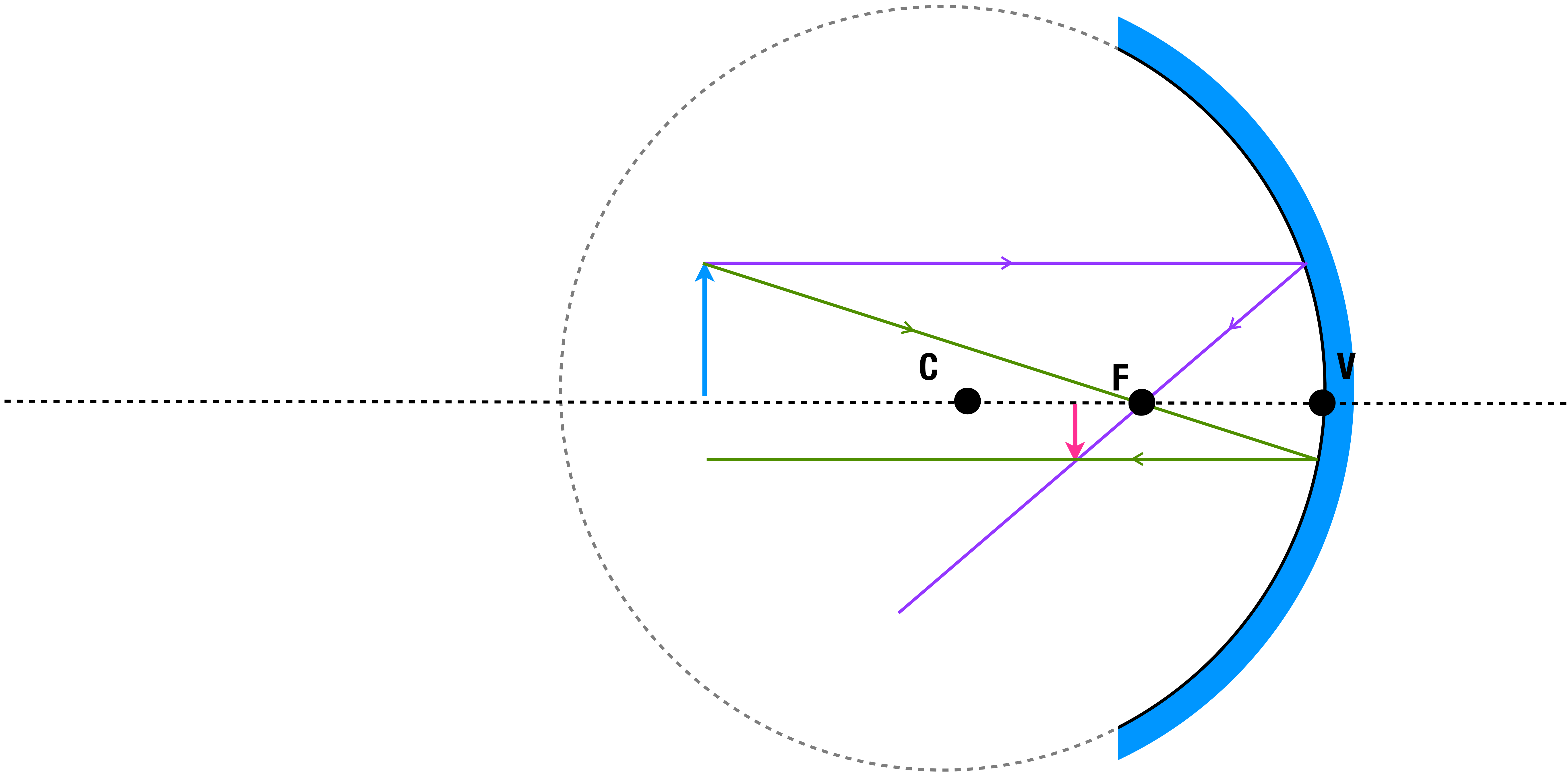


graphical method - principal ray tracing



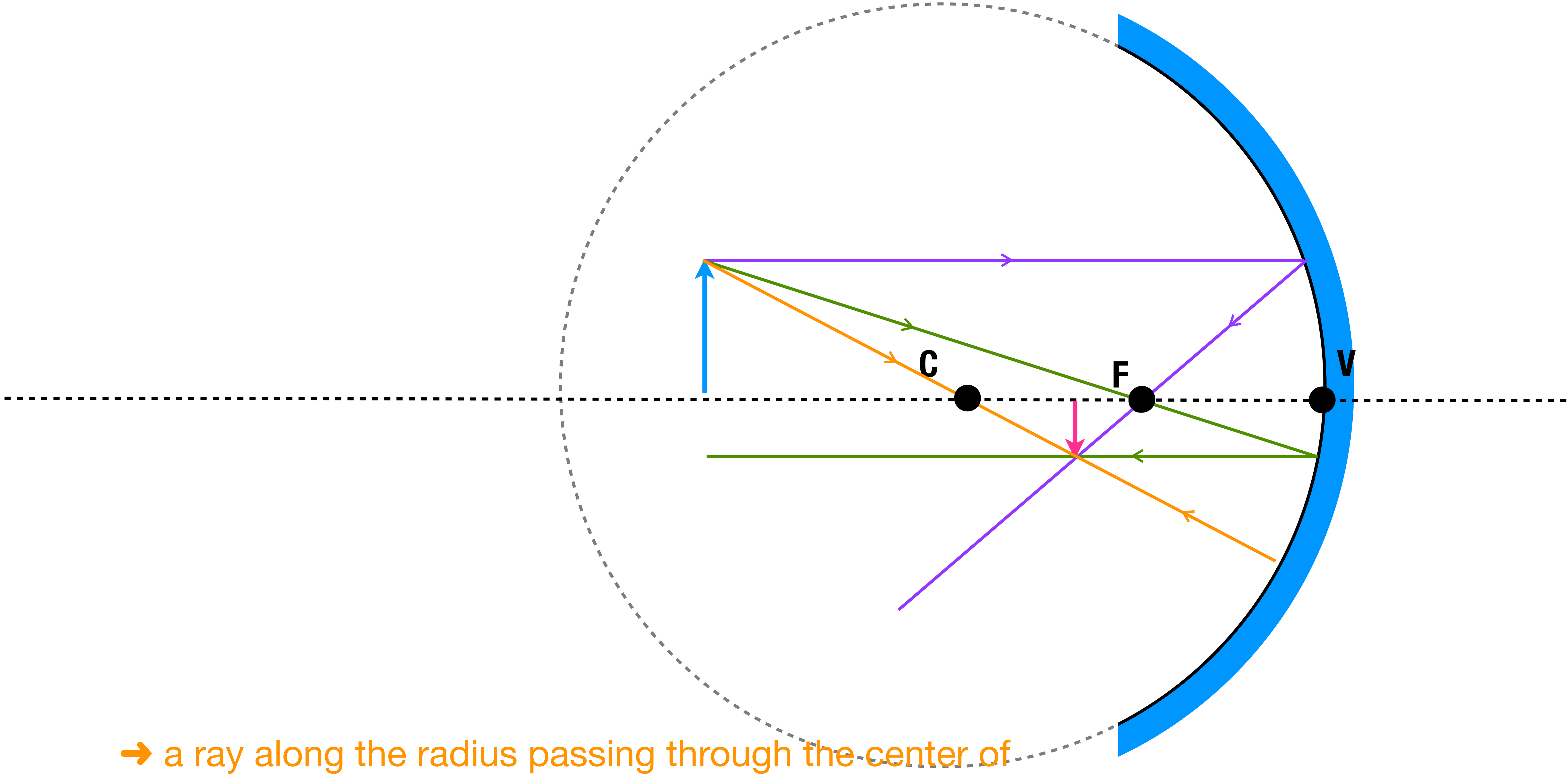
➜ a ray parallel to the optic axis is reflected through the focal point of the mirror

graphical method - principal ray tracing



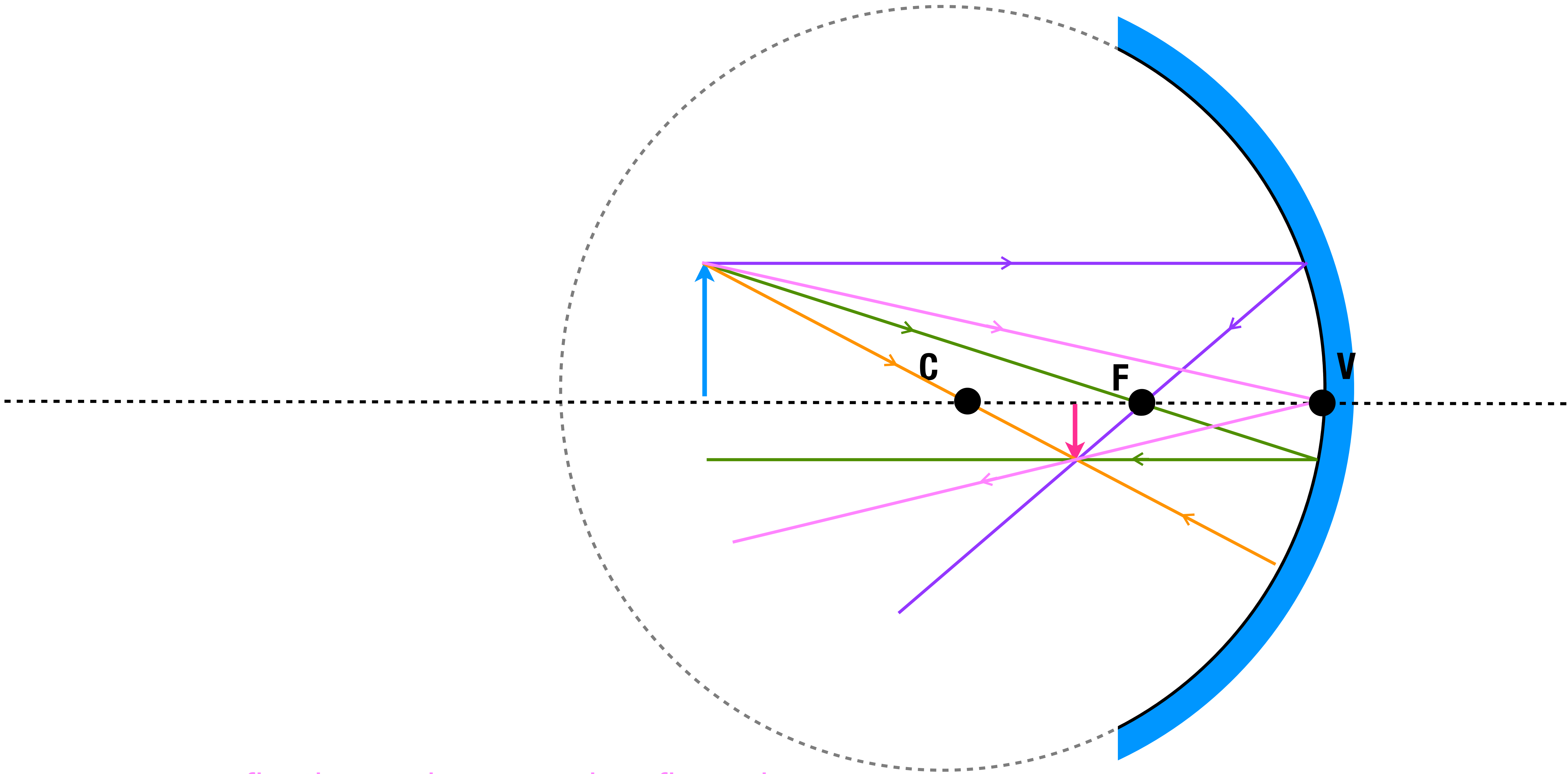
➜ a ray through the focal point is reflected parallel to the optic axis

graphical method - principal ray tracing



➜ a ray along the radius passing through the center of curvature is reflected back along the same line

graphical method - principal ray tracing



➜ a ray reflecting at the vertex is reflected forming an equal angle to its original direction

image in a concave mirror

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$ $$f=\frac{R}{2}$$ $$m\equiv\frac{y'}{y}=-\frac{s'}{s}$$

A concave mirror has a radius of curvature of absolute value 20 cm. Find the position, reality, magnification and orientation of the image in the case that the object distance is 30 cm

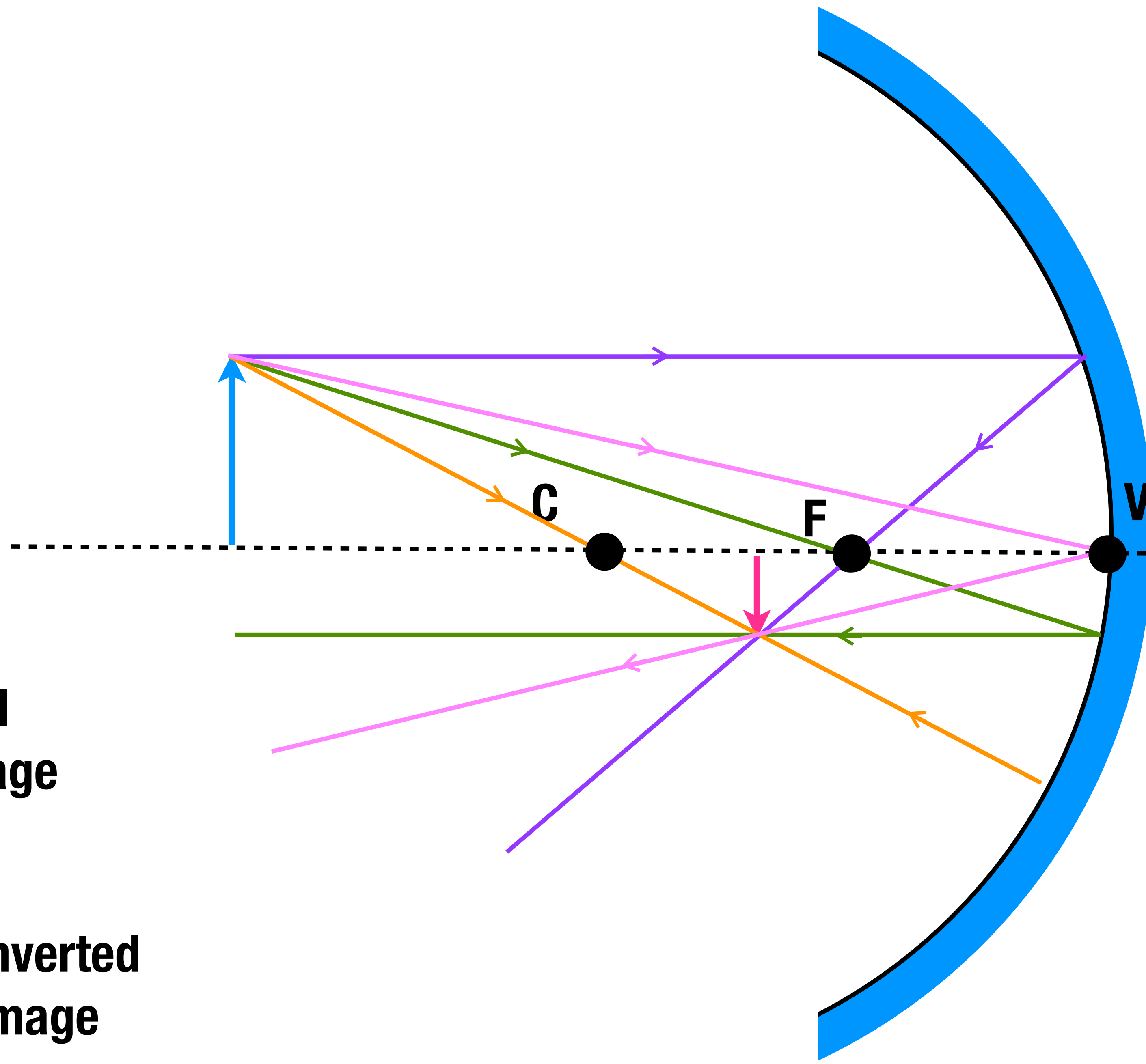
$$R=+20\,\mathrm{cm}$$

$$f=+10\,\mathrm{cm}$$

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$$
$$=\frac{1}{10\,\mathrm{cm}}-\frac{1}{30\,\mathrm{cm}}$$
$$=\frac{3}{30\,\mathrm{cm}}-\frac{1}{30\,\mathrm{cm}}$$
$$=\frac{2}{30\,\mathrm{cm}}$$
$$\frac{1}{s'}=\frac{1}{15\,\mathrm{cm}}$$

$$s'=+15\,\mathrm{cm}$$

real image



$$m=-\frac{s'}{s}=-\frac{15\,\mathrm{cm}}{30\,\mathrm{cm}}$$
$$m=-\tfrac{1}{2}$$

$$m=-\frac{1}{2}$$

inverted image

reduced by a factor of 2

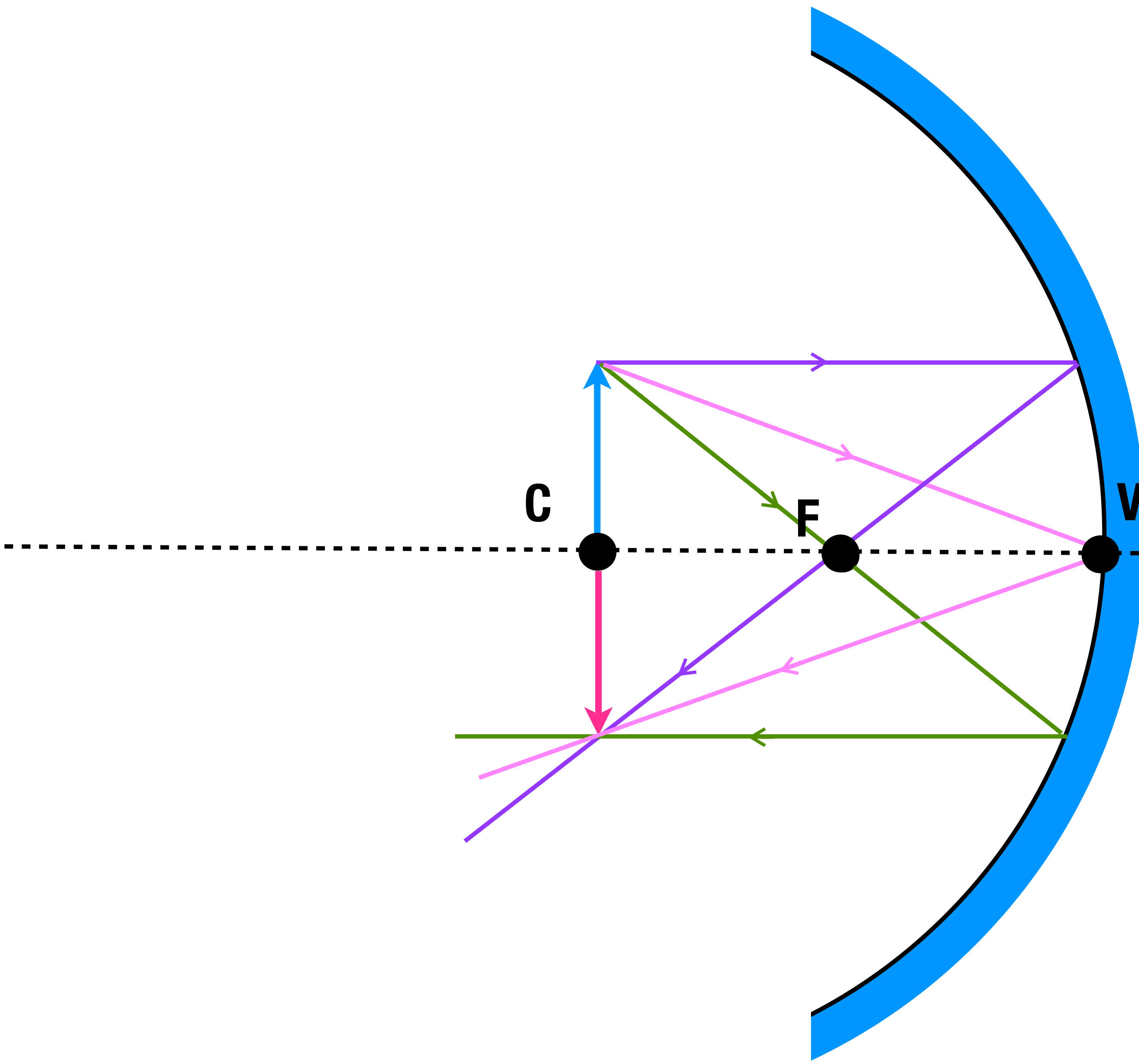
image in a concave mirror

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$ $$f=\frac{R}{2}$$ $$m\equiv\frac{y'}{y}=-\frac{s'}{s}$$

A concave mirror has a radius of curvature of absolute value 20 cm. Find the position, reality, magnification and orientation of the image in the case that the object distance is 20 cm

$$R=+20\,\mathrm{cm}$$

$$f=+10\,\mathrm{cm}$$



$$s'=+20\,\mathrm{cm}$$ **real image**

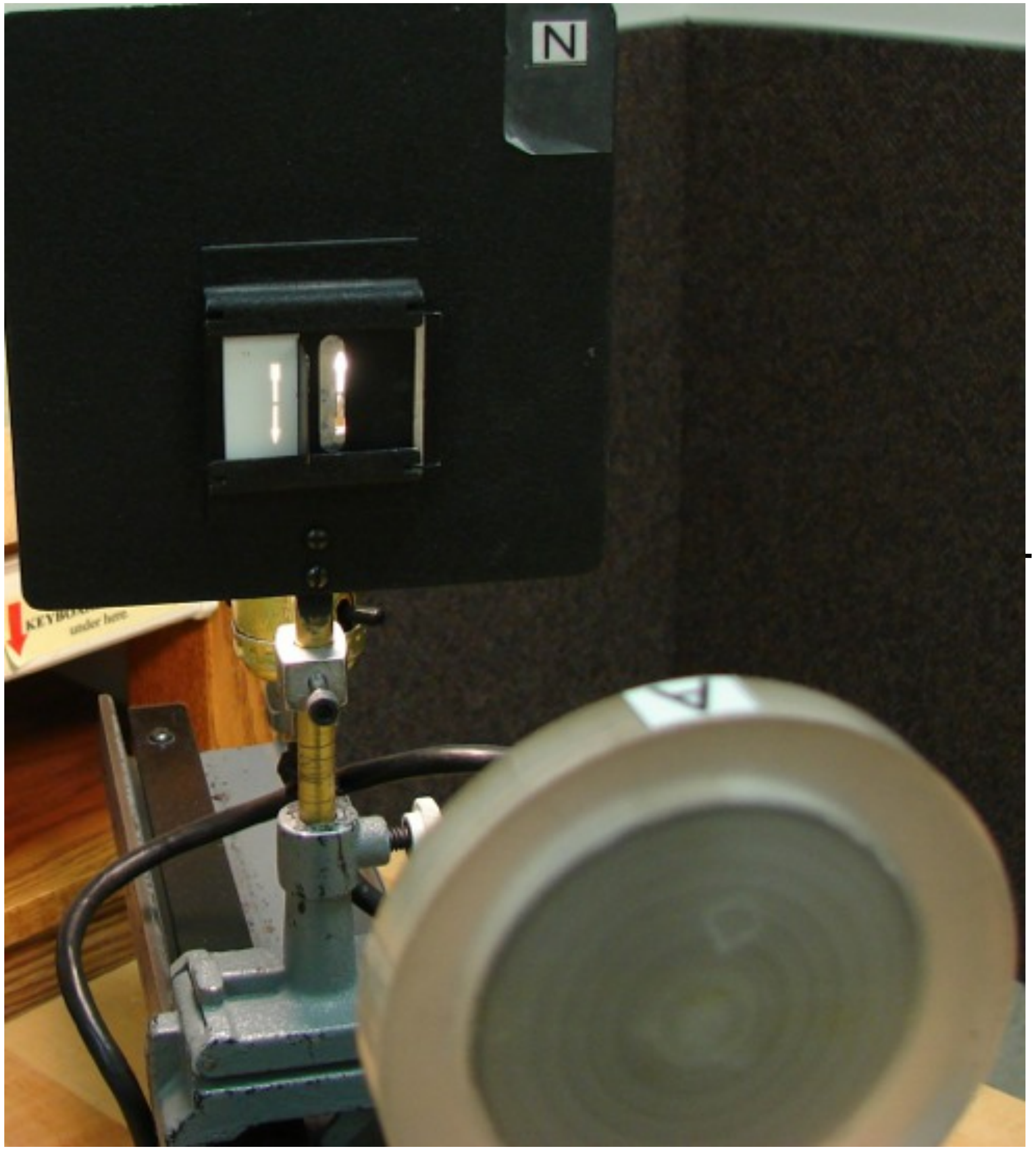
$$m=-1$$ **inverted image**

same size

image in a concave mirror

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$ $$f = \frac{R}{2}$$ $$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

real image **inverted image** **same size**



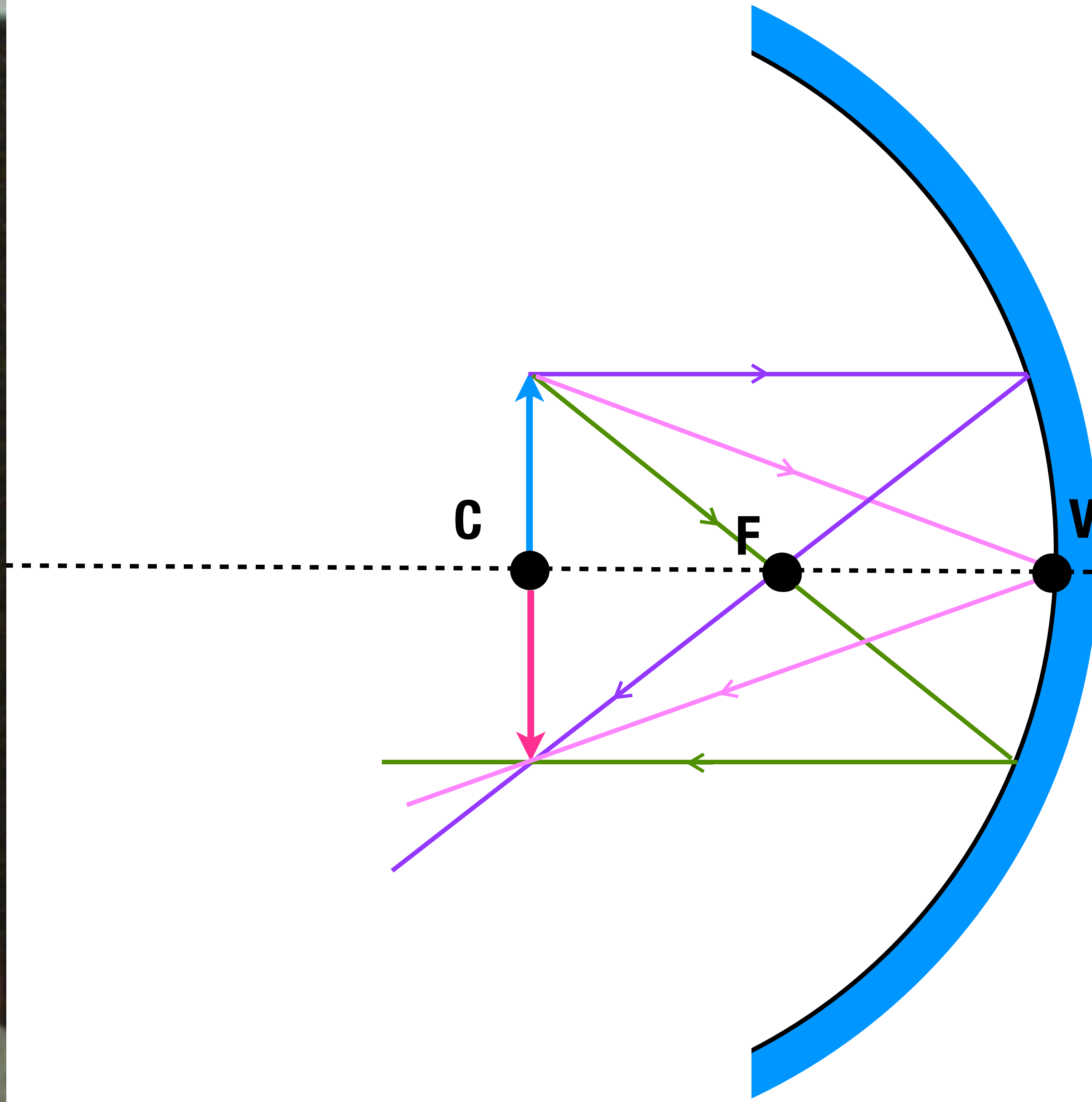


image in a concave mirror

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$ $$f = \frac{R}{2}$$ $$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

A concave mirror has a radius of curvature of absolute value 20 cm. Find the position, reality, magnification and orientation of the image in the case that the object distance is 10 cm

$$R = +20\,\text{cm}$$

$$f = +10\,\text{cm}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{10\,\text{cm}} - \frac{1}{10\,\text{cm}} = 0$$

$$s' \to \infty$$

real image at infinity

an observer at any finite distance will just see a blur

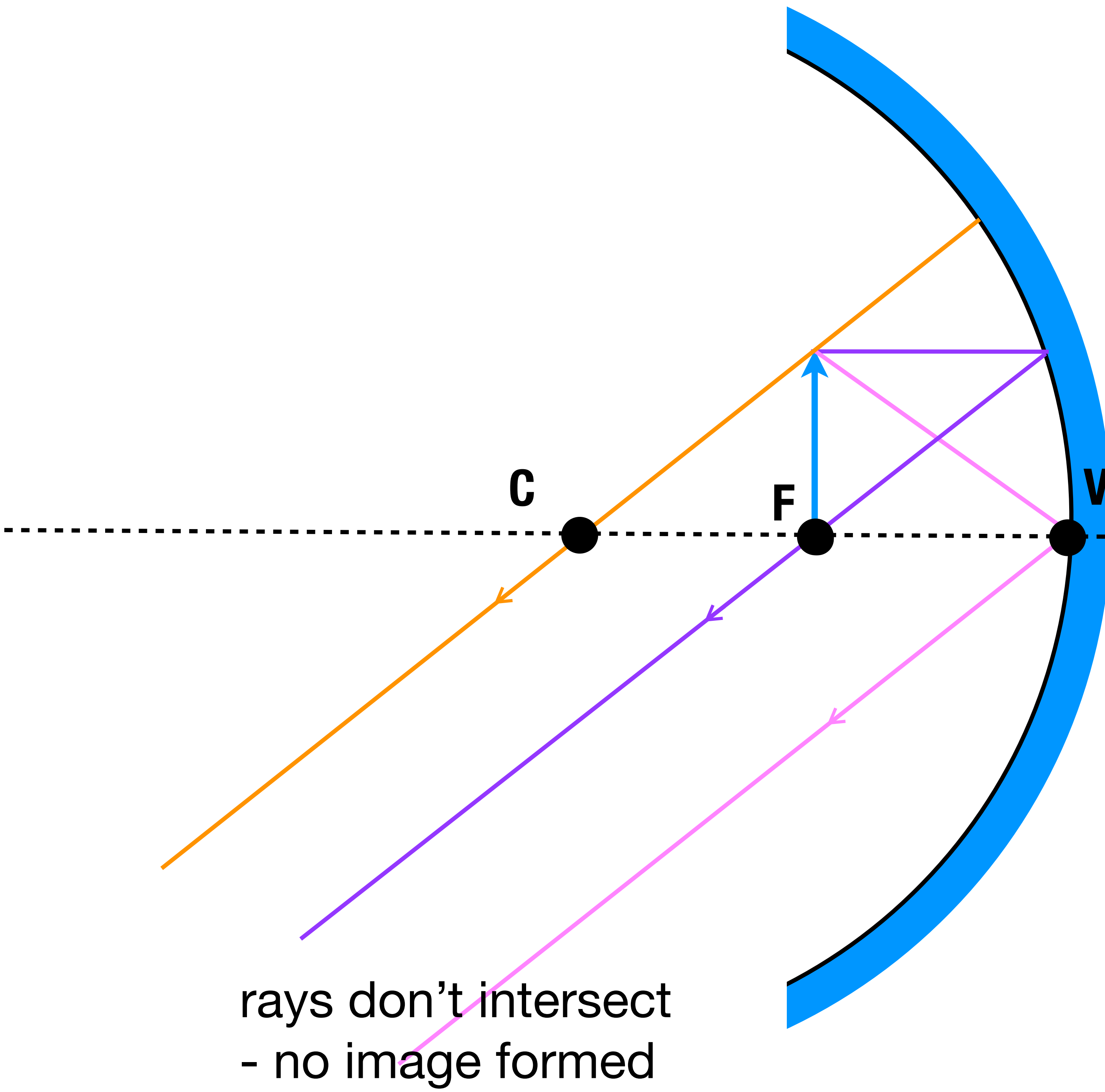


image in a concave mirror

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$ $$f=\frac{R}{2}$$ $$m\equiv\frac{y'}{y}=-\frac{s'}{s}$$

A concave mirror has a radius of curvature of absolute value 20 cm. Find the position, reality, magnification and orientation of the image in the case that the object distance is 5 cm

$$R=+20\,\text{cm}$$

$$f=+10\,\text{cm}$$

$$s'=-10\,\text{cm}$$ **virtual image**

$$m=+2$$ **upright magnified ×2**

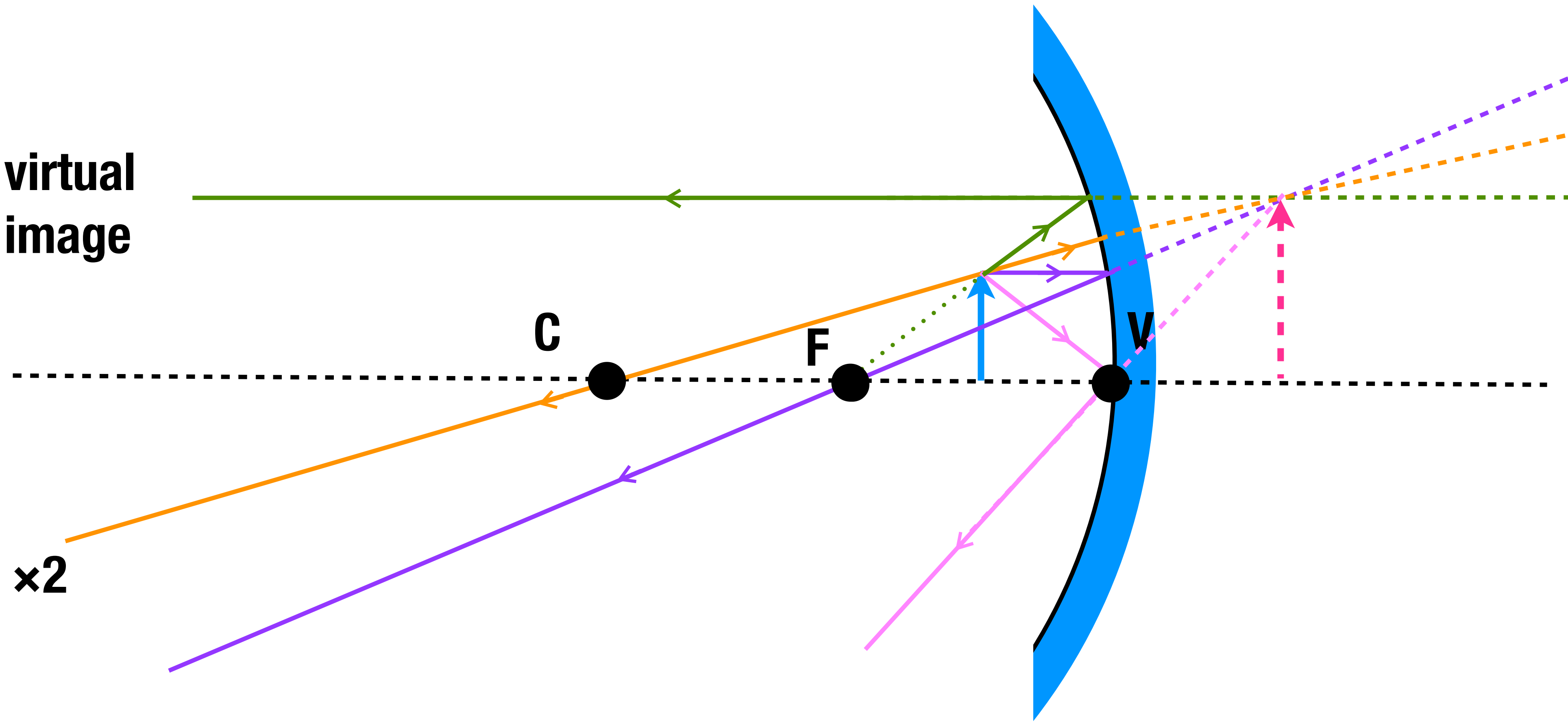


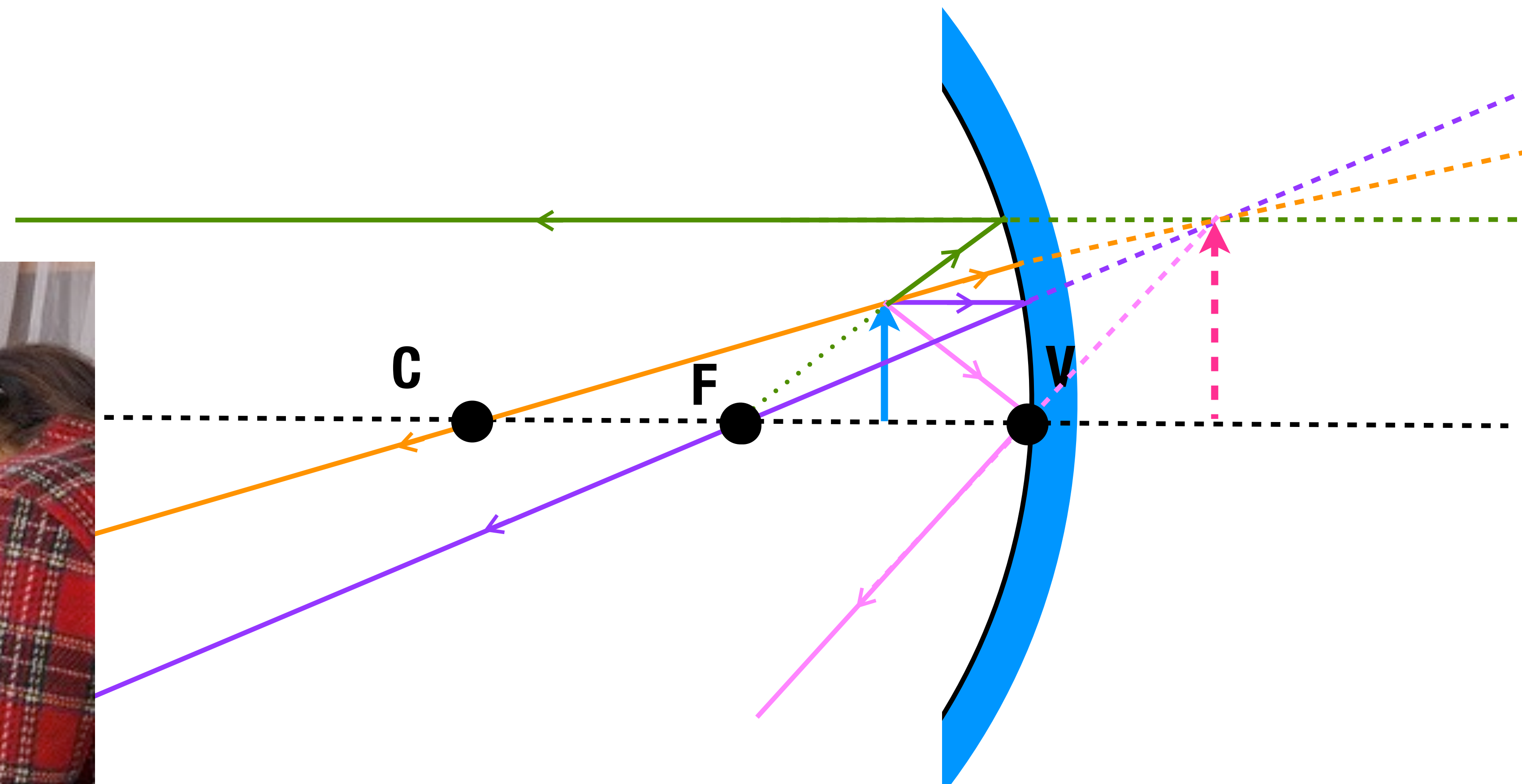
image in a concave mirror

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$f = \frac{R}{2}$$

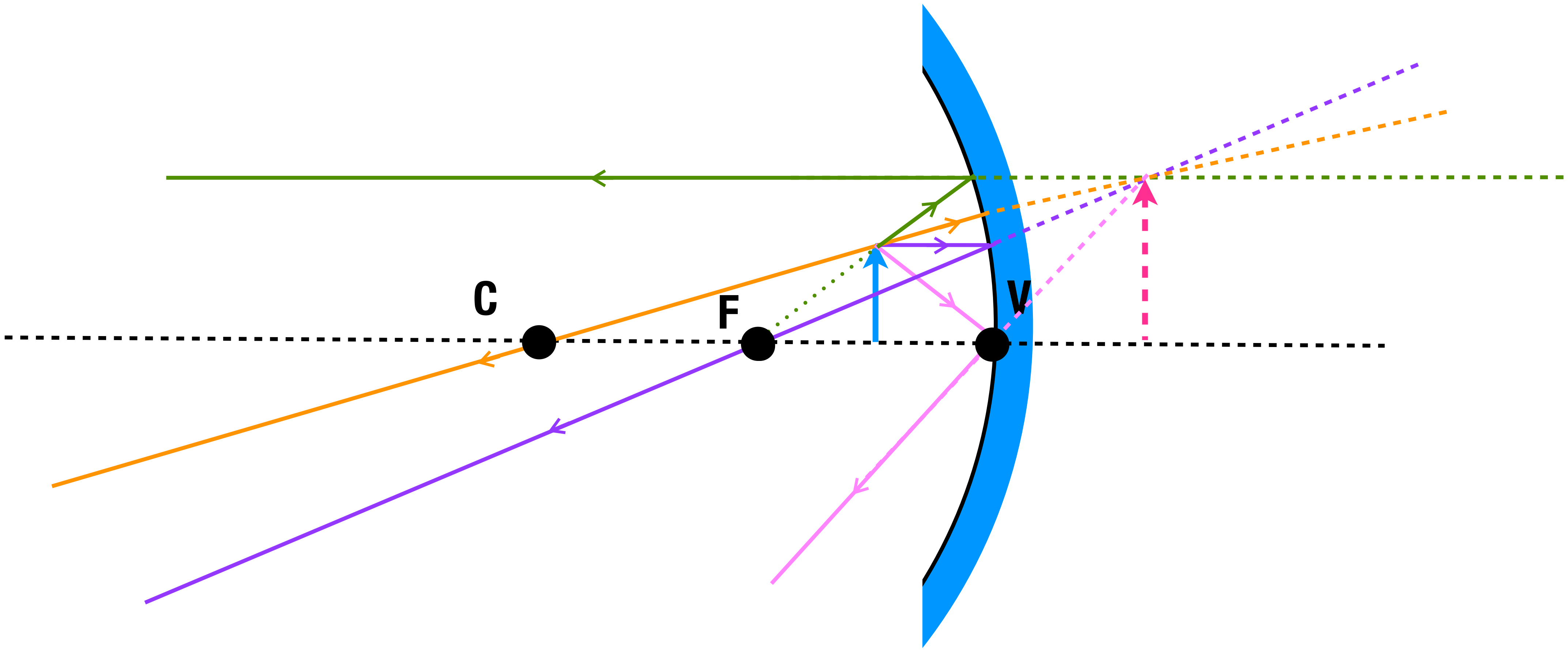
$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

virtual image | **upright magnified**



non-principal rays

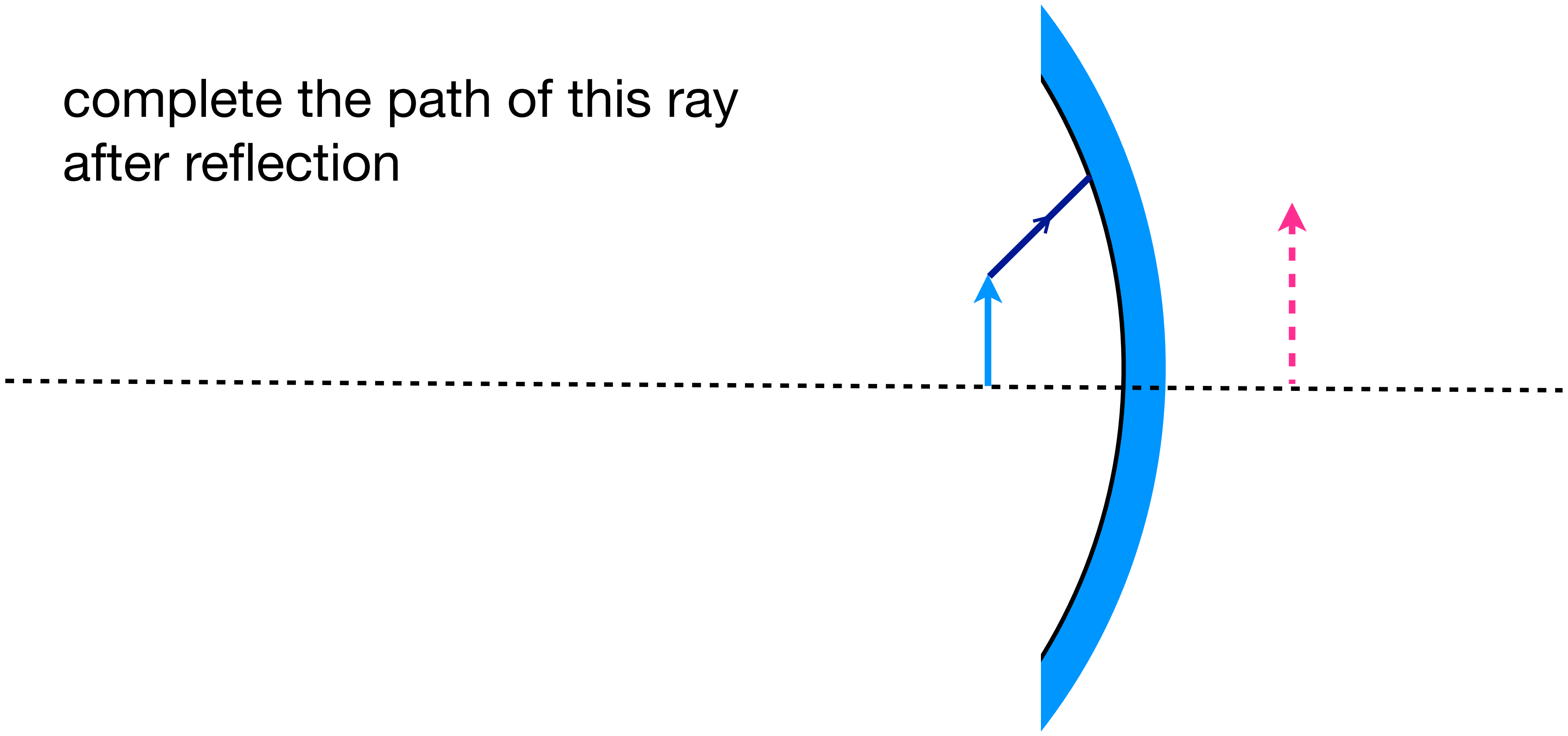
principal rays are easy - but once we've got the image we should be able to work out the path of any other rays



non-principal rays

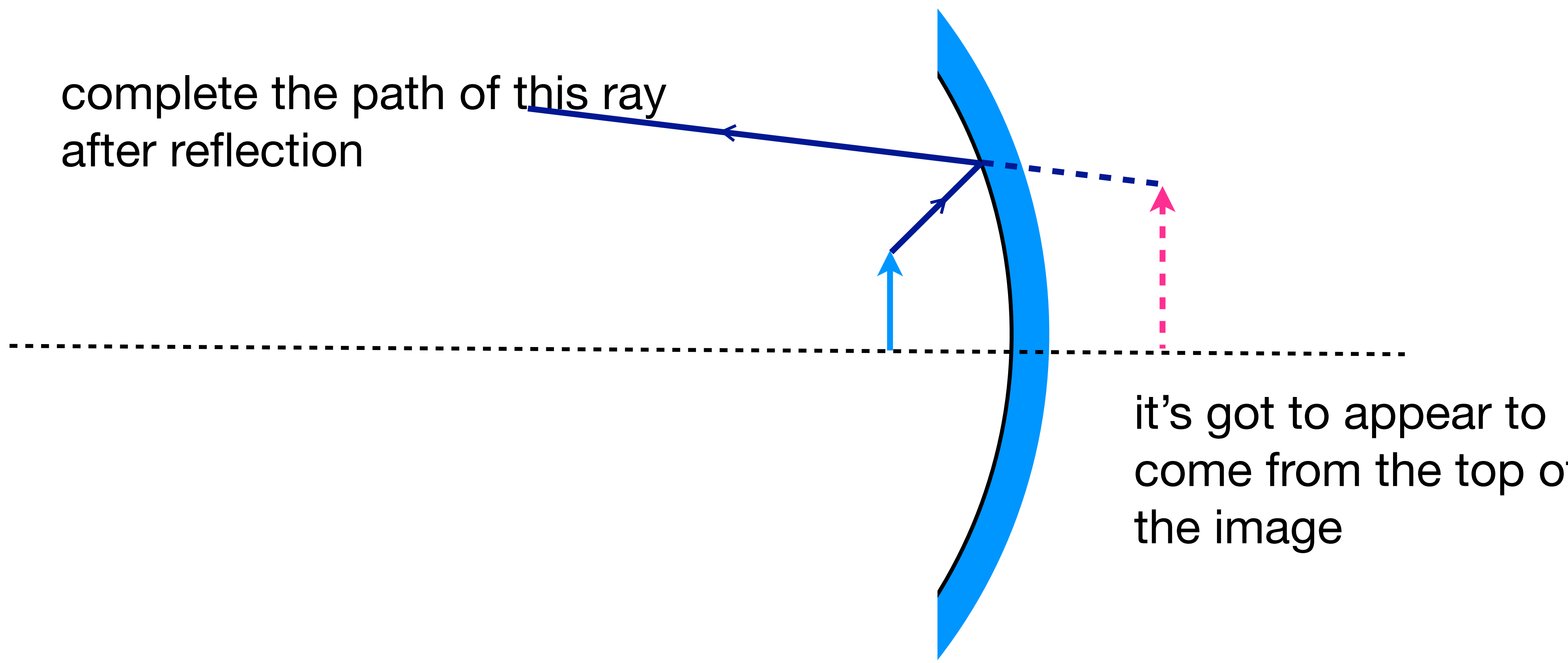
principal rays are easy - but once we've got the image we should be able to work out the path of any other rays

complete the path of this ray after reflection



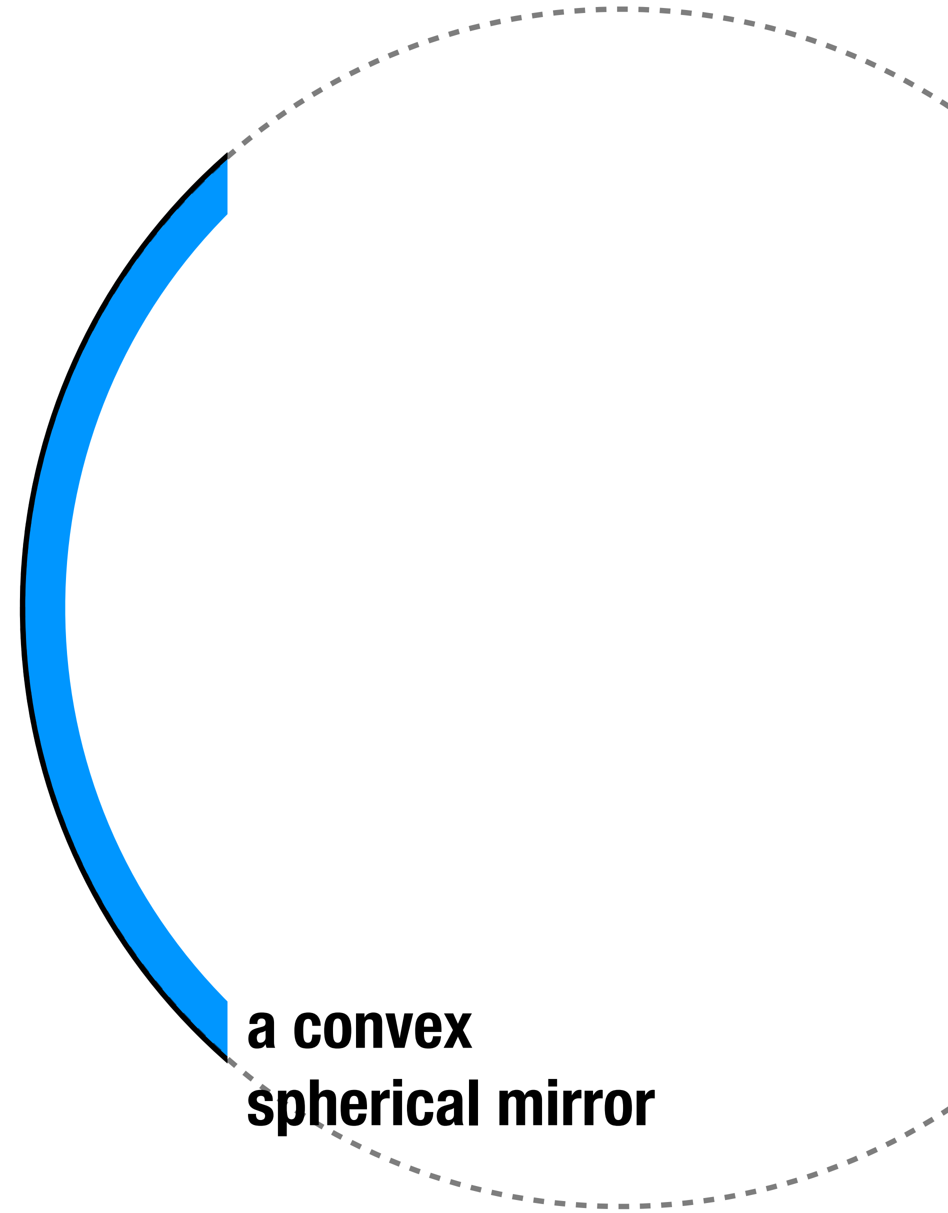
non-principal rays

principal rays are easy - but once we’ve got the image we should be able to work out the path of any other rays



reflection from a convex spherical mirror

➜ a spherical mirror is shown and an object is placed to the left of the mirror’s surface

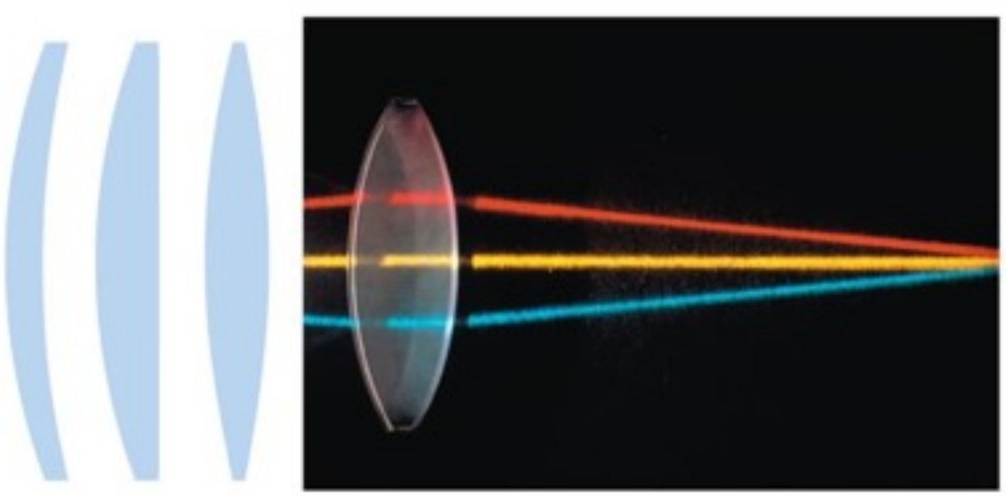


‘thin’ lenses

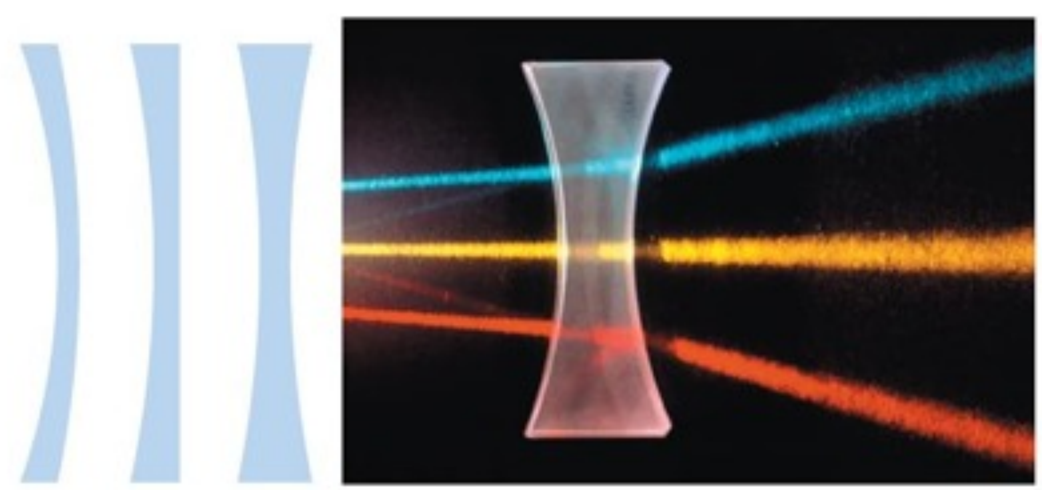
➜ these are the classic optical device - used very widely (how many of you wear glasses or contacts?) - we should learn about their properties

➜ two spherical surfaces close enough together that we can neglect the distance between them

(a) Converging lenses, which are thicker in the center than at the edges, refract parallel rays toward the optical axis.



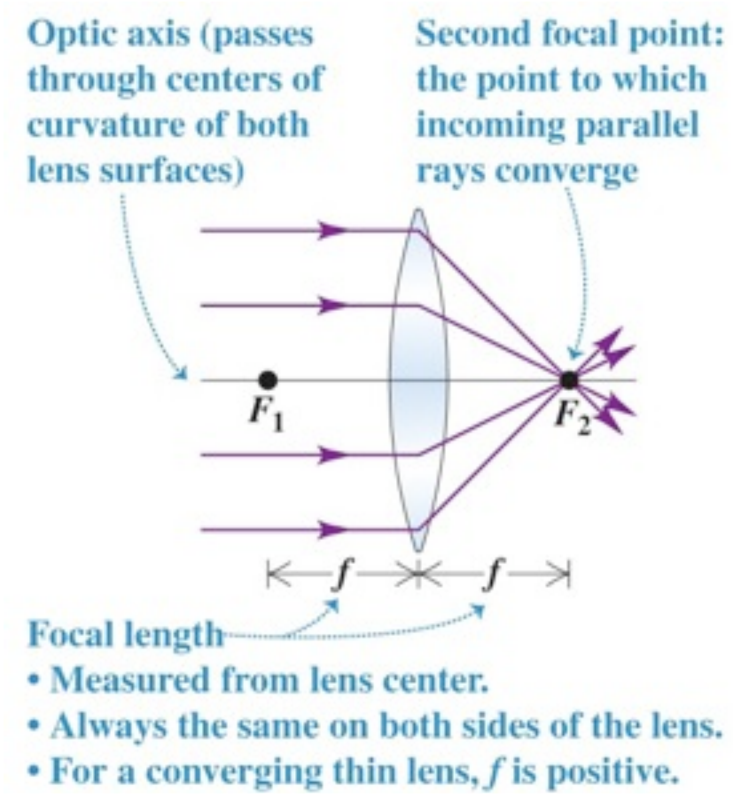
(b) Diverging lenses, which are thinner in the center than at the edges, refract parallel rays away from the optical axis.

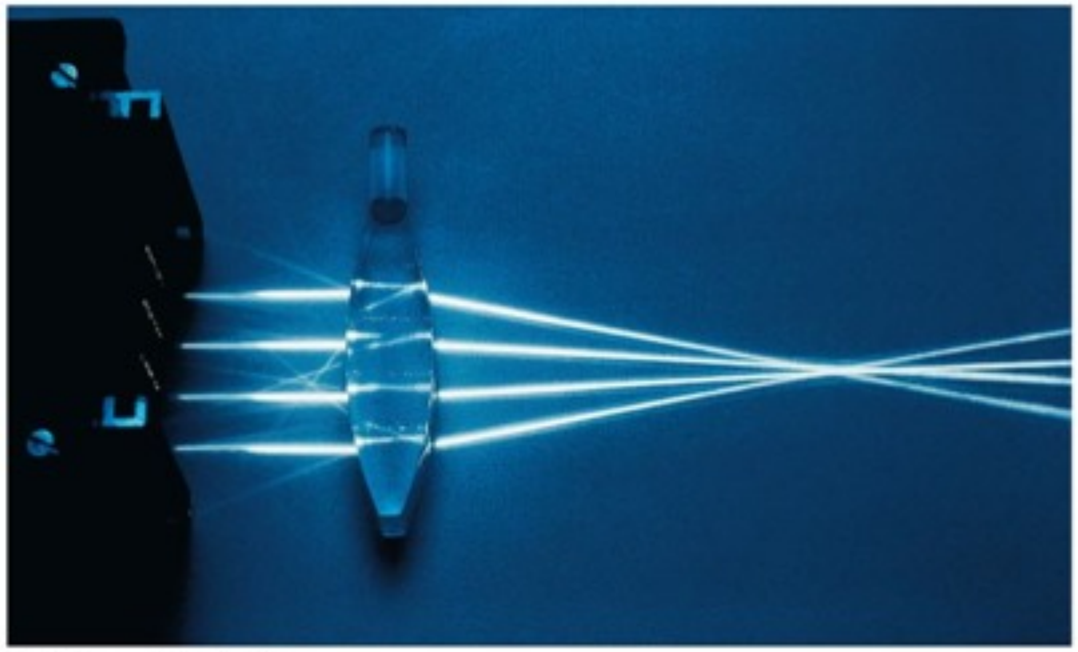


focal point of ‘thin’ lenses

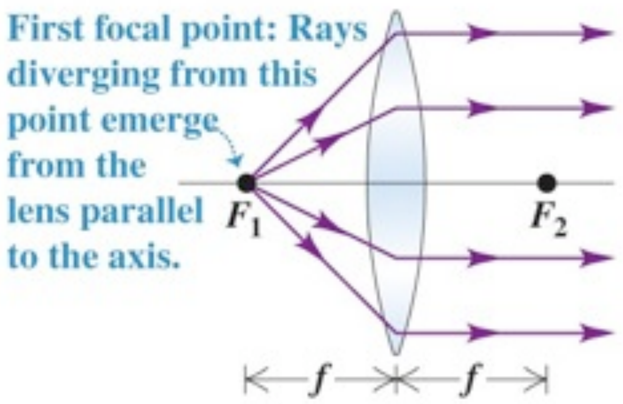
➜ consider parallel rays entering the lens - from either side

➜ technically two focal points - focal lengths the same for a ‘thin’ lens





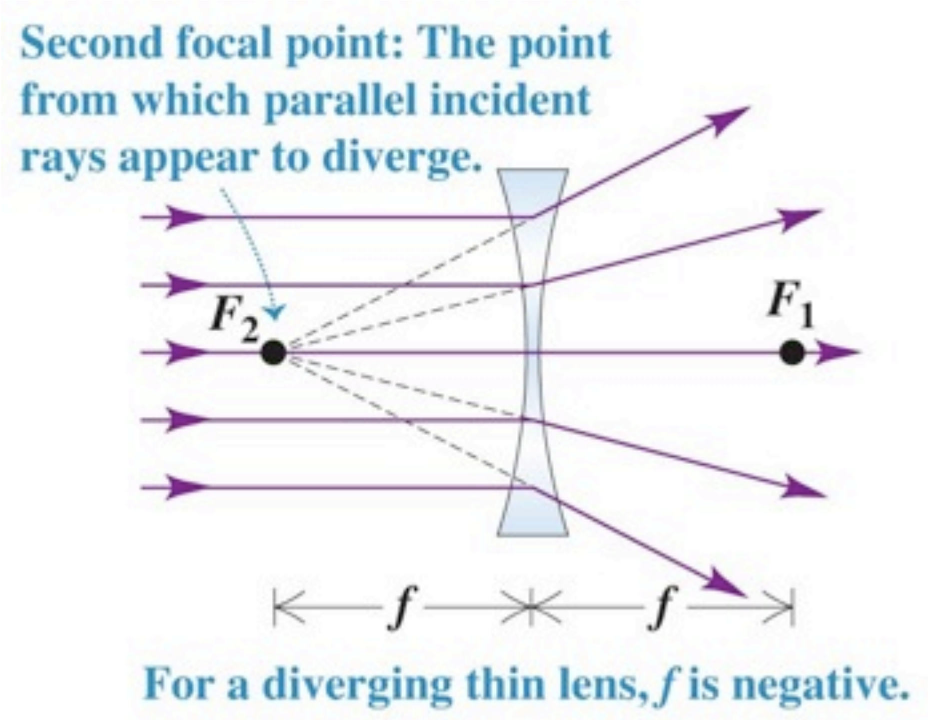
➜ as for mirrors, rays diverging from a focal point are parallel after refraction

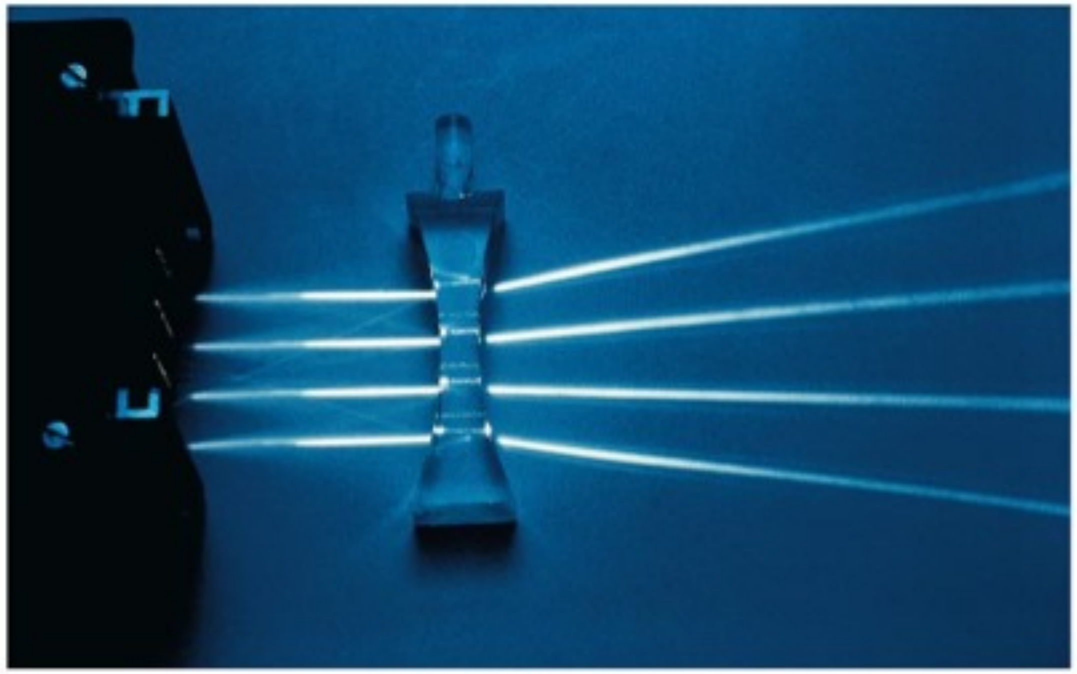


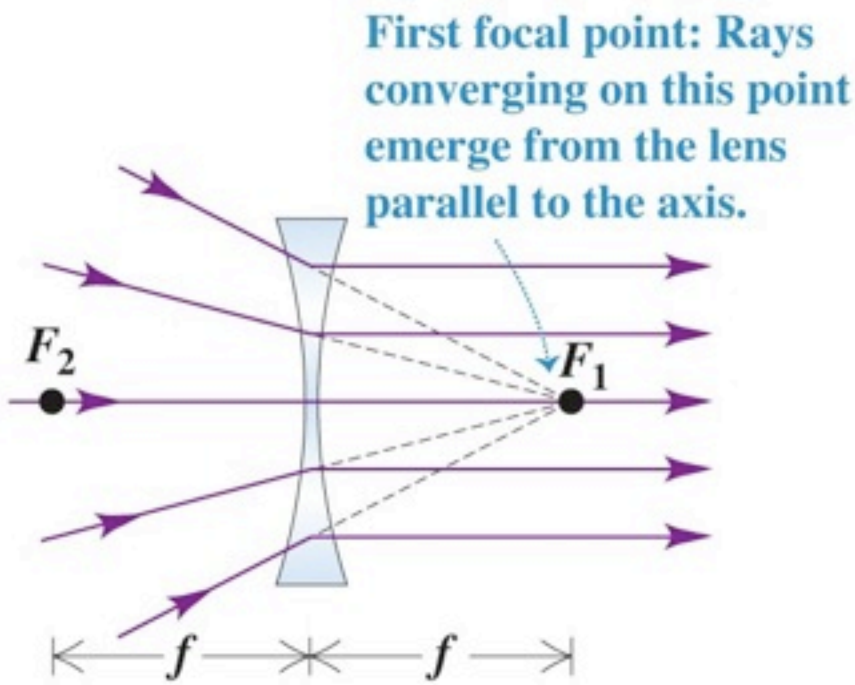
focal point of ‘thin’ lenses

➜ consider parallel rays entering the lens - from either side

➜ technically two focal points - focal lengths the same for a ‘thin’ lens



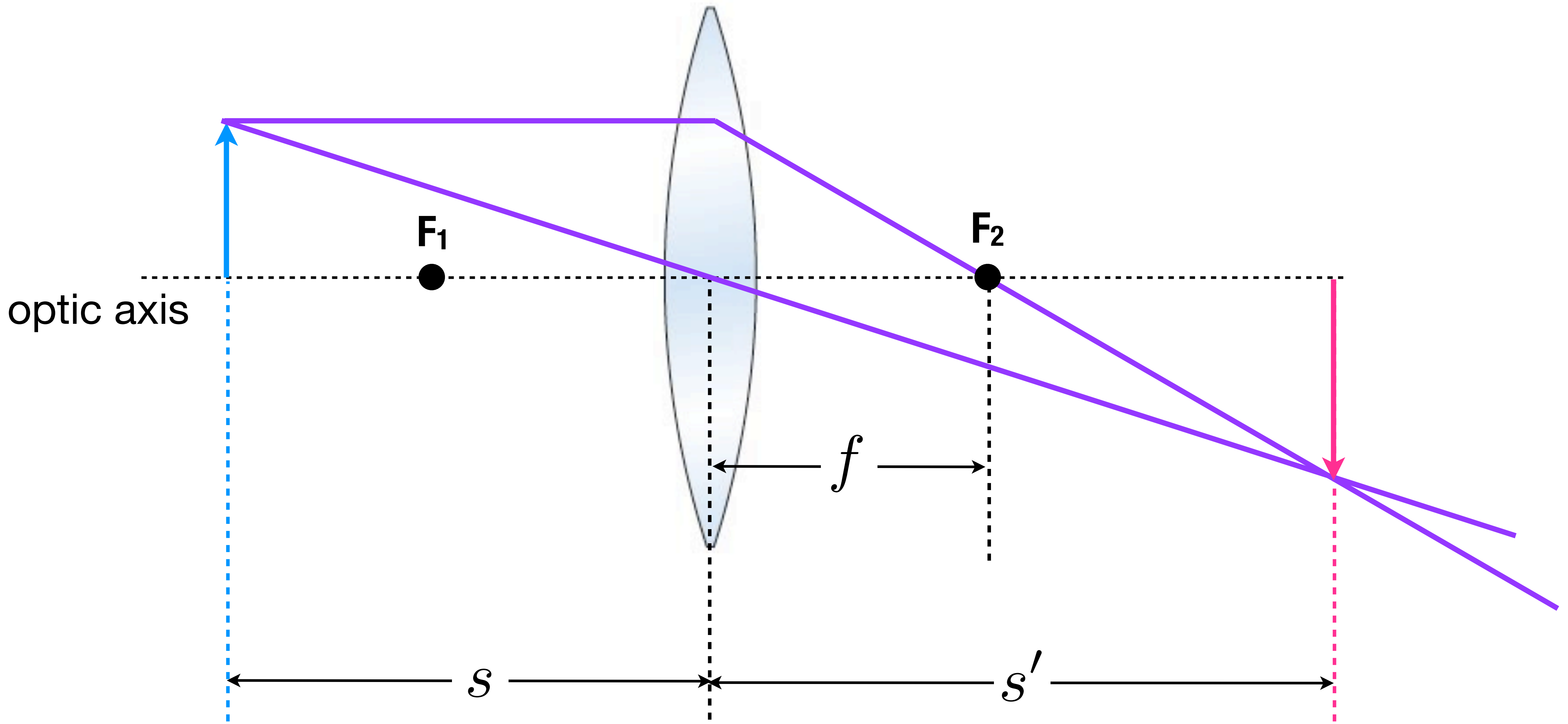




➜ as for mirrors, rays diverging from a focal point are parallel after refraction

image formation in a converging lens

➜ we draw the rays as though they refract at the center - this is OK for 'thin' lenses



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

focal lengths

➜ sign convention:

➜ converging lenses have positive focal length
➜ diverging lenses have negative focal length

➜ the focal length of a thin lens can be calculated if you know the refractive index of the lens material and the radii of curvature of the lens faces

graphical method - principal ray tracing

➜ we'll use the formulas soon, but first let's explore a graphical technique

➜ we'll single out some special rays whose path is easy to work out

➜ we call them **principal rays**

➜ an incident ray parallel to the optic axis refracts to pass through the second focal point

➜ a ray through the center of the lens does not deviate

➜ a ray through the first focal point refracts parallel to the optic axis

image formation in a converging lens

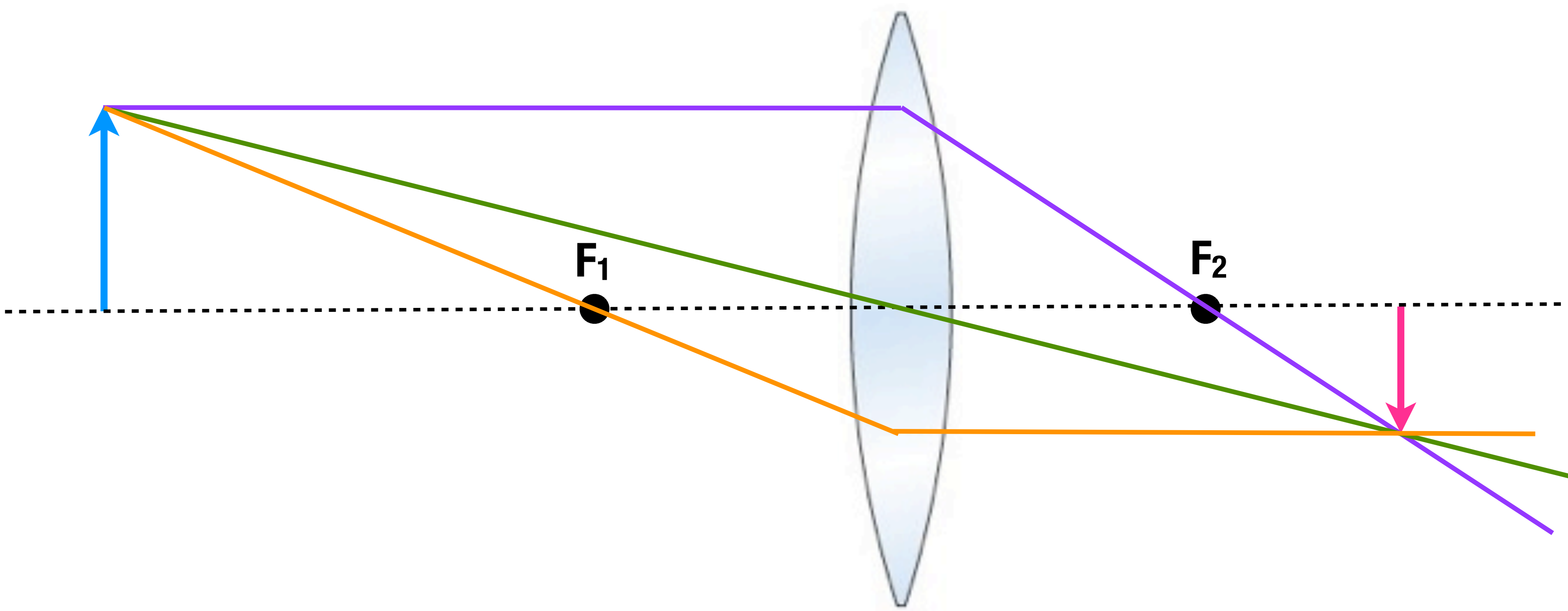
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$ $$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

A converging lens has a focal length of 20 cm. Find the position, reality, magnification and orientation of the image in the case that the object distance is 50 cm

$$\begin{aligned}\frac{1}{s'} &= \frac{1}{f} - \frac{1}{s} \\ &= \frac{1}{20\,\text{cm}} - \frac{1}{50\,\text{cm}} \\ &= \frac{5}{100\,\text{cm}} - \frac{2}{100\,\text{cm}} \\ &= \frac{3}{100\,\text{cm}}\end{aligned}$$

$$s' = 33.3\,\text{cm}$$

real image



$$m = -\frac{33.3\,\text{cm}}{50\,\text{cm}}$$

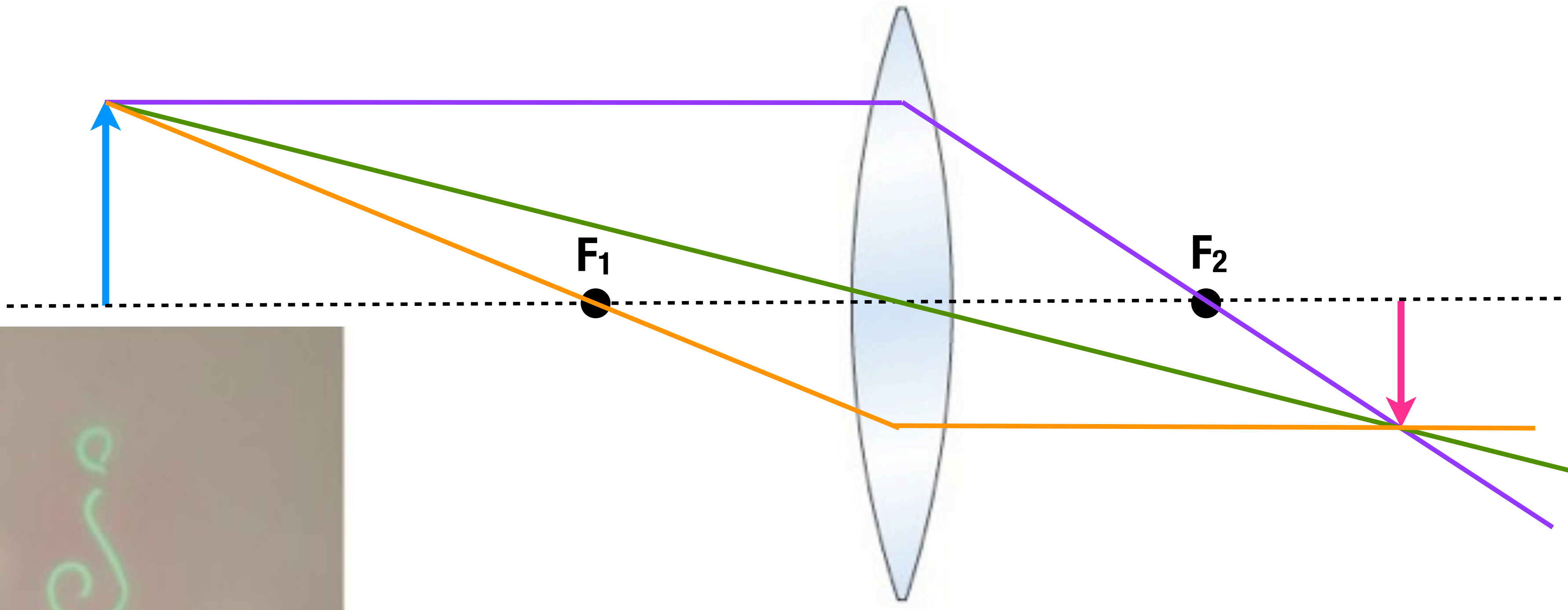
$$m = -0.666$$

inverted & reduced

image formation in a converging lens

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ $m \equiv \frac{y'}{y} = -\frac{s'}{s}$

real image **inverted & reduced**



real image formation and 'focus'

➜ what if we place a screen somewhere other than the image position ?

➜ the rays from a single point in the object aren't meeting at a single point on the screen

➜ fuzzy "out of focus" image

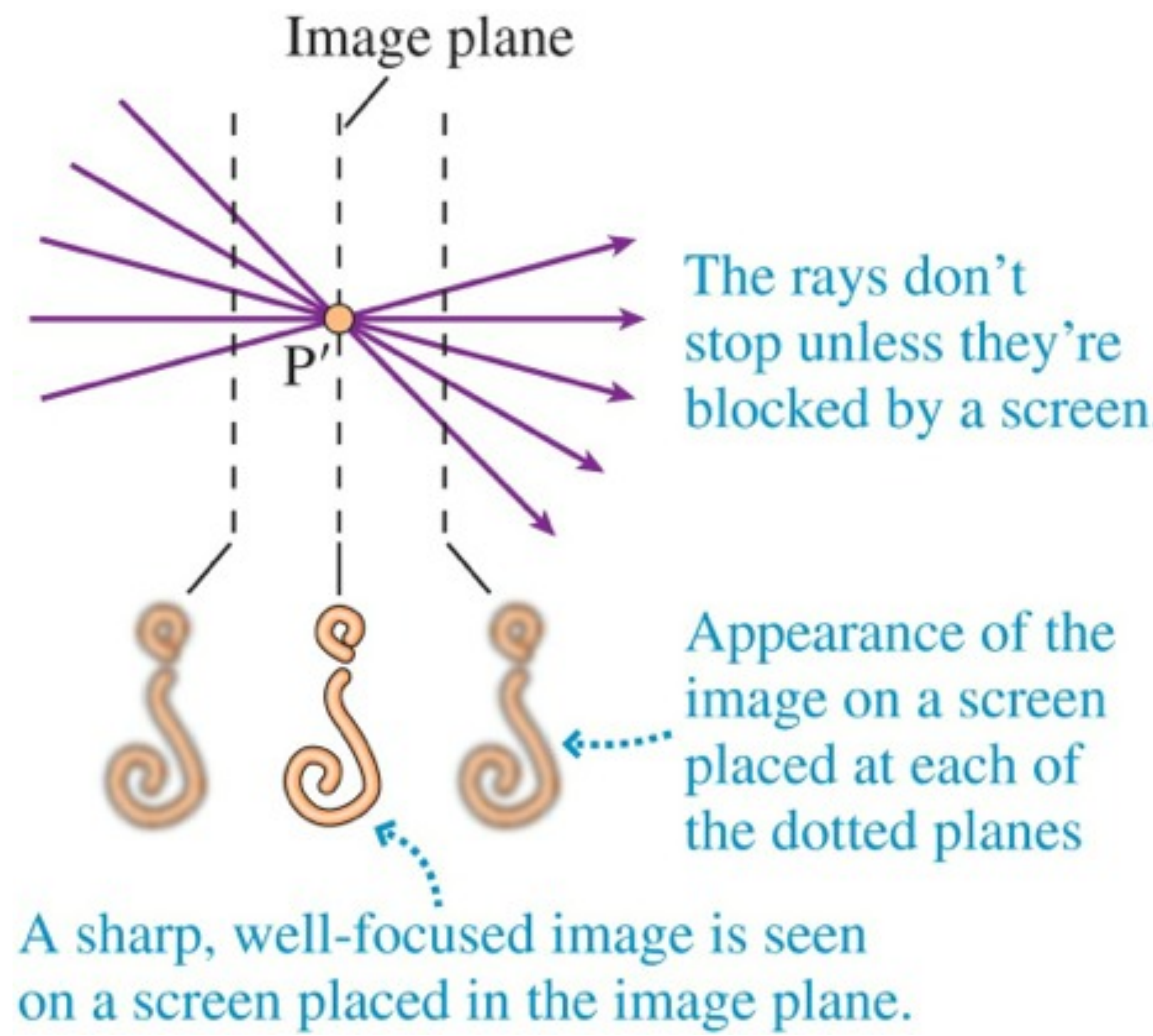
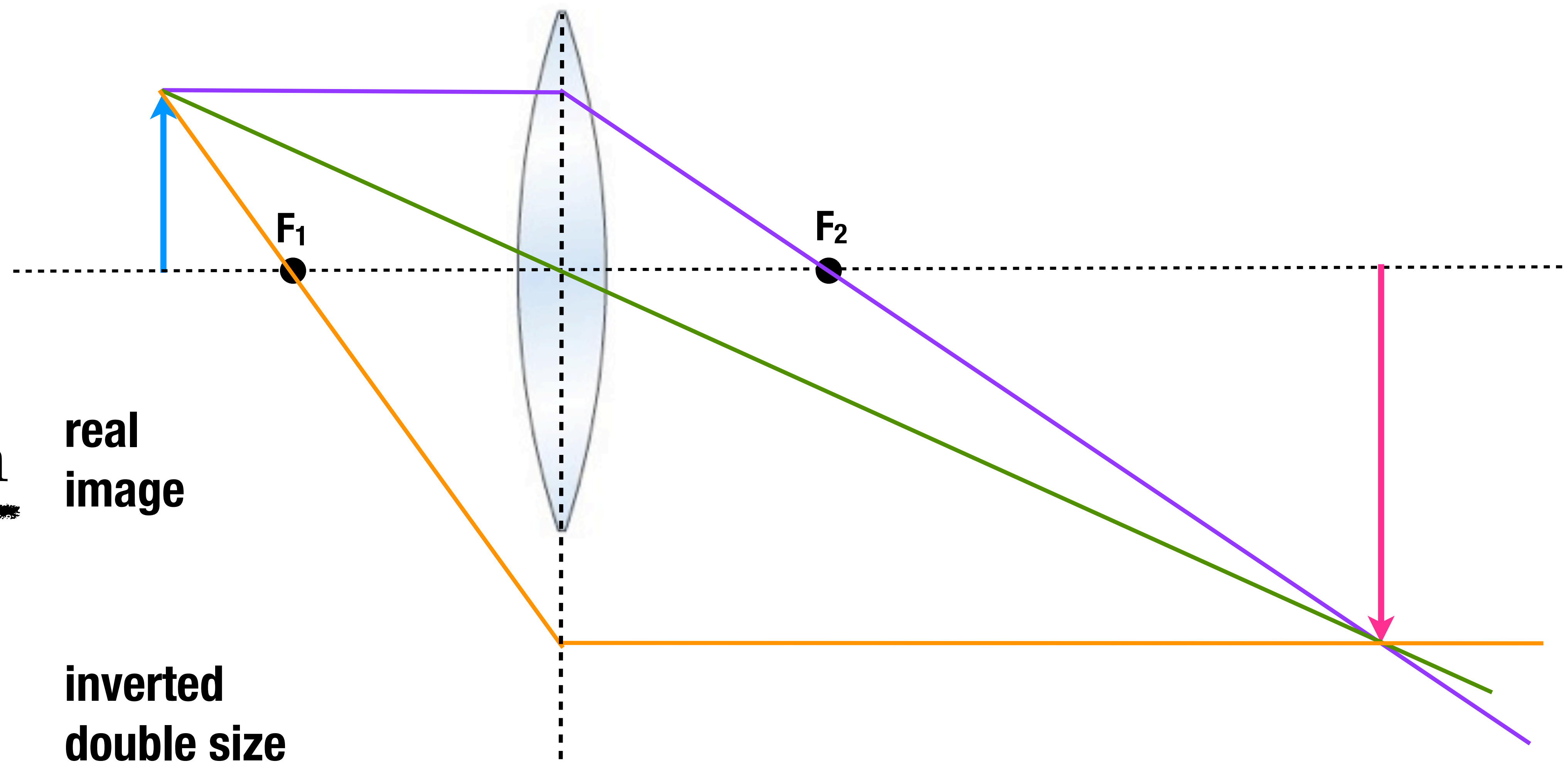


image formation in a converging lens

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

A converging lens has a focal length of 20 cm. Find the position, reality, magnification and orientation of the image in the case that the object distance is 30 cm



$$s' = +60\,\mathrm{cm}$$

real image

$$m = -2$$

inverted double size

image formation in a converging lens

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

A converging lens has a focal length of 20 cm. Find the position, reality, magnification and orientation of the image in the case that the object distance is 10 cm

$$s' = -20\,\text{cm}$$

virtual image

$$m = +2$$

upright double size

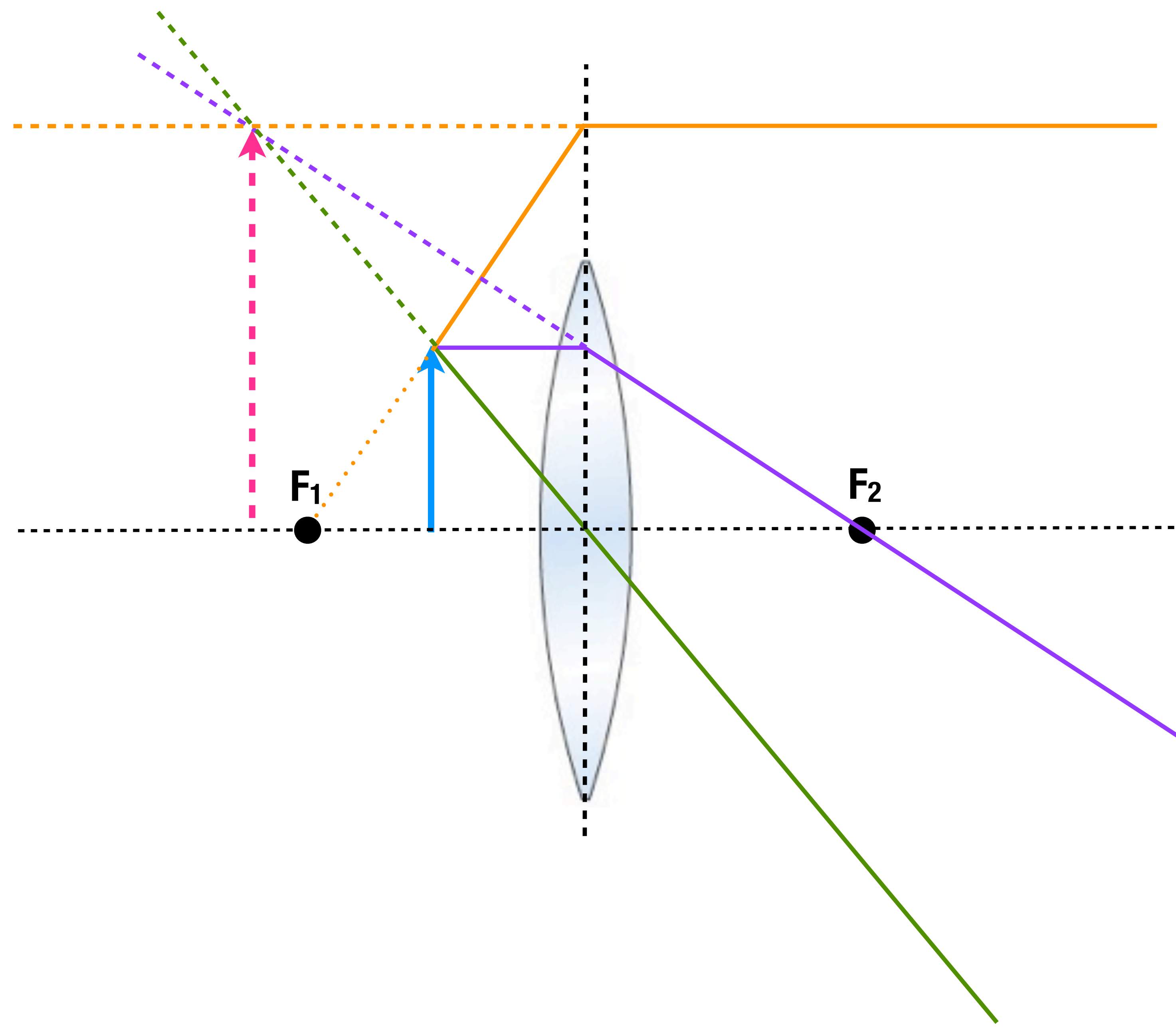


image formation in a converging lens

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

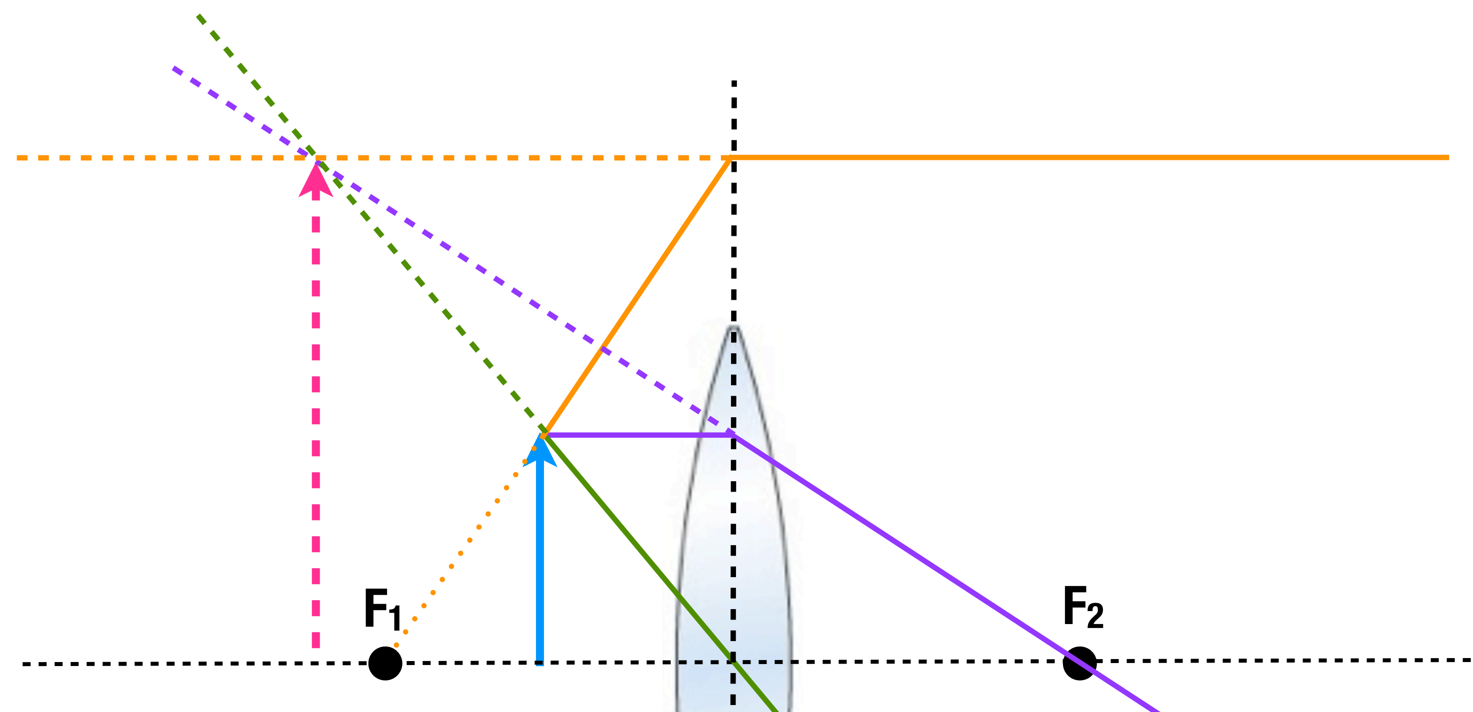
A converging lens has a focal length of 20 cm. Find the position, reality, magnification and orientation of the image in the case that the object distance is 10 cm

$$s' = -20\,\text{cm}$$

virtual image

$$m = +2$$

upright double size



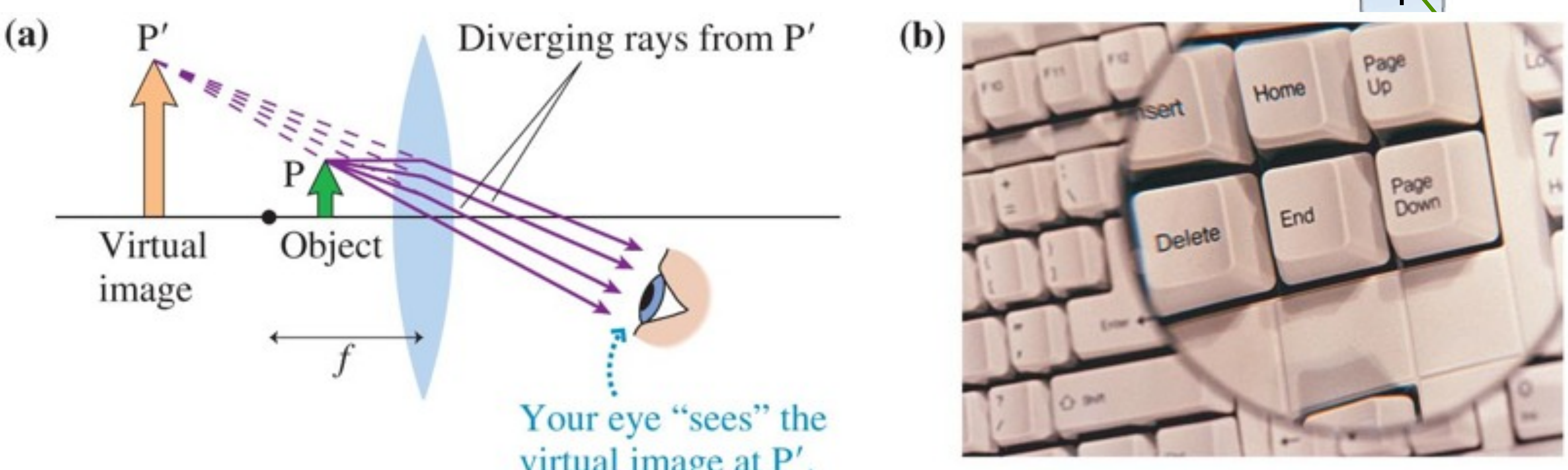


image formation in a diverging lens

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ $m \equiv \frac{y'}{y} = -\frac{s'}{s}$

A diverging lens has a focal length of 20 cm. If you want an upright image two-thirds the height of the object, where should you place the object ?

$$f = -20.0\,\mathrm{cm}$$

diverging lens

$$m = +\frac{2}{3} = -\frac{s'}{s}$$

$$s' = -\tfrac{2}{3}s$$

$$\frac{1}{s} + \frac{1}{-\frac{2}{3}s} = \frac{1}{f}$$

$$\frac{1}{s} - \frac{3}{2s} = \frac{1}{f}$$

$$\frac{2}{2s} - \frac{3}{2s} = \frac{1}{f}$$

$$-\frac{1}{2s} = \frac{1}{f}$$

$$s = -\tfrac{1}{2}f = +10\,\mathrm{cm}$$

image formation in a diverging lens

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$ $$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

A diverging lens has a focal length of 20 cm. If you want an upright image two-thirds the height of the object, where should you place the object ?

$$f = -20.0\,\mathrm{cm}$$ **diverging lens**

$$m = +\frac{2}{3} = -\frac{s'}{s}$$

$$s' = -\tfrac{2}{3}s$$

$$\frac{1}{s} + \frac{1}{-\frac{2}{3}s} = \frac{1}{f}$$
$$\frac{1}{s} - \frac{3}{2s} = \frac{1}{f}$$
$$\frac{2}{2s} - \frac{3}{2s} = \frac{1}{f}$$
$$-\frac{1}{2s} = \frac{1}{f}$$

$$s = -\tfrac{1}{2}f = +10\,\mathrm{cm}$$

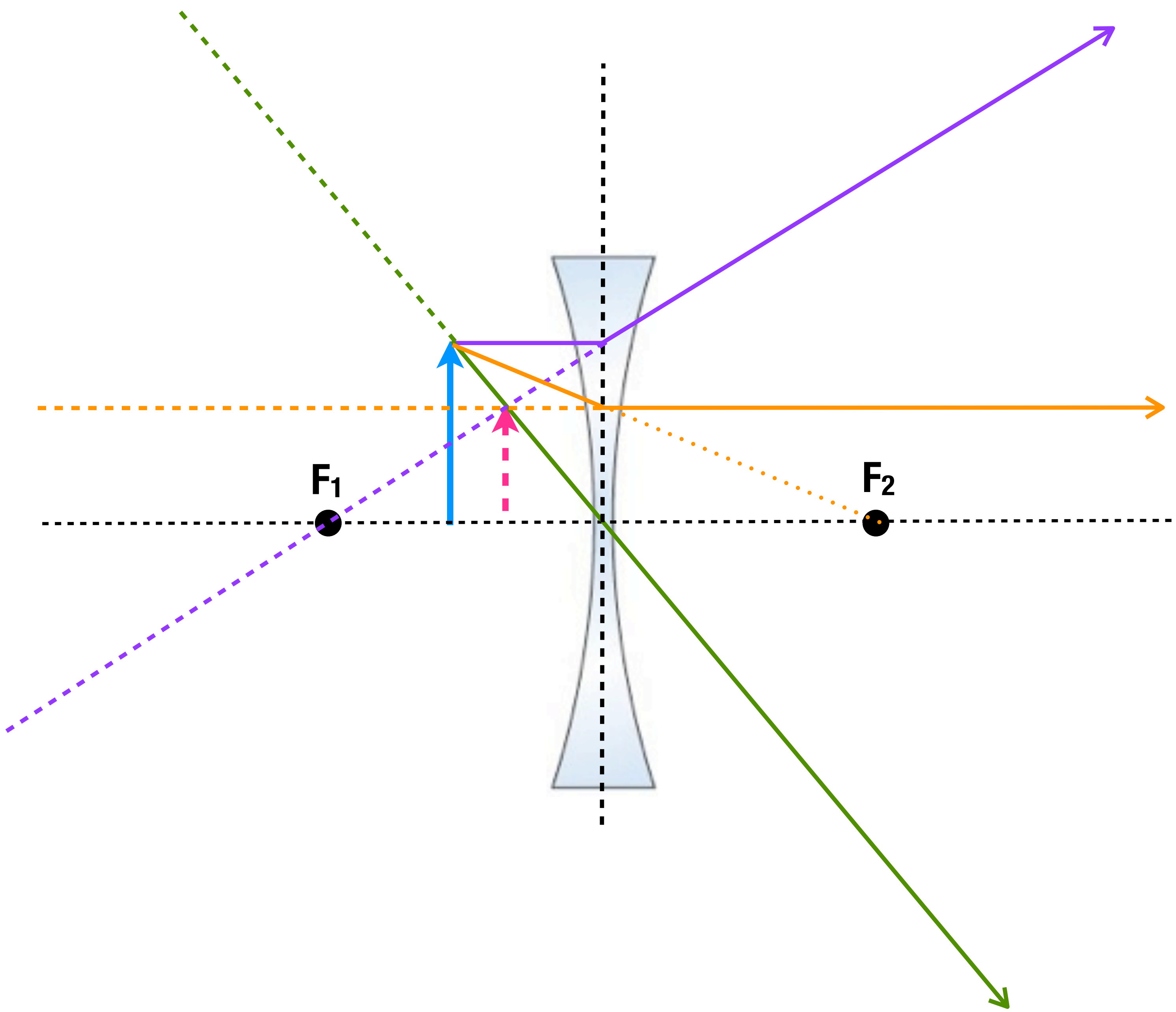
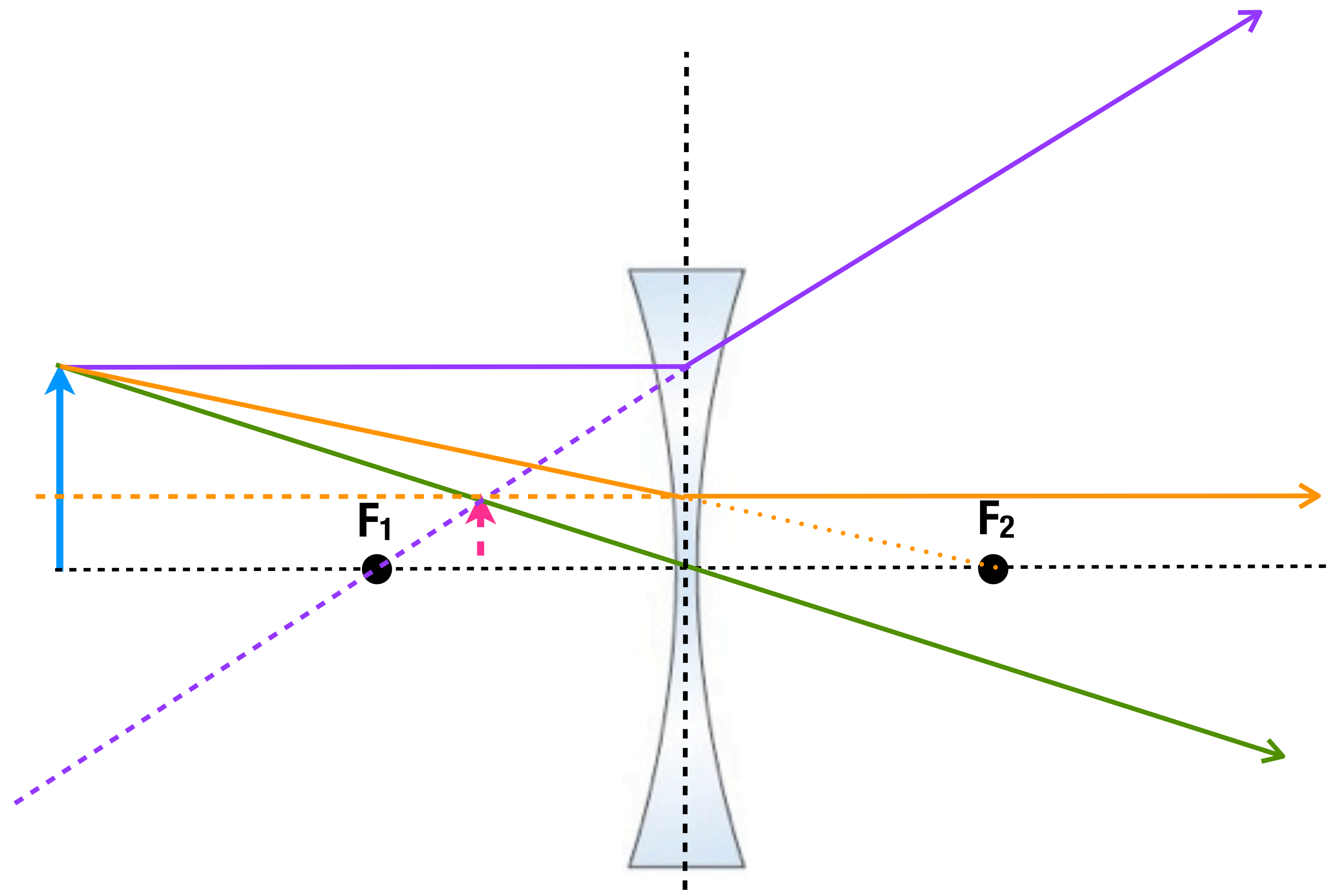


image formation in a diverging lens

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

virtual image **upright image**



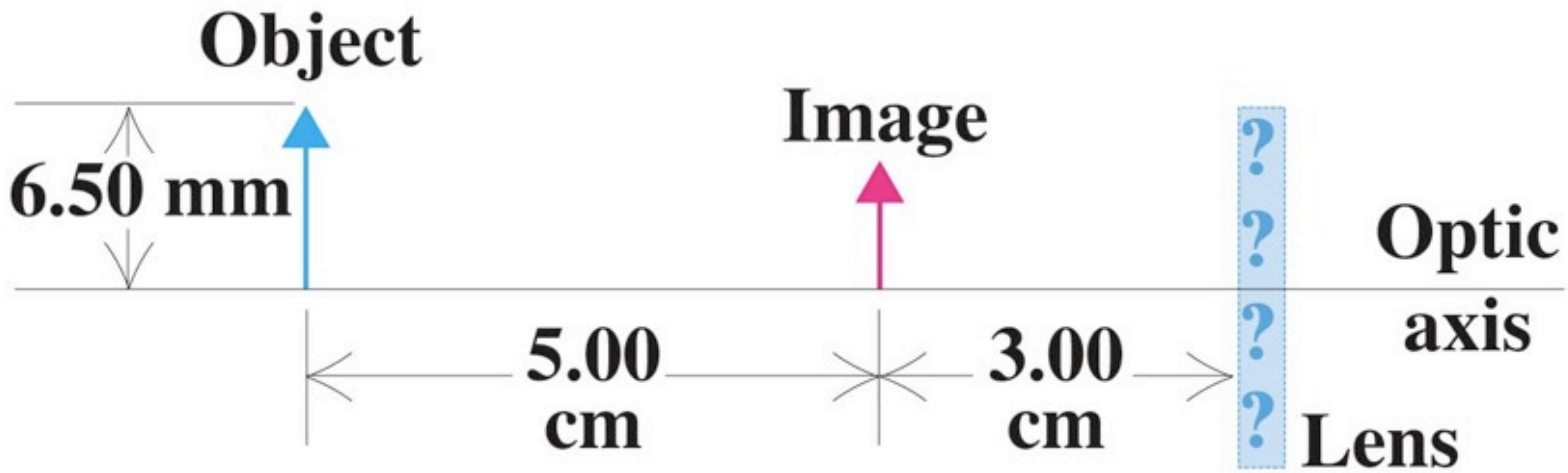
imaging from a lens

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$ $$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

The figure shows an object and its image formed by a thin lens.

(a) what is the focal length of the lens and is the lens converging or diverging ?

(b) what is the height of the image and is it real or virtual ?

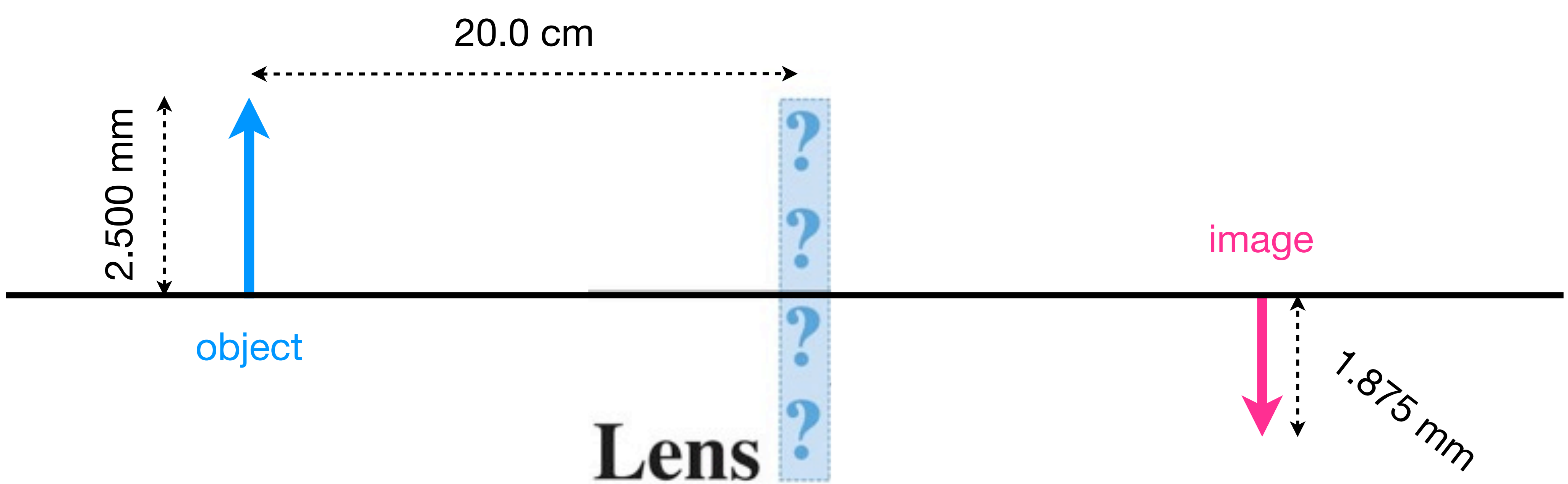


imaging from a lens

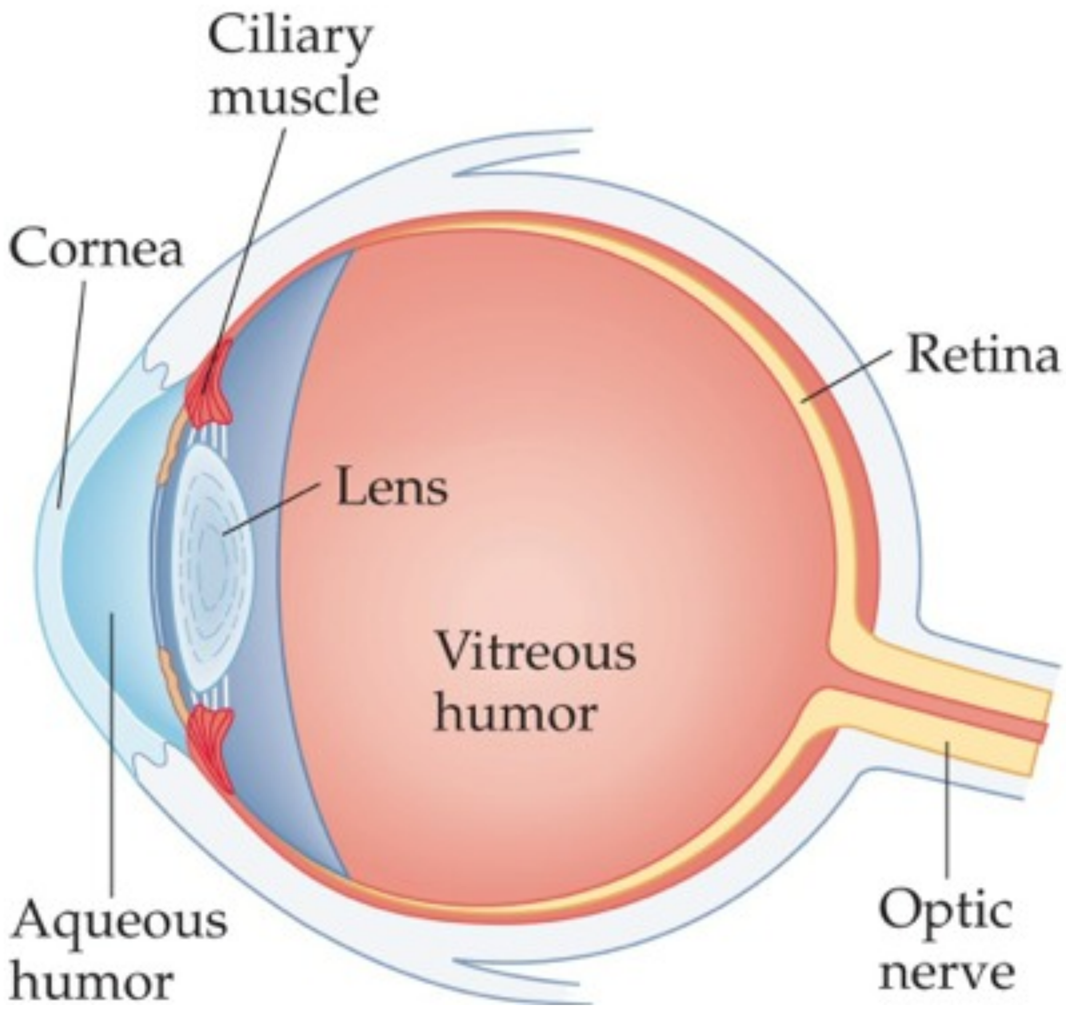
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

The figure shows an object and its image formed by a thin lens.
What is the focal length of the lens and is the lens converging or diverging ?

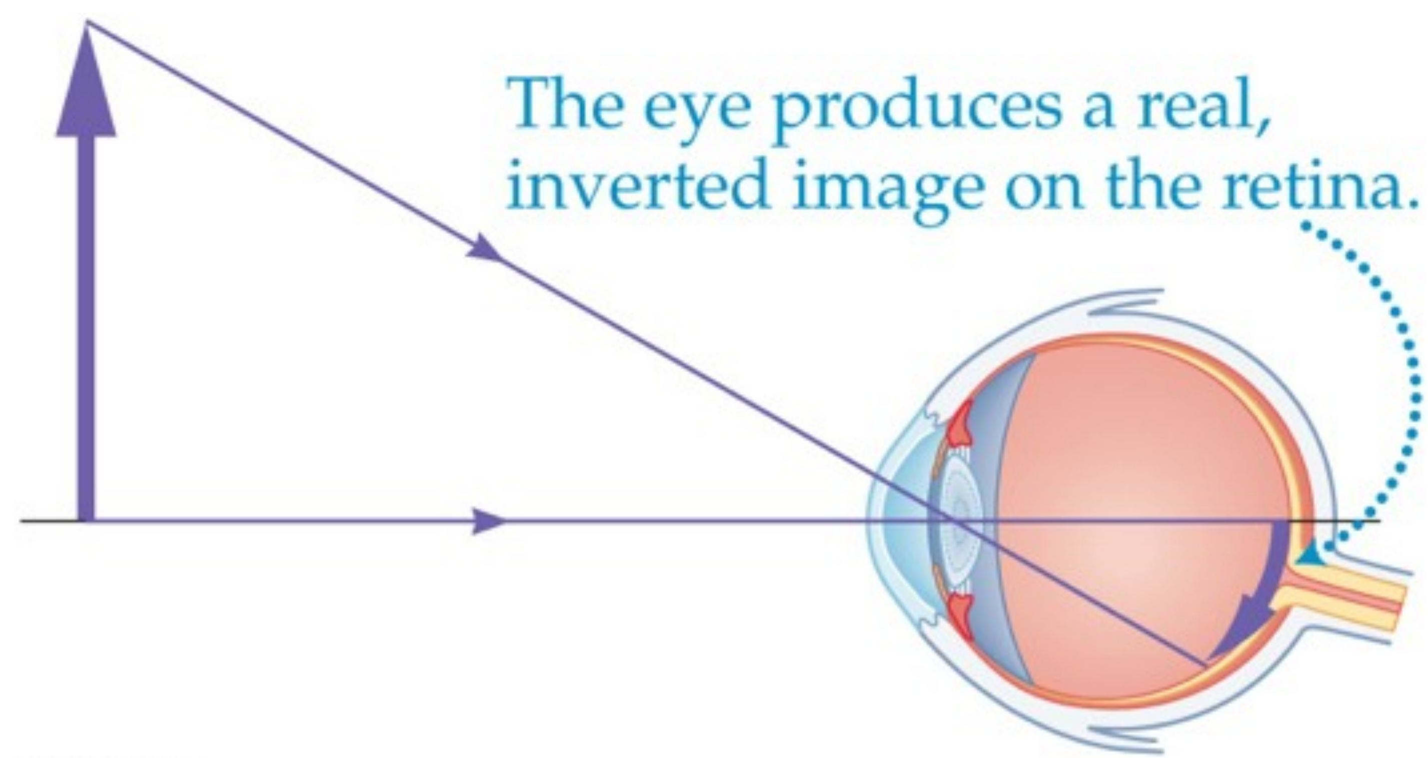


an important optical instrument - the eye



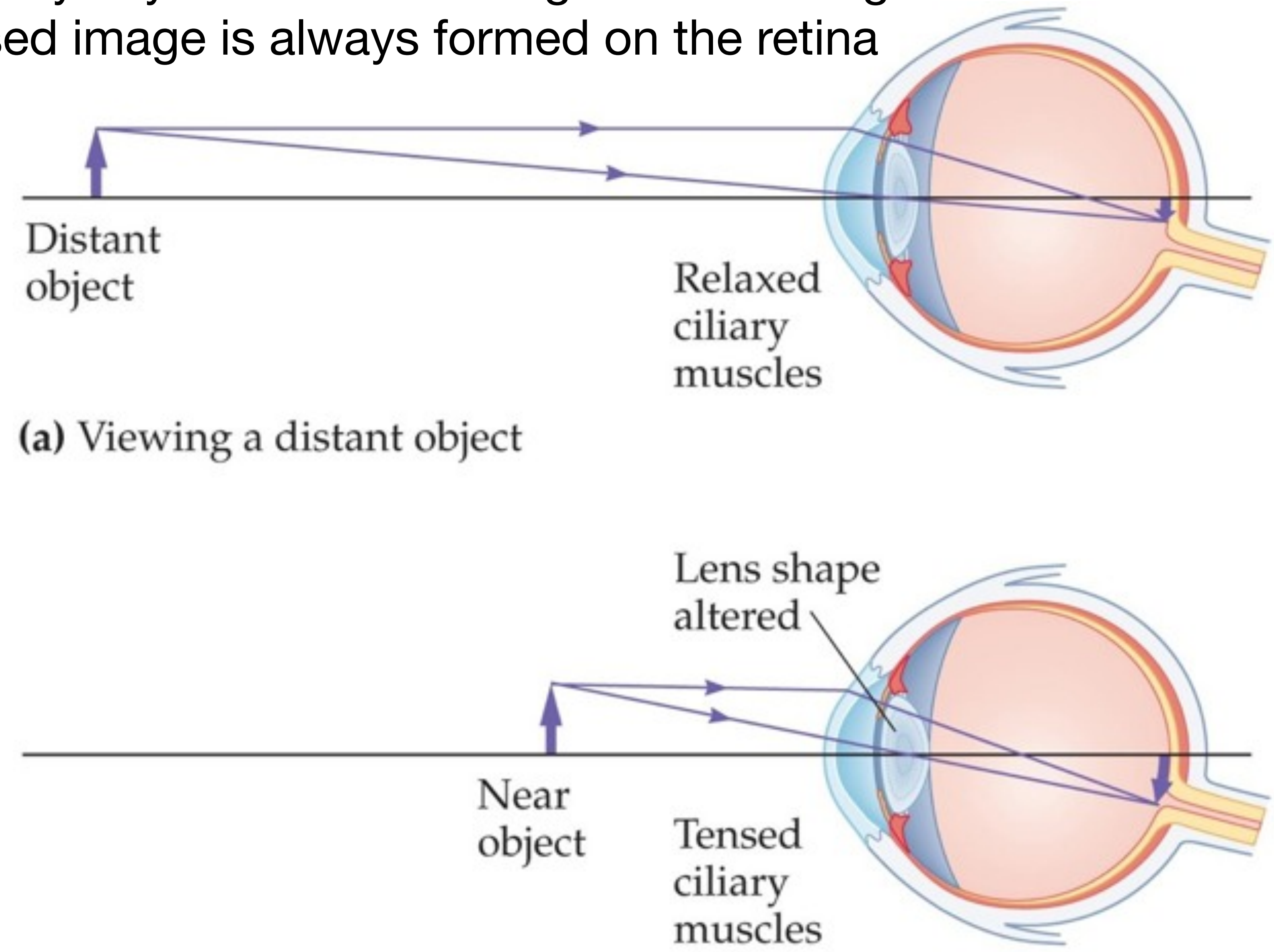
➜ most of the refraction is done by the cornea

➜ the lens can change its shape and thus its focal length



ocular accommodation

➜ the eye's lens can be deformed by tiny muscles to change its focal length
➜ this attempts to ensure a focused image is always formed on the retina
adjust **f** to keep **s′** constant



(a) Viewing a distant object

(b) Viewing a near object

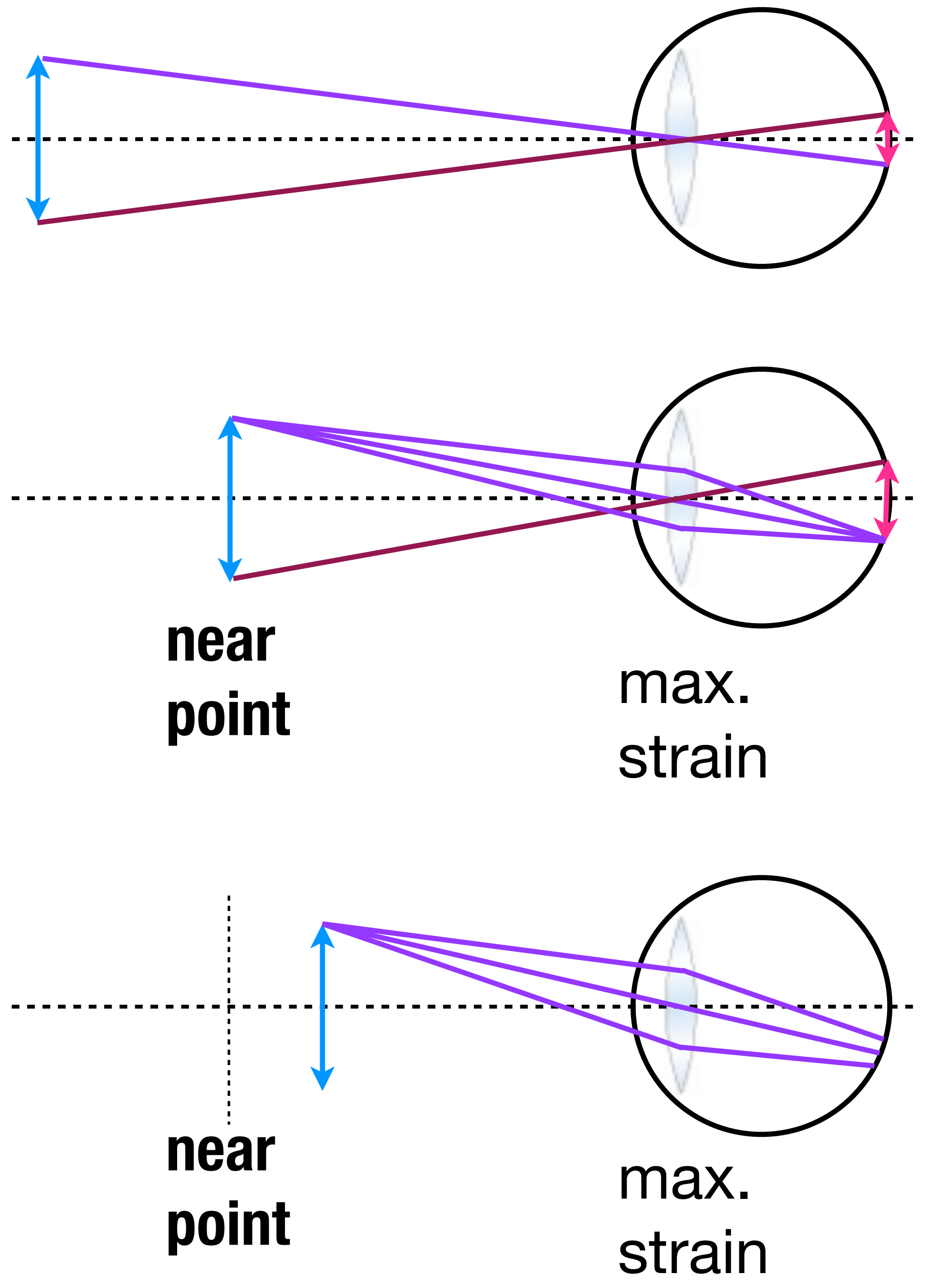
➜ the lens has a limited ability to adjust
- if the object is too close it cannot focus on the retina.

➜ the closest point is called the '**near point**'
- can vary between about 7 cm for teenagers up to 100 cm for people over 60.

near point

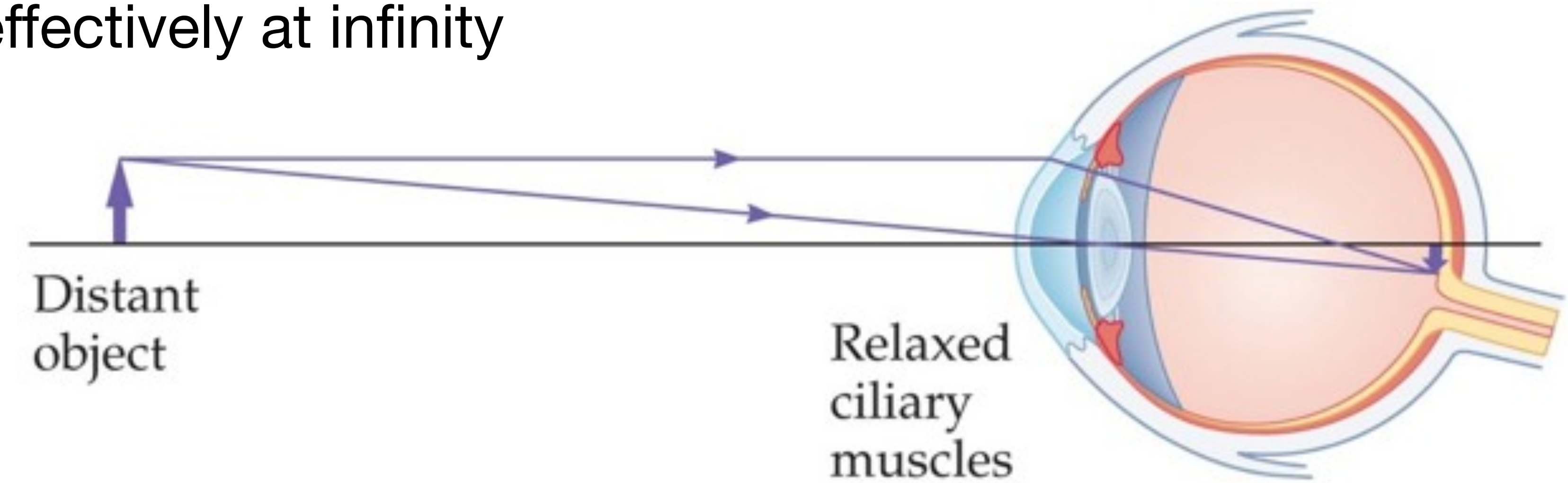
➜ we usually make an object seem larger by bringing it closer to our eye (this allows us to resolve more detail)

➜ however, at some point we reach the near point of our eye, and bringing the object any closer causes it to blur

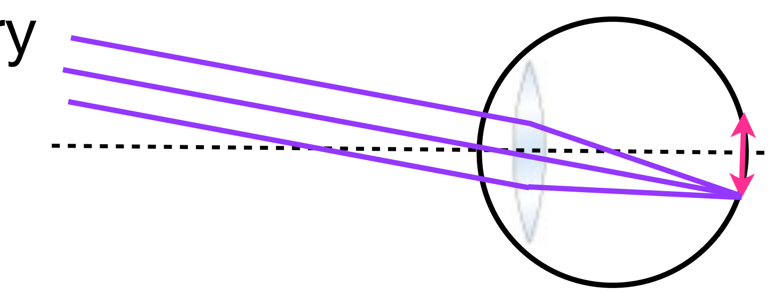


ocular accommodation

➜ the 'unaccommodated' eye (the lens under no 'strain') has what is called a **'far point'** - this is the maximum distance of an object focused on the retina
➜ for the 'common' eye this is effectively at infinity



(a) Viewing a distant object



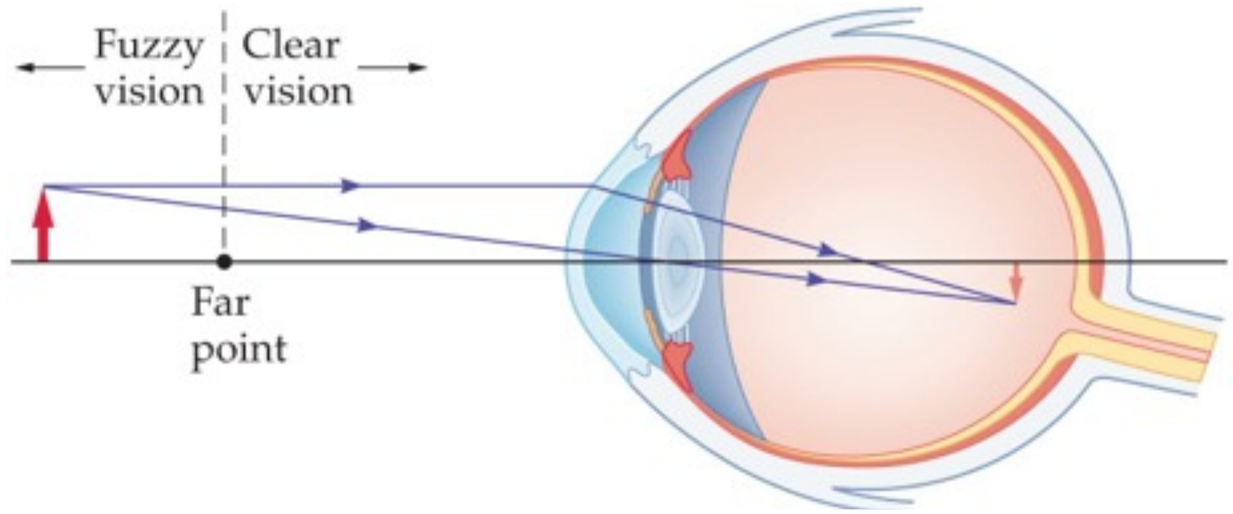
➜ a good proportion of eyes are not 'common' however
- they have abnormalities in shape or refracting mechanism

➜ two relatively common problems are myopia and hyperopia

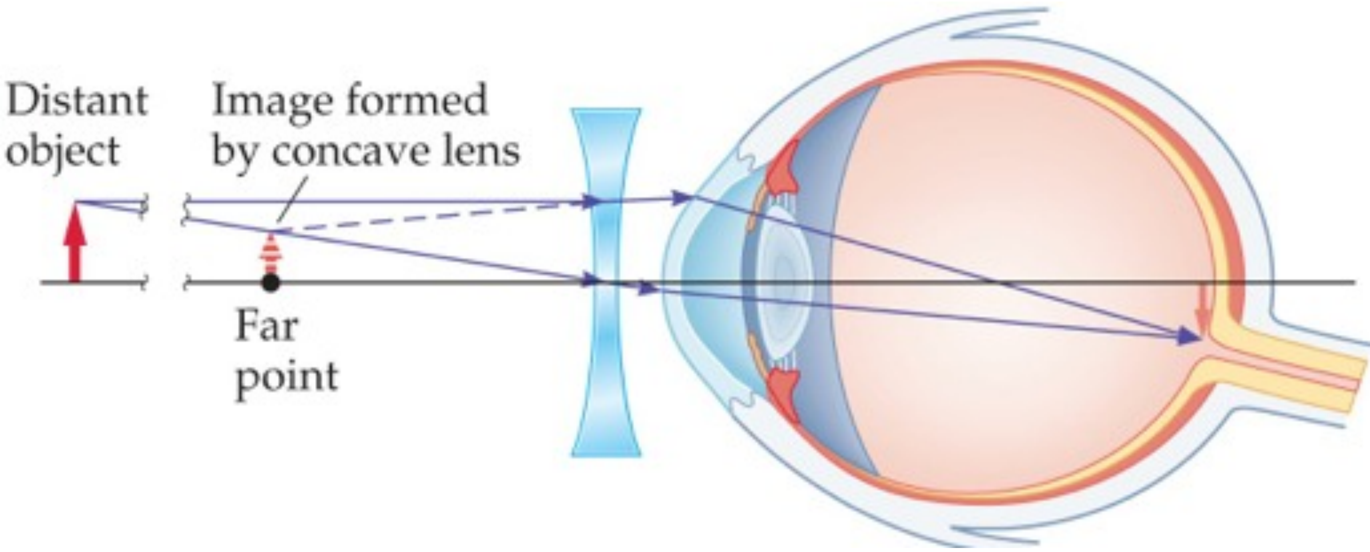
myopia - “nearsightedness”

➜ far point is not at infinity

➜ rays from objects past the far point get bent **too much** by the relaxed lens



➜ can be adjusted for by using a diverging lens to diverge the rays enough to place an intermediate image at the far point

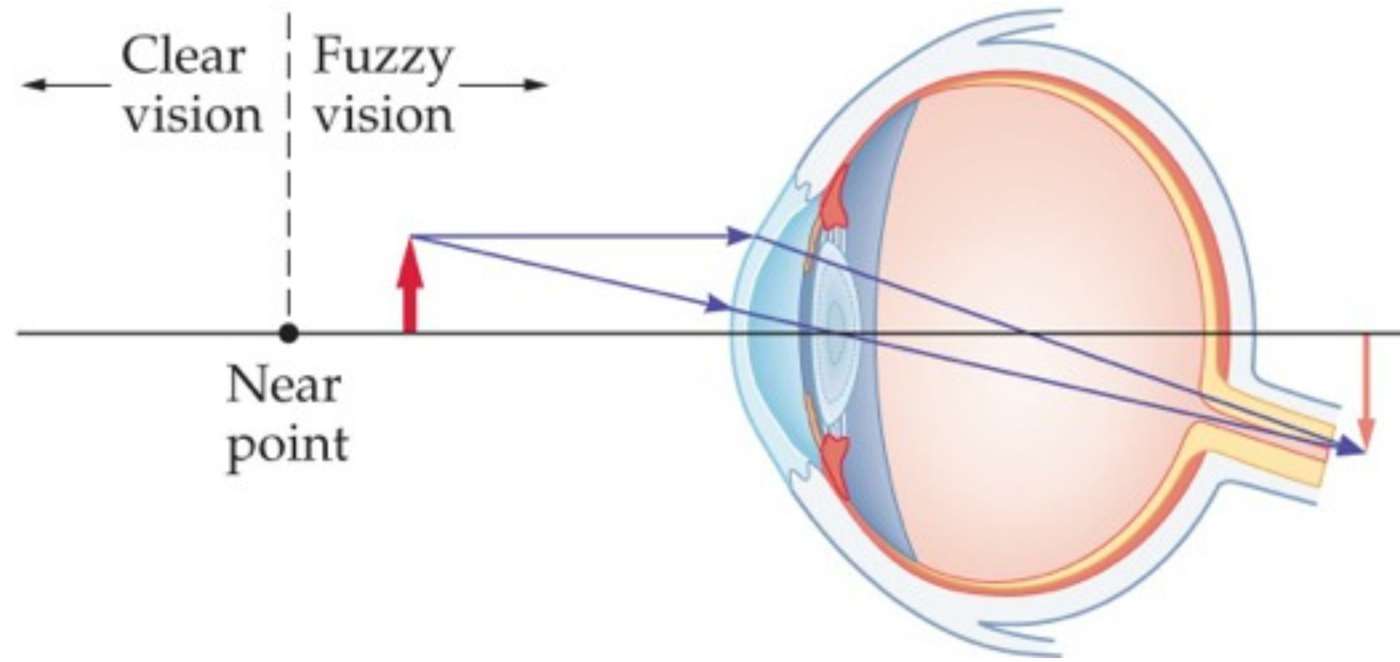


➜ if you’re nearsighted you should be wearing glasses/contacts featuring diverging lenses

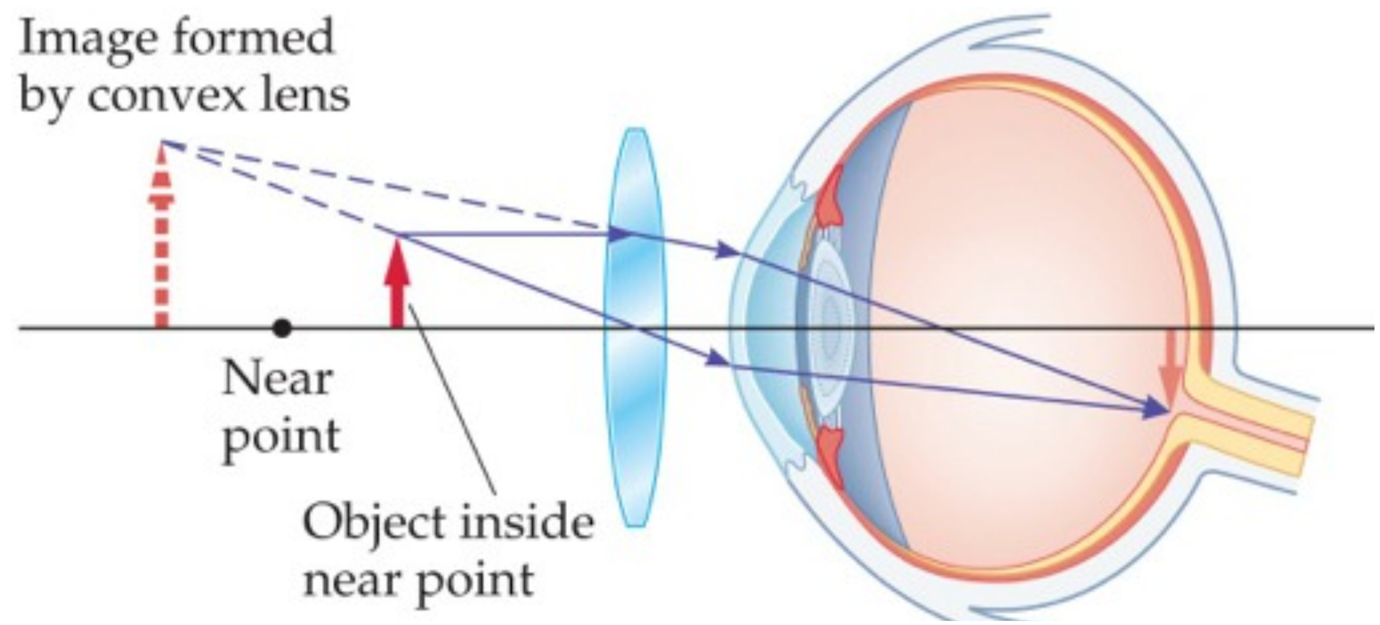
hyperopia - “farsightedness”

➜ near point is far away from the eye

➜ rays from objects inside the near point **can’t be bent enough** even by the lens under maximum strain



➜ can be adjusted for by using a converging lens to converge the rays enough to place an intermediate image outside the near point

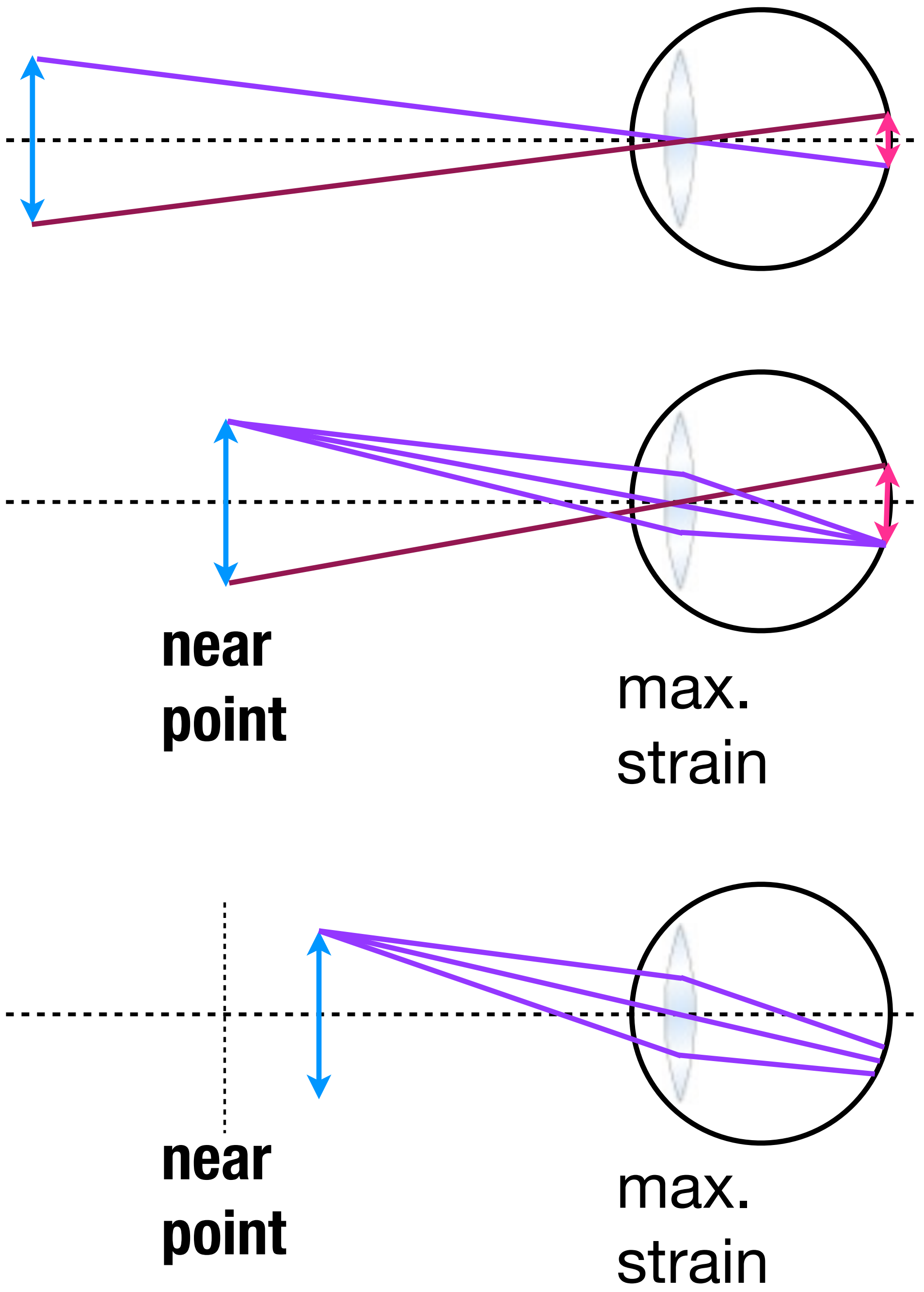


➜ if you’re farsighted you should be wearing glasses/contacts featuring converging lenses

magnifying glass

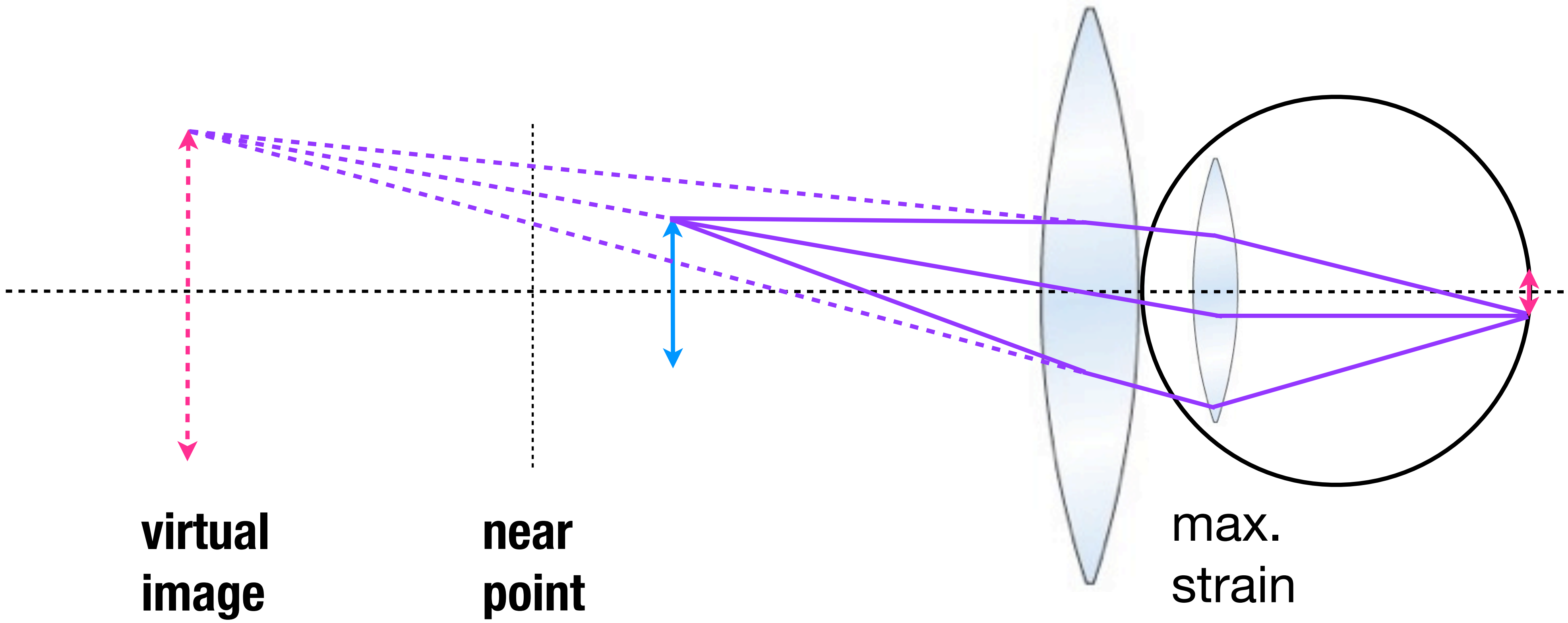
➜ we usually make an object seem larger by bringing it closer to our eye (this allows us to resolve more detail)

➜ however, at some point we reach the near point of our eye, and bringing the object any closer causes it to blur



magnifying glass

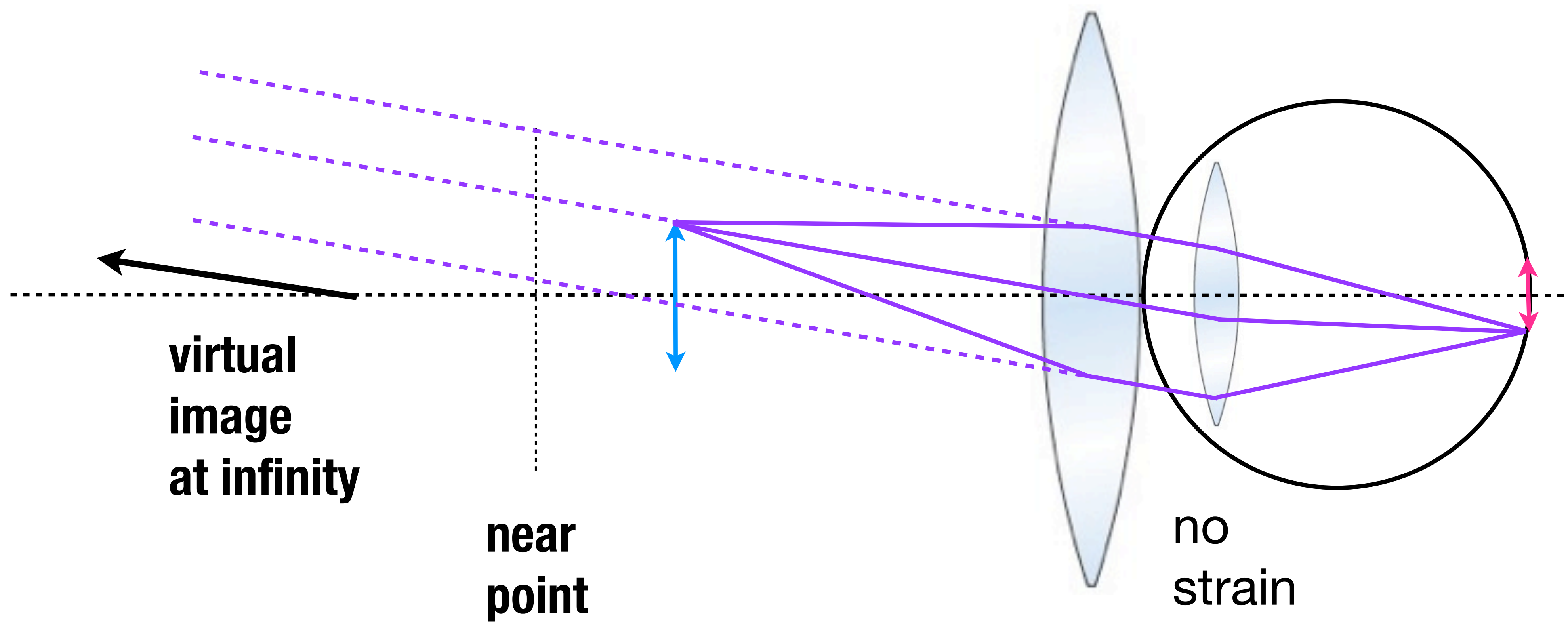
➜ we can bring the object closer than the near point if we bring in some external focusing power - a converging lens will do the trick



magnifying glass

➜ if we put the virtual image at the far point (infinity for most people), then the eye can be relaxed

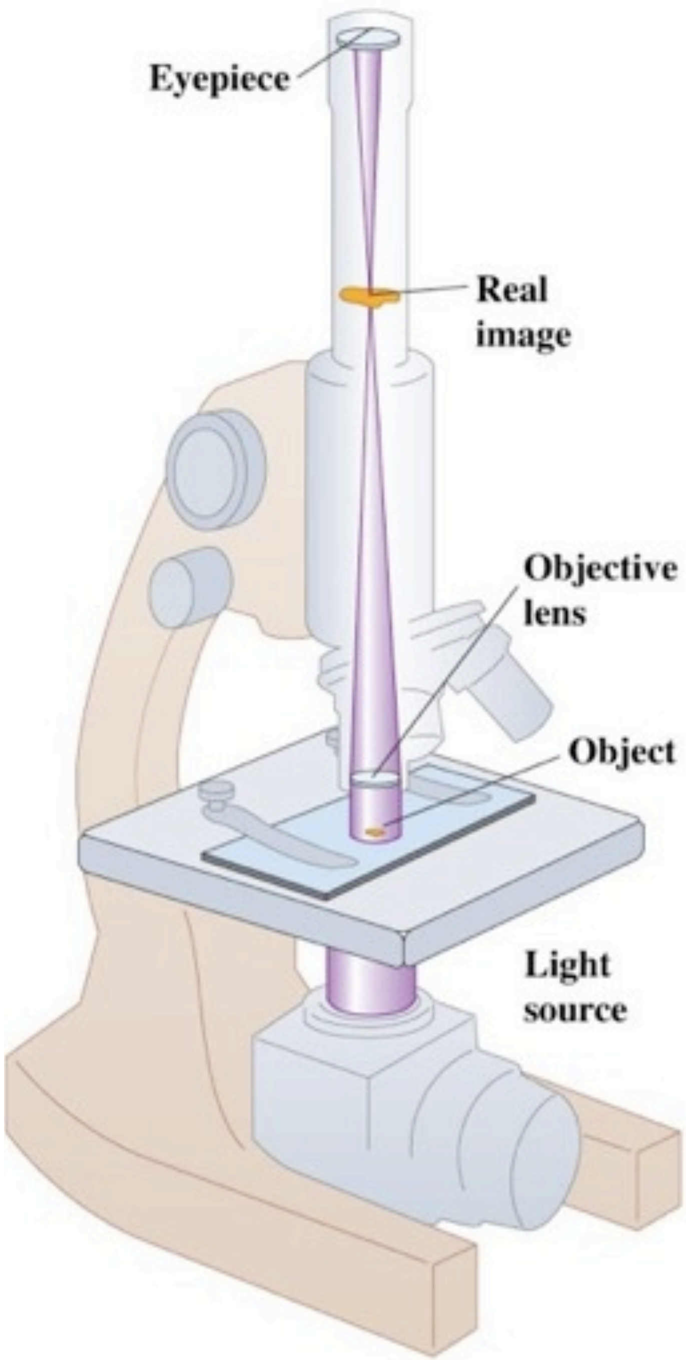
➜ achieved if the object is at the focal point of the magnifying lens

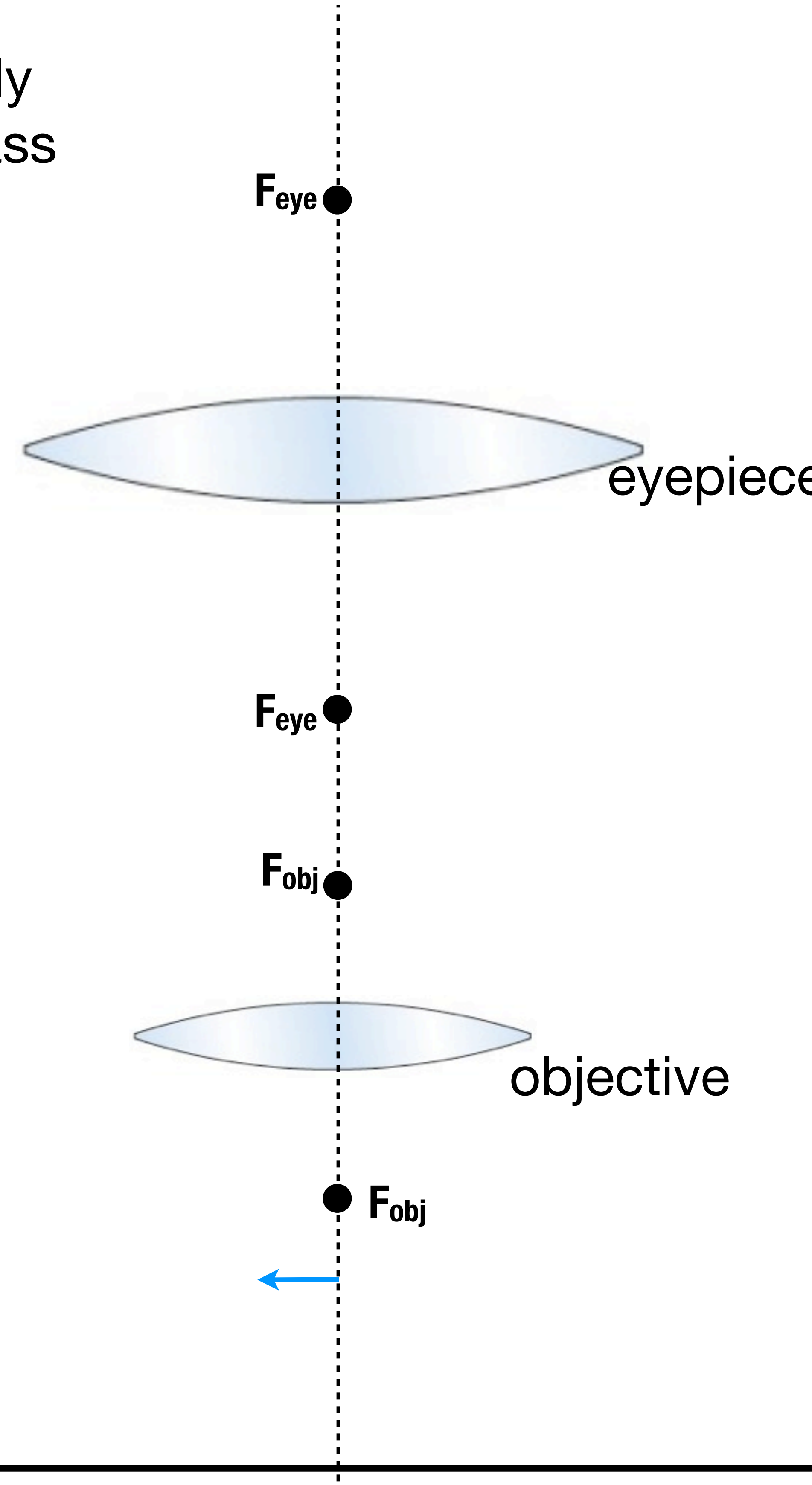


➜ see the textbook for a discussion of how much magnification we can get

the microscope

➜ physical limitations of lensmaking prevent really large magnifications with a simple magnifying glass - the microscope beats this by using two lenses

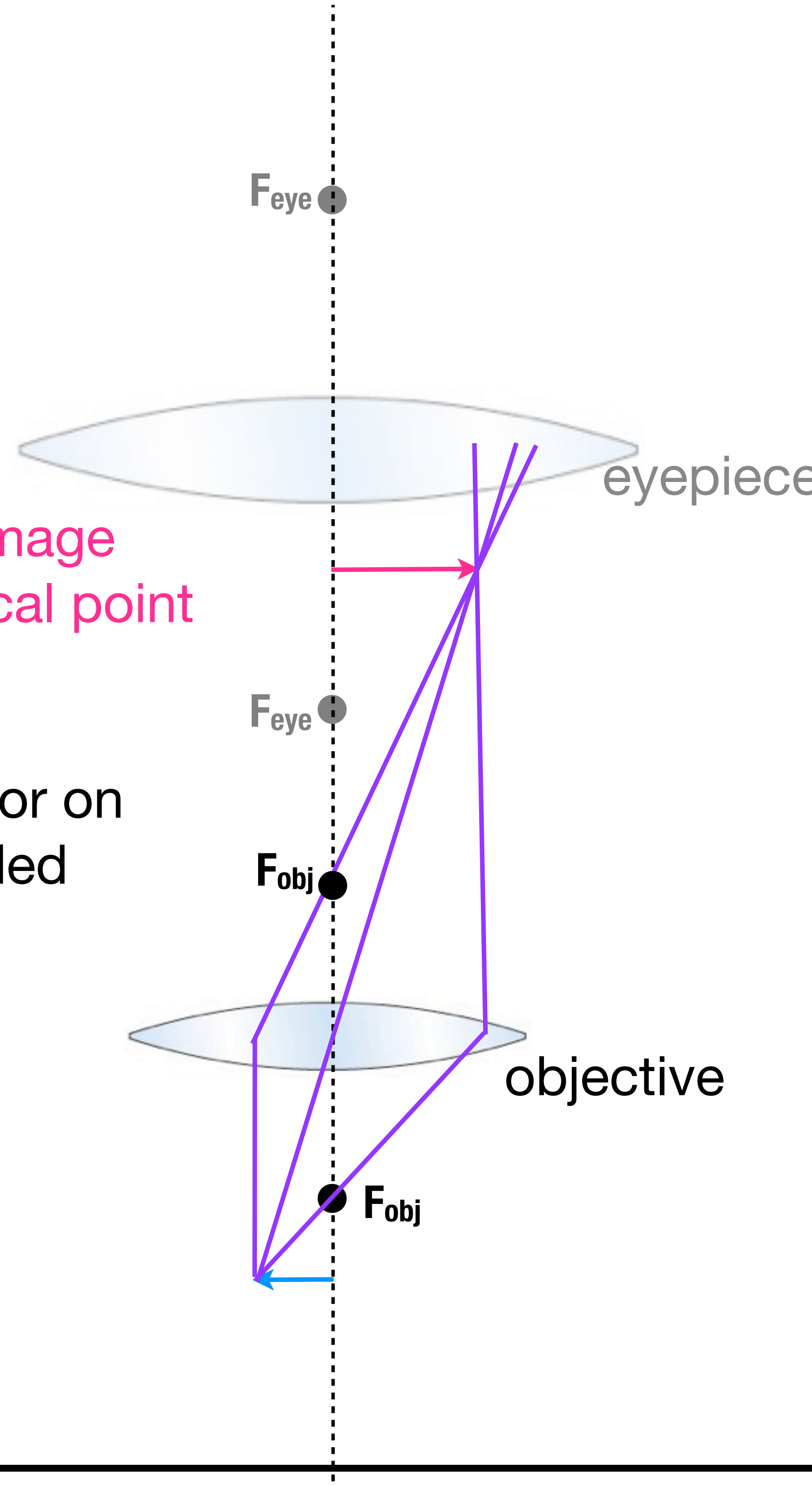




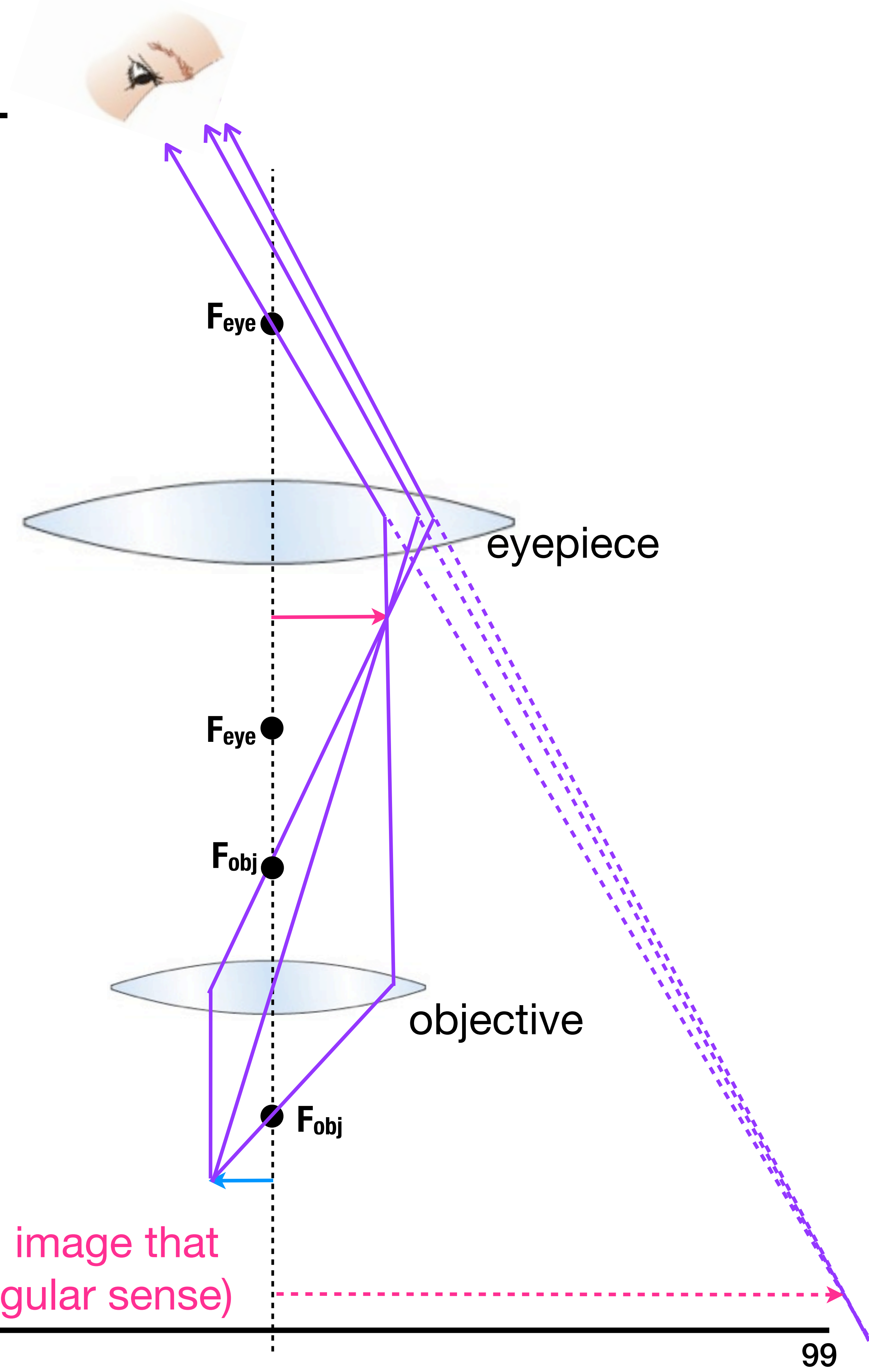
the microscope

the objective lens produces a real image of an object placed at or near its focal point

this image should be formed inside or on the focal point of a second lens, called the eyepiece

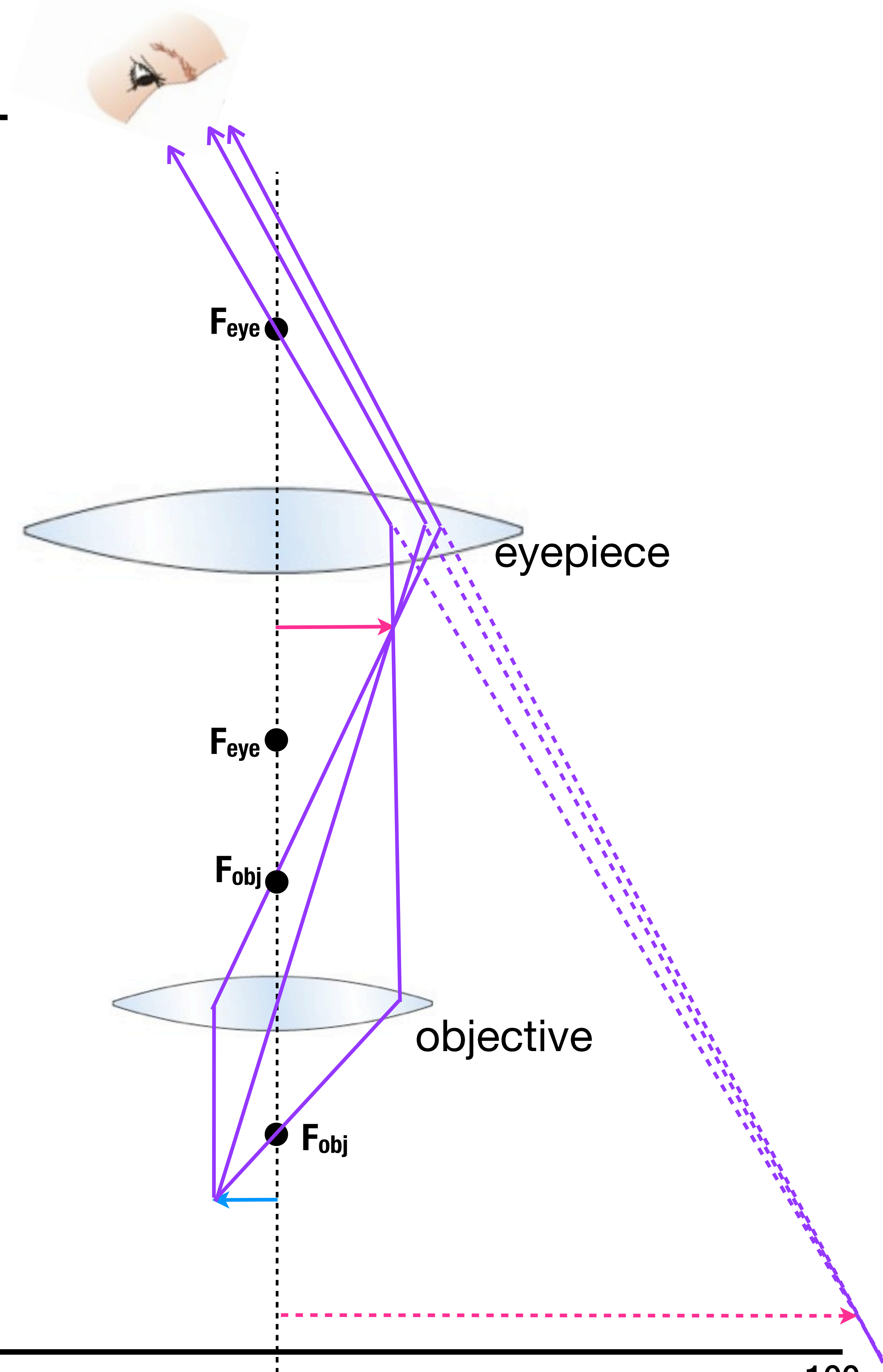


the microscope



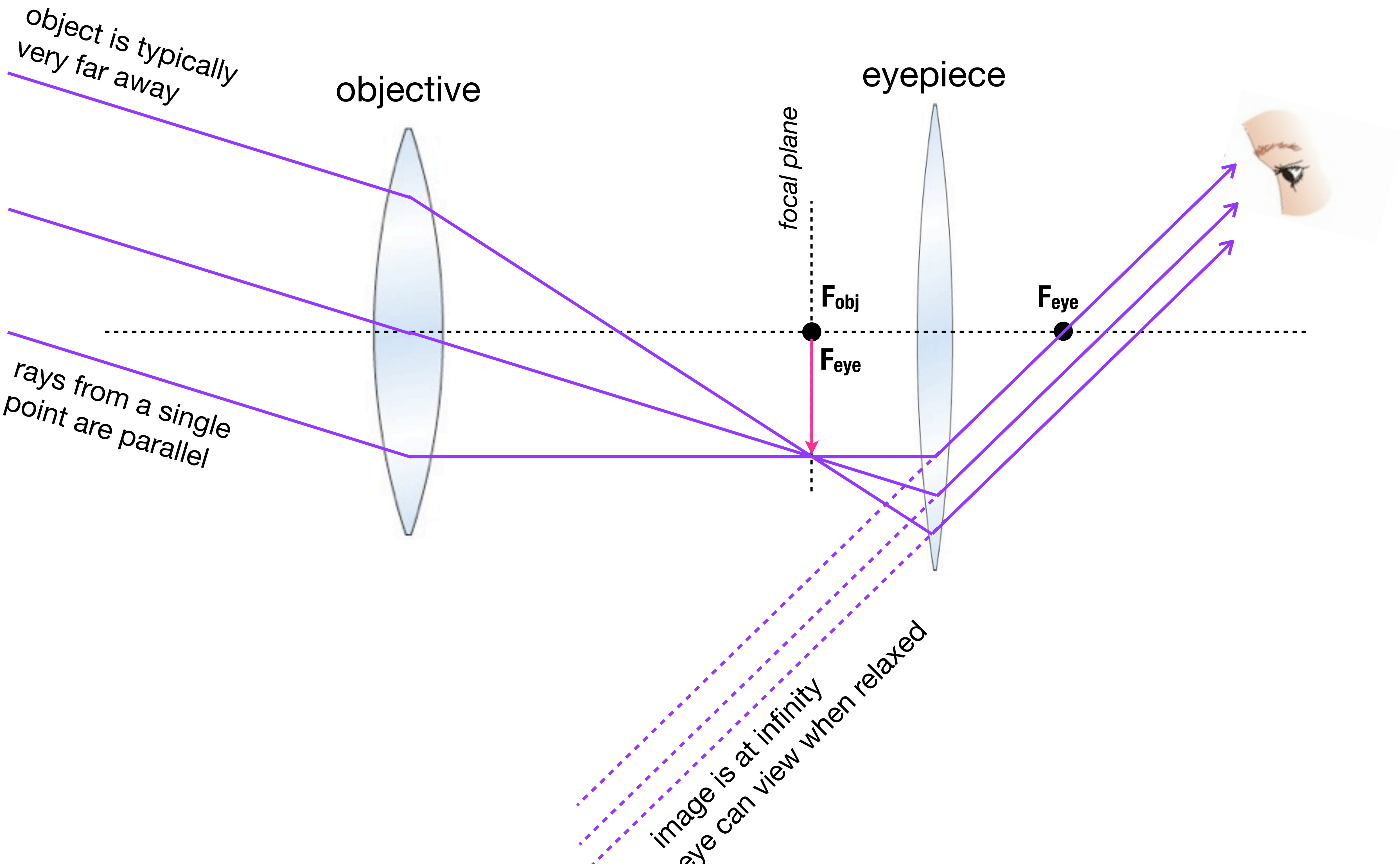
the eyepiece forms a virtual image that
is much magnified (in an angular sense)

the microscope

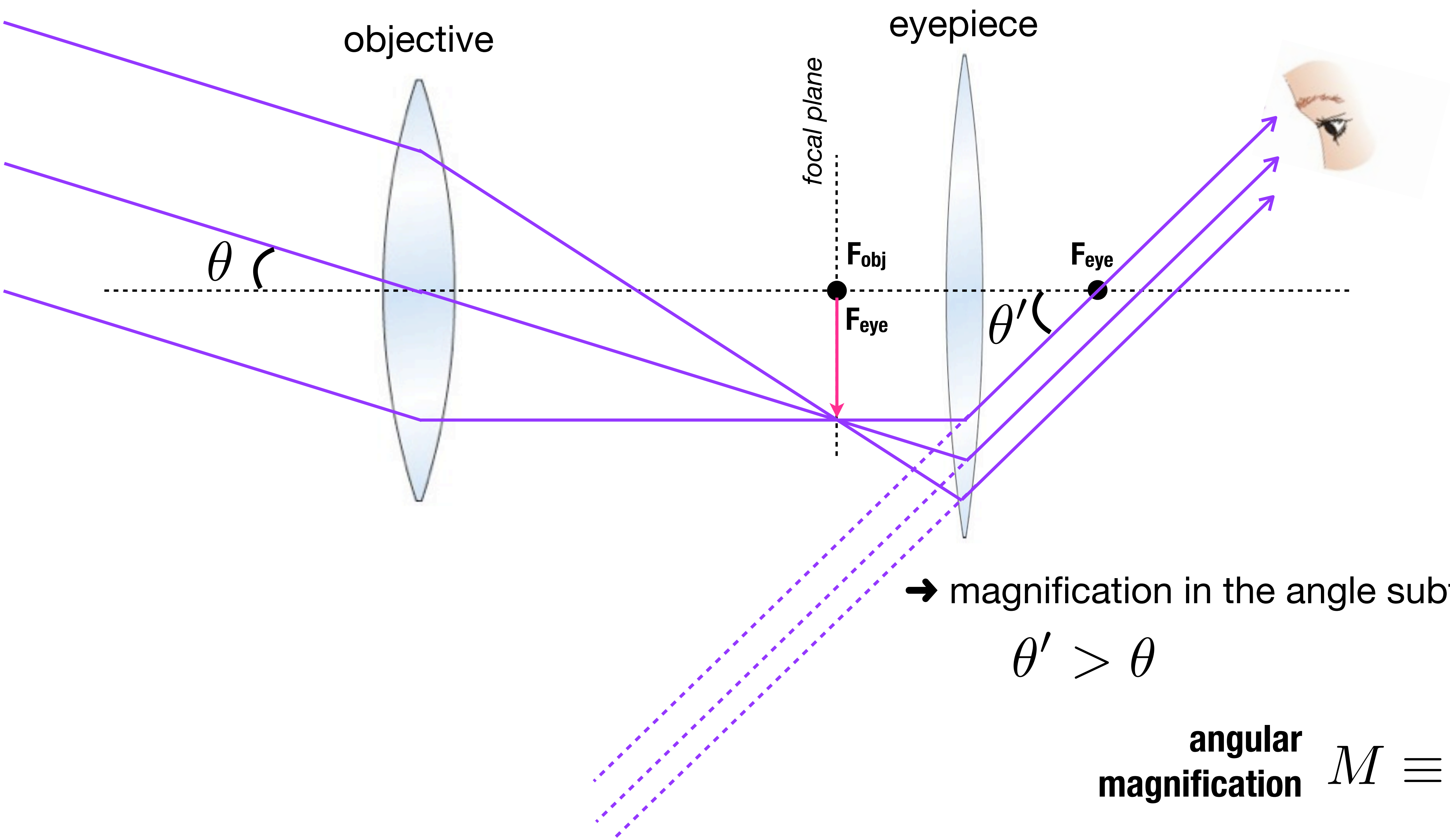


➜ see the textbook for a discussion of how much magnification we can get from a microscope

the telescope



the telescope

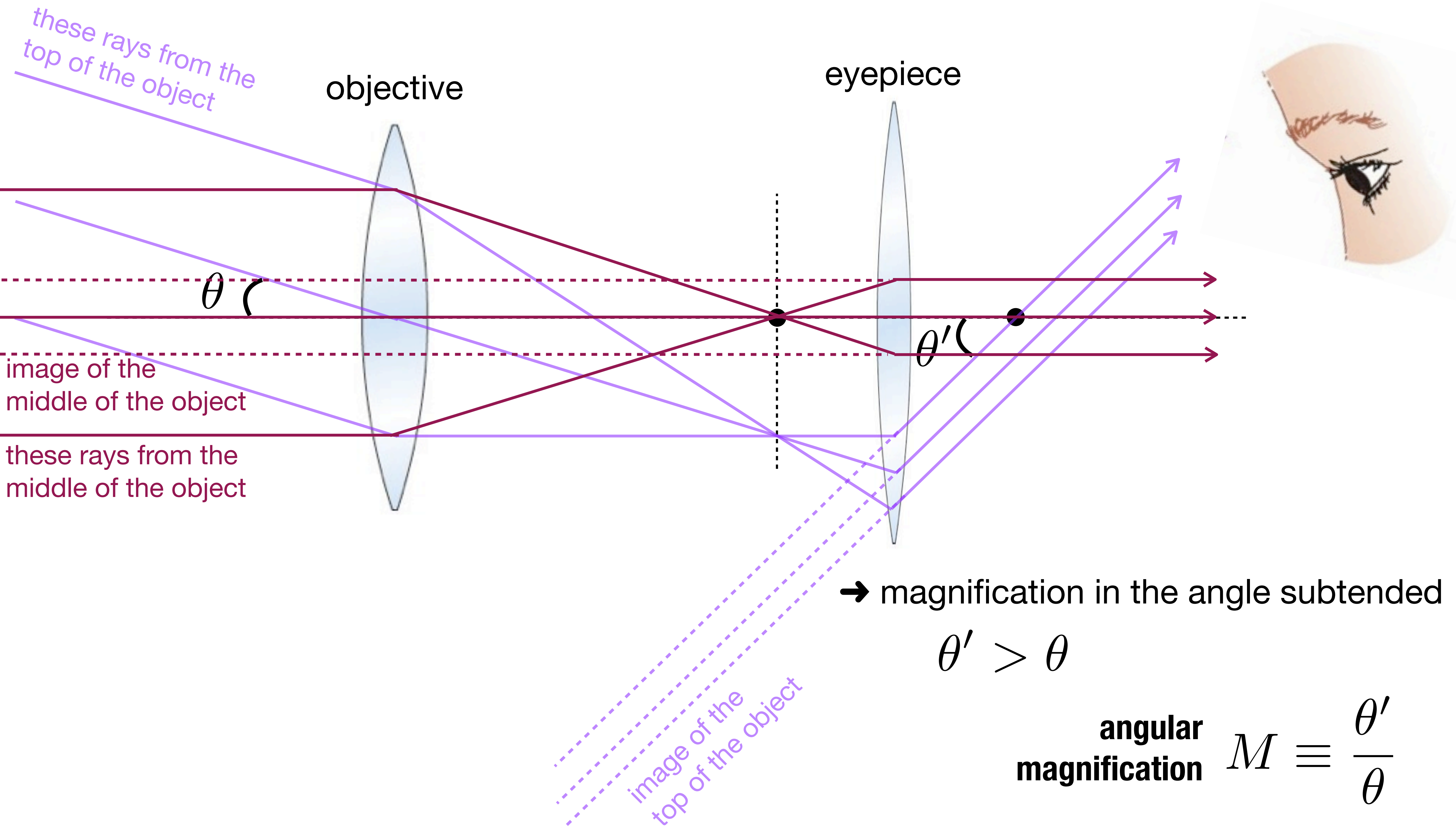


➜ magnification in the angle subtended

$$\theta' > \theta$$

angular magnification $$M \equiv \frac{\theta'}{\theta}$$

the telescope



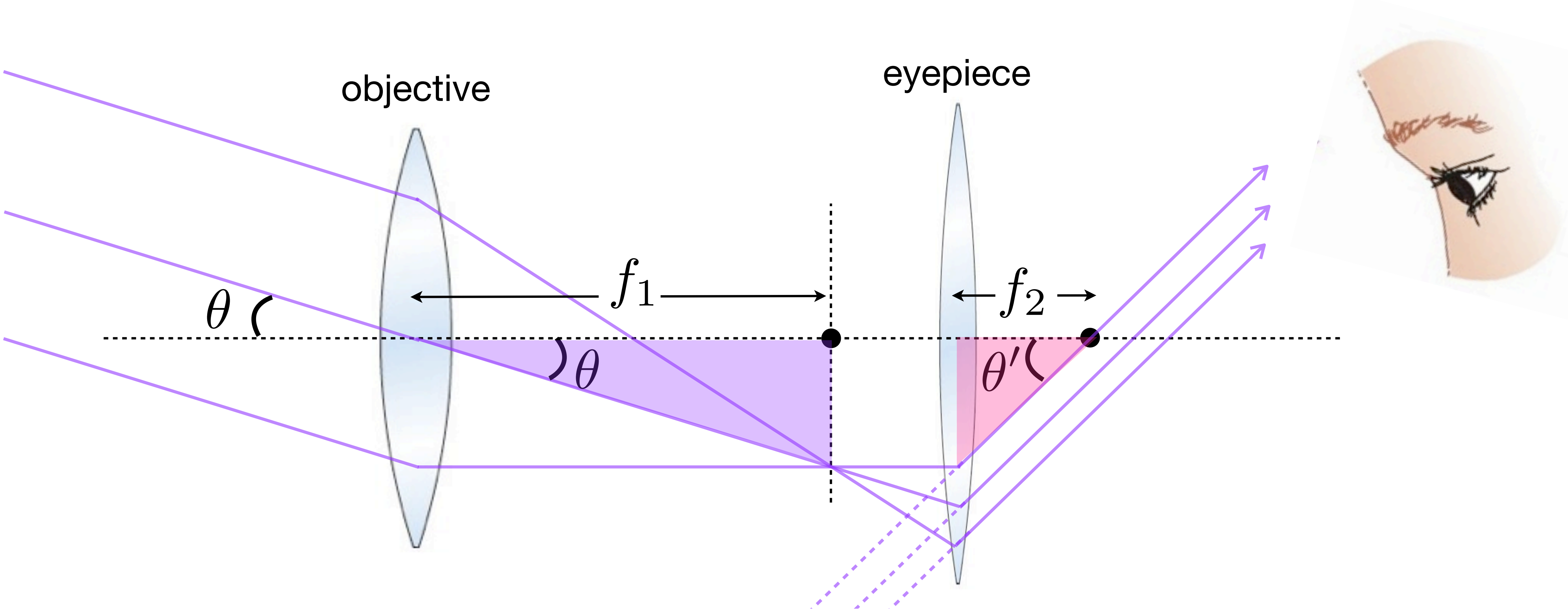
➜ magnification in the angle subtended

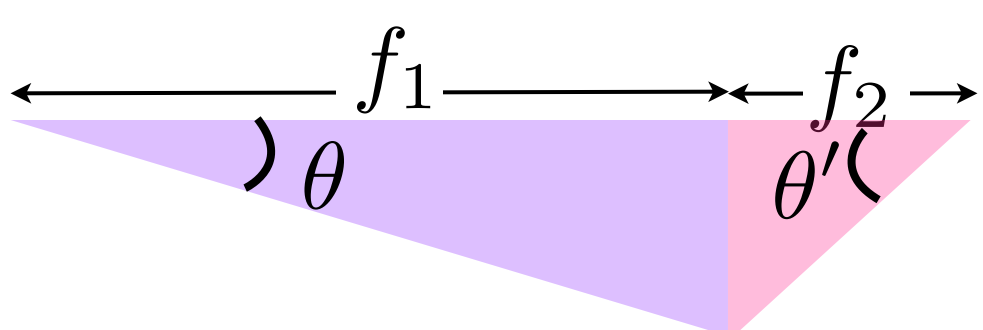
$$\theta' > \theta$$

angular magnification $$M \equiv \frac{\theta'}{\theta}$$

the telescope

angular magnification $M \equiv \dfrac{\theta'}{\theta}$





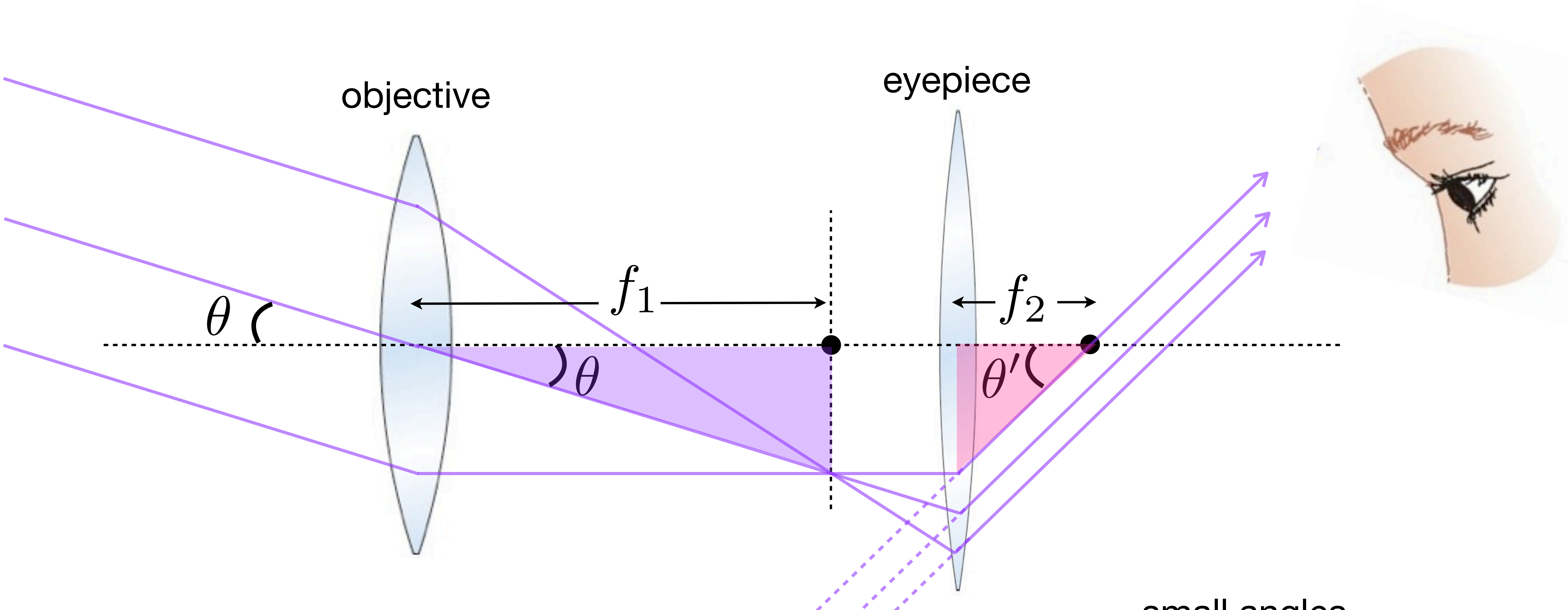
$$\tan\theta = \frac{y}{f_1}$$

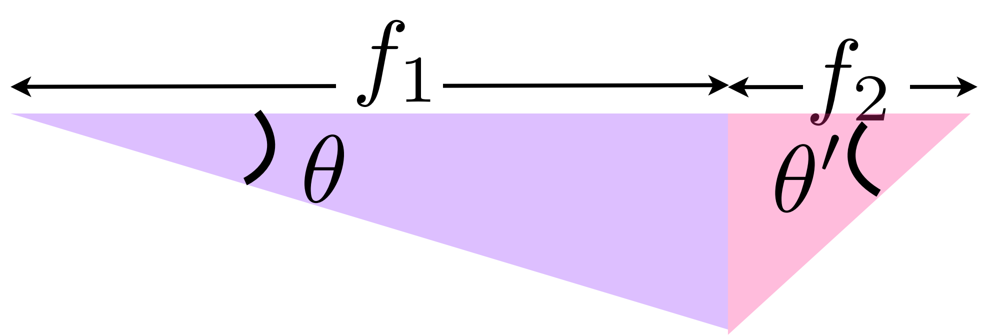
$$\tan\theta' = \frac{y}{f_2}$$

$$\frac{\tan\theta'}{\tan\theta} = \frac{f_1}{f_2}$$

the telescope

angular magnification $M \equiv \dfrac{\theta'}{\theta}$





$$\frac{\tan\theta'}{\tan\theta} = \frac{f_1}{f_2}$$

small angles
& inverted image

$$M = \frac{\theta'}{\theta} = -\frac{f_1}{f_2}$$

the telescope

➜ a very large refracting telescope has an objective lens of focal length 19.4m.

➜ suppose we want Jupiter viewed through the telescope to look the same size as the Moon looks to the naked eye, what focal length eyepiece lens do we require ?

diameter of Jupiter = 1.38×10^5 km
distance to Jupiter = 6.28×10^8 km

diameter of the Moon = 3.58×10^3 km
distance to the Moon = 3.84×10^5 km

$$M = \frac{\theta'}{\theta} = -\frac{f_1}{f_2}$$