## Chemical Equilibrium

Chapter 13

## CHEMICAL EQUILIBRIUM

## OBJECTIVES:

- Be able to apply stoichiometry to an equilibrium mixture to determine equilibrium concentrations of reactants and products
- Be able to write equilibrium constant expressions from a chemical equation
- Be able to obtain the equilibrium constant from the reaction composition at equilibrium
- Be able to apply LeChatlier's Principle to decide the effects of changing reaction condition(s) on the equilibrium concentrations of reactants and products

Figure 13.2: The changes in concentrations with time for the reaction:

$$
\mathrm{H}_{2} \mathrm{O}(g)+\mathrm{CO}(g) \rightleftharpoons \mathrm{H}_{2}(g)+\mathrm{CO}_{2}(g)
$$

when equimolar quantities of $\mathrm{H}_{2} \mathrm{O}(g)$ and $\mathrm{CO}(g)$ are mixed.


Figure 13.3: (a) $\mathrm{H}_{2} \mathrm{O}$ and CO are mixed in equal numbers and begin to react (b) to form $\mathrm{CO}_{2}$ and $\mathrm{H}_{2}$. After time has passed, equilibrium is reached (c) and the numbers of reactant and product molecules then remain constant over time (d).


## Chemical Equilibrium

-The state where the concentrations of all reactants and products remain constant with time.

- On the molecular level, there is frantic activity. Equilibrium is not static, but is a highly dynamic situation.

Figure 13.4: The changes with time in the rates of forward and reverse reactions for $\mathrm{H}_{2} \mathrm{O}(g)+\mathrm{CO}(g) \rightleftharpoons \mathrm{H}_{2}(g)+\mathrm{CO}_{2}(g)$ when equimolar quantities of $\mathrm{H}_{2} \mathrm{O}(g)$ and $\mathrm{CO}(g)$ are mixed. The rates do not change in the same way with time because the forward reaction has a much larger rate constant than the reverse reaction.


## The Law of Mass Action

-For

$$
\bullet j \mathrm{~A}+k \mathrm{~B} \leftrightarrow l \mathrm{C}+m \mathrm{D}
$$

-The law of mass action is represented by the equilibrium expression:

$$
K=\frac{[\mathrm{C}]^{l}[\mathrm{D}]^{m}}{[\mathrm{~A}]^{j}[\mathrm{~B}]^{k}}
$$

## Equilibrium Expression

- $4 \mathrm{NH}_{3}(g)+7 \mathrm{O}_{2}(g) \leftrightarrow 4 \mathrm{NO}_{2}(g)+6 \mathrm{H}_{2} \mathrm{O}(g)$

$$
K=\frac{\left[\mathrm{NO}_{2}\right]^{4}\left[\mathrm{H}_{2} \mathrm{O}\right]^{6}}{\left[\mathrm{NH}_{3}\right]^{4}\left[\mathrm{O}_{2}\right]^{7}}
$$

TABLE 13.1 Results of Three Experiments for the Reaction $\mathrm{N}_{2}(g)+$ $3 \mathrm{H}_{2}(g) \rightleftharpoons 2 \mathrm{NH}_{3}(g)$

| Experiment | Initial <br> Concentrations | Equilibrium Concentrations | $K=\frac{\left[\mathrm{NH}_{3}\right]^{2}}{\left[\mathrm{~N}_{2}\right]\left[\mathrm{H}_{2}\right]^{3}}$ |
| :---: | :---: | :---: | :---: |
| I | $\begin{aligned} & {\left[\mathrm{N}_{2}\right]_{0}=1.000 \mathrm{M}} \\ & {\left[\mathrm{H}_{2}\right]_{0}=1.000 \mathrm{M}} \\ & {\left[\mathrm{NH}_{3}\right]_{0}=0} \end{aligned}$ | $\begin{aligned} & {\left[\mathrm{N}_{2}\right]=0.921 \mathrm{M}} \\ & {\left[\mathrm{H}_{2}\right]=0.763 \mathrm{M}} \\ & {\left[\mathrm{NH}_{3}\right]=0.157 \mathrm{M}} \end{aligned}$ | $K=6.02 \times 10^{-2}$ |
| II | $\begin{aligned} & {\left[\mathrm{N}_{2}\right]_{0}=0} \\ & {\left[\mathrm{H}_{2} \mathrm{l}_{0}=0\right.} \\ & {\left[\mathrm{NH}_{3}\right]_{0}=1.000 \mathrm{M}} \end{aligned}$ | $\begin{aligned} & {\left[\mathrm{N}_{2}\right]=0.399 \mathrm{M}} \\ & {\left[\mathrm{H}_{2}\right]=1.197 \mathrm{M}} \\ & {\left[\mathrm{NH}_{3}\right]=0.203 \mathrm{M}} \end{aligned}$ | $K=6.02 \times 10^{-2}$ |
| III | $\begin{aligned} & {\left[\mathrm{N}_{2}\right]_{0}=2.00 \mathrm{M}} \\ & {\left[\mathrm{H}_{2} \mathrm{l}_{\mathrm{l}}=1.00 \mathrm{M}\right.} \\ & \mid \mathrm{NH}_{3} \mathrm{l}_{0}=3.00 \mathrm{M} \end{aligned}$ | $\begin{aligned} & {\left[\mathrm{N}_{2}\right]=2.59 \mathrm{M}} \\ & {\left[\mathrm{H}_{2}\right]=2.77 \mathrm{M}} \\ & \left\|\mathrm{NH}_{3}\right\|=1.82 \mathrm{M} \end{aligned}$ | $K=6.02 \times 10^{-2}$ |

## Applying Stoichiometry to a Reaction Mixture in Equilibrium

1. Must know the balanced equation for the reaction
2. Must know the molar concentrations of reactants and products after reaction has reached equilibrium in order to calculate the equilibrium constant.
3. If we know concentrations of all but one of the components we can calculate the concentration of the missing component, if we know $K$.

Figure 13.5: A concentration profile for the reaction $\mathrm{N}_{2}(g)+3 \mathrm{H}_{2}(g) \rightleftharpoons 2 \mathrm{NH}_{3}(g)$ when only $\mathrm{N}_{2}(g)$ and $\mathrm{H}_{2}(g)$ are mixed initially.


## A Problem to Consider

- Applying Stoichiometry to an Equilibrium Mixture.
- Suppose we place $1.000 \mathrm{~mol} \mathrm{~N}_{2}$ and 3.000 mol $\mathrm{H}_{2}$ in a reaction vessel at $450{ }^{\circ} \mathrm{C}$ and 10.0 atmospheres of pressure. The reaction is

$$
\mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NH}_{3}(\mathrm{~g})
$$

- What is the composition of the equilibrium mixture if it contains $0.080 \mathrm{~mol} \mathrm{NH}_{3}$ ?


## A Problem to Consider

- Using the information given, set up the following table.

$$
\mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NH}_{3}(\mathrm{~g})
$$

| Starting | 1.000 | 3.000 | 0 |
| :---: | :---: | :---: | :---: |
| Change | -x | -3 x | +2 x |
| Equilibrium | $1.000-\mathrm{x}$ | $3.000-3 \mathrm{x}$ | $\mathbf{2 x}=\mathbf{0 . 0 8 0} \mathbf{~ m o l}$ |

- The equilibrium amount of $\mathrm{NH}_{3}$ was given as 0.080 mol . Therefore, $2 \mathrm{x}=\mathbf{0 . 0 8 0} \mathrm{mol} \mathrm{NH}_{3}$ ( $\mathrm{x}=0.040 \mathrm{~mol}$ ).

Figure 15.4: The concentration of a gas at a given temperature is proportional to the pressure.

$$
\mathbf{P}=(\mathbf{n} / \mathbf{V}) \mathbf{R T}
$$

## The Equilibrium Constant, $\mathbf{K}_{\mathbf{p}}$

- If we express a gas-phase equilibria in terms of partial pressures, we obtain $K_{p}$.
- Consider the reaction below.
$\mathrm{CO}(\mathrm{g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftarrows \mathrm{CH}_{4}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$
- The equilibrium-constant expression in terms of partial pressures becomes:

$$
\mathrm{K}_{\mathrm{p}}=\frac{\boldsymbol{P}_{\mathrm{CH}_{4}} \boldsymbol{P}_{\mathrm{H}_{2} \mathrm{O}}}{\boldsymbol{P}_{\mathrm{CO}} \boldsymbol{P}_{\mathrm{H}_{2}}{ }^{3}}
$$

## The Equilibrium Constant, $\mathbf{K}_{\mathbf{p}}$

- In general, the numerical value of $\mathrm{K}_{\mathrm{p}}$ differs from that of $\mathrm{K}_{\mathrm{c}}$.
- From the relationship $n / V=P / R T$, we can show that

$$
\mathbf{K}_{\mathrm{p}}=\mathbf{K}_{\mathrm{c}}(\mathbf{R T})^{\Delta \mathrm{n}}
$$

where $\Delta \mathrm{n}$ is the sum of the moles of gaseous products in a reaction minus the sum of the moles of gaseous reactants.
$\mathbf{K}_{\mathbf{P}}=\left(\mathbf{M}_{\mathbf{p}} / \mathbf{M}_{\mathbf{R}}\right)(\mathbf{R T})^{\text {(moles Products - moles Reactants) }}$

$$
\boldsymbol{K} v . \boldsymbol{K}_{\mathrm{p}}
$$

- For $j \mathrm{~A}+k \mathrm{~B} \leftrightarrow l \mathrm{C}+m \mathrm{D}$
- $K_{\mathrm{p}}=K_{C}(R T)^{\Delta n}$
$\square \quad \Delta n=$ sum of coefficients of gaseous products minus sum of coefficients of gaseous reactants.


## A Problem to Consider

- Consider the reaction

$$
2 \mathrm{SO}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{SO}_{3}(\mathrm{~g})
$$

- Since

$$
\begin{gathered}
\mathbf{K}_{\mathbf{p}}=\mathbf{K}_{\mathbf{c}}(\mathbf{R T})^{\Delta \mathrm{n}} \quad \Delta \mathrm{n}=(2-3) \\
K_{p}=2.8 \times 10^{2}\left(0.08206 \frac{\mathrm{~L} \cdot a t \mathrm{~m}}{\mathrm{~mol} \cdot \mathrm{~K}} \times 1273 \mathrm{~K}\right)^{-1}=2.7
\end{gathered}
$$

## A Problem to Consider

- Consider the reaction

$$
2 \mathrm{SO}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{SO}_{3}(\mathrm{~g})
$$

- $\mathrm{K}_{\mathrm{c}}$ for the reaction is $2.8 \times 10^{2}$ at $1000^{\circ} \mathrm{C}$. Calculate $\mathrm{K}_{\mathrm{p}}$ for the reaction at this temperature.


## Equilibrium Constant for the Sum of Reactions

- Similar to the method of combining reactions that we saw using Hess's law in Chapter 6, we can combine equilibrium reactions whose $K_{c}$ values are known to obtain $\mathrm{K}_{\mathrm{c}}$ for the overall reaction.
- With Hess's law, when we reversed reactions or multiplied them prior to adding them together, we had to manipulate the $\Delta \mathrm{H}$ 's values to reflect what we had done.
- The rules are a bit different for manipulating $\mathrm{K}_{\mathrm{c}}$.


## Equilibrium Constant for the Sum of Reactions

- If you reverse a reaction, invert the value of $\mathrm{K}_{\mathrm{c}}$.

2. If you multiply each of the coefficients in an equation by the same factor $(2,3, \ldots)$, raise $\mathbf{K}_{\mathbf{c}}$ to the same power ( $2,3, \ldots$ ).
3. If you divide each coefficient in an equation by the same factor $(2,3, \ldots)$, take the corresponding root of $K_{c}$ (i.e., square root, cube root, ...).
4. When you finally combine (that is, add) the individual equations together, take the product of the equilibrium constants to obtain the overall $\mathbf{K}_{\mathrm{c}}$.

$$
\begin{aligned}
& \begin{array}{ll}
\mathrm{A}+2 \mathrm{~B} \rightleftharpoons 3 \mathrm{C} & \frac{[\mathrm{C}]^{3}}{[\mathrm{~A}][\mathrm{B}]^{2}}=\mathrm{K}_{\mathrm{F}} \\
3 \mathrm{C} \rightleftharpoons \mathbf{A}+2 \mathbf{B} & \frac{[\mathrm{~A}][\mathrm{B}]^{2}}{[\mathrm{C}]^{3}}=\mathbf{K}_{\mathrm{R}}=\mathbf{1} / \mathbf{K}_{\mathbf{F}} \\
2 \mathrm{~A}+4 \mathrm{~B} \rightleftharpoons 6 \mathrm{C} & \frac{[\mathrm{C}]^{6}}{[\mathrm{~A}]^{2}[\mathrm{~B}]^{4}}=\mathbf{K}_{\mathrm{F}}{ }^{2}
\end{array} \\
& \mathrm{H}_{2} \mathrm{O}+\mathrm{HCN} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{CN}^{-} \quad \mathrm{K}_{\mathrm{A}}=\underline{\mathrm{H}}_{3} \underline{\mathrm{O}^{+}}[\mathrm{CNN}] \\
& {\left[\mathrm{H}_{2} \mathrm{O}\right][\mathrm{HCN}]} \\
& \underline{H}_{2} \mathrm{O}+\mathrm{CN}^{-}-\mathrm{HCN}+\mathrm{OH}^{-} \quad \mathrm{K}_{\mathrm{B}}=\frac{[\mathrm{HCN}]\left[\mathrm{OH}^{-}\right.}{\left[\mathrm{H}_{2} \mathrm{O}\right][\mathrm{CN}-]} \\
& 2 \mathrm{H}_{2} \mathrm{O}+\mathrm{HCN}+\mathrm{CN}^{-} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{HCN}+\mathrm{CN}^{-}+\mathrm{OH}^{-} \\
& {\left[\mathrm{H}_{3} \mathrm{O}^{+} \amalg \mathrm{HCN} \| \mathrm{CN}-\amalg \mathrm{OH} \mathrm{~L}=\mathrm{K}_{\mathrm{A}} \cdot \mathrm{~K}_{\mathrm{B}}\right.} \\
& {\left[\mathrm{H}_{2} \mathrm{O}\right]^{2}[\mathrm{HCN}][\mathrm{CN}-]} \\
& {\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right]=\mathrm{K}_{\mathrm{W}}}
\end{aligned}
$$

## Equilibrium Constant for the

## Sum of Reactions

- To combine equations (1) and (2) to obtain equation (3), we must first reverse equation (2). When we do we must also take the reciprocal of its $\mathrm{K}_{\mathrm{c}}$ value.
(1)

$$
\mathrm{N}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NO}(\mathrm{~g}) \quad \mathrm{K}_{\mathrm{c}}=4.1 \times 10^{-31}
$$

(2) $\quad \mathrm{N}_{2} \mathrm{O}(\mathrm{g}) \rightleftarrows \mathrm{N}_{2}(\mathrm{~g})+\frac{1}{2} \mathrm{O}_{2}(\mathrm{~g}) \quad \mathrm{K}_{\mathrm{c}}=\frac{1}{2.4 \times 10^{-18}}$
(3) $\quad \mathrm{N}_{2} \mathrm{O}(\mathrm{g})+\frac{1}{2} \mathrm{O}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NO}(\mathrm{g})$
$\mathrm{K}_{\mathrm{c}}($ overall $)=\left(4.1 \times 10^{-31}\right) \times\left(\frac{1}{2.4 \times 10^{-18}}\right)=1.7 \times 10^{-13}$

## Equilibrium Constant for the Sum of Reactions

- Similar to the method of combining reactions that we saw using Hess's law in Chapter 6, we can combine equilibrium reactions whose $K_{c}$ values are known to obtain $K_{c}$ for the overall reaction.
- With Hess's law, when we reversed reactions or multiplied them prior to adding them together, we had to manipulate the $\Delta \mathrm{H}$ 's values to reflect what we had done.
- The rules are a bit different for manipulating $\mathrm{K}_{\mathrm{c}}$.


## The Equilibrium Constant for the Sum of Reactions

When two reactions are added together to give an overall reaction, the new equilibrium constant for this reaction is the product of the equilibrium constants for the original reactions.

$$
\begin{aligned}
& \mathbf{A}+\mathbf{B} \neq \mathbf{C}+\mathbf{D} \quad \mathbf{K}_{1} \\
& \mathbf{A}+\mathbf{E} \neq \mathbf{F} \\
& \mathbf{2} \mathbf{A + \mathbf { B } + \mathbf { E } \neq \mathbf { C } + \mathbf { D } + \mathbf { F }} \mathbf{K _ { 2 }} \mathbf{K}=\mathbf{K}_{1} \mathbf{K}_{2}
\end{aligned}
$$

## Heterogeneous Equilibria

-. . . are equilibria that involve more than one phase.

$$
\begin{gathered}
\cdot \mathrm{CaCO}_{3}(s) \leftrightarrow \mathrm{CaO}(s)+\mathrm{CO}_{2}(g) \\
\cdot K=\left[\mathrm{CO}_{2}\right]
\end{gathered}
$$

-The position of a heterogeneous equilibrium does not depend on the amounts of pure solids or liquids present.

## Heterogeneous Equilibria

- A heterogeneous equilibrium is an equilibrium that involves reactants and products in more than one phase.
- The equilibrium of a heterogeneous system is unaffected by the amounts of pure solids or liquids present, as long as some of each is present.
- The concentrations of pure solids and liquids are always considered to be " 1 " and therefore, do not appear in the equilibrium expression.

Figure 13.6: The nosition of the equilibrium $\mathrm{CaCO}_{3}(s) \rightarrow$ $\mathrm{CaO}(s)+\mathrm{CO}_{2}(g)$ does not depend on the amounts of $\mathrm{CaCO}_{3}(s)$ and $\mathrm{CaO}(s)$ present.


## Heterogeneous Equilibria

- Consider the reaction below.

$$
\mathrm{C}(\mathrm{~s})+\mathrm{H}_{2} \mathrm{O}(\mathrm{~g}) \rightleftharpoons \mathrm{CO}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g})
$$

- The equilibrium-constant expression contains terms for only those species in the homogeneous gas phase $\ldots \mathrm{H}_{2} \mathrm{O}, \mathrm{CO}$, and $\mathrm{H}_{2}$.


Hydrated copper (II) sulfate on the left. Water applied to anhydrous copper (II) sulfate, on the right, forms the hydrated compound.


## Reaction Quotient

- . . . helps to determine the direction of the move toward equilibrium.
-The law of mass action is applied with initial concentrations.

Figure 13.7:
(a) A physical analogy illustrating the difference between
thermodynamic and kinetic stabilities. (b) The reactants H2 and O 2 have a strong tendency to form H2O.


## Predicting the Direction of

## Reaction

- How could we predict the direction in which a reaction at non-equilibrium conditions will shift to reestablish equilibrium?
- To answer this question, substitute the current concentrations into the reaction quotient expression and compare it to $\mathrm{K}_{\mathrm{c}}$.
- The reaction quotient, $\mathbf{Q}_{\mathrm{c}}$, is an expression that has the same form as the equilibriumconstant expression but whose concentrations are not necessarily at equilibrium.


## Predicting the Direction of

## Reaction

- For the general reaction

$$
\mathrm{aA}+\mathrm{bB} \rightleftarrows \mathrm{cC}+\mathrm{dD}
$$

the $Q_{c}$ expression would be:

$$
\mathbf{Q}_{\mathrm{c}}=\frac{[\mathbf{C}]_{i}^{\mathrm{c}}[\mathbf{D}]_{i}^{\mathrm{d}}}{[\mathrm{~A}]_{i}^{\mathrm{a}}[\mathbf{B}]_{i}^{\mathrm{b}}}
$$

## Predicting the Direction of

## Reaction

- For the general reaction

$$
\mathbf{a A}+\mathrm{bB} \rightleftarrows \mathrm{cC}+\mathrm{dD}
$$

- If $\mathrm{Q}_{\mathrm{c}}>\mathrm{K}_{\mathrm{c}}$, the reaction will shift left...toward reactants.
- If $\mathrm{Q}_{\mathrm{c}}<\mathrm{K}_{\mathrm{c}}$, the reaction will shift right... toward products.
- If $\mathrm{Q}_{\mathrm{c}}=\mathrm{K}_{\mathrm{c}}$, then the reaction is at equilibrium.


## A Problem to Consider

- Consider the following equilibrium.

$$
\mathbf{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NH}_{3}(\mathrm{~g})
$$

- A 50.0 L vessel contains $1.00 \mathrm{~mol} \mathrm{~N}_{2}, 3.00 \mathrm{~mol} \mathrm{H}_{2}$, and $0.500 \mathrm{~mol} \mathrm{NH}_{3}$. In which direction (toward reactants or toward products) will the system shift to reestablish equilibrium at $400^{\circ} \mathrm{C}$ ?
- $\mathrm{K}_{\mathrm{c}}$ for the reaction at $400^{\circ} \mathrm{C}$ is 0.500 .


## A Problem to Consider

- First, calculate concentrations from moles of substances.

$$
\begin{aligned}
& \mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NH}_{3}(\mathrm{~g}) \\
& \frac{1.00 \mathrm{~mol}}{50.0 \mathrm{~L}} \frac{3.00 \mathrm{~mol}}{50.0 \mathrm{~L}}
\end{aligned}
$$

## A Problem to Consider

- First, calculate concentrations from moles of substances.

$$
\begin{array}{lc}
\mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) & 2 \mathrm{NH}_{3}(\mathrm{~g}) \\
0.0200 \mathrm{M} & 0.0600 \mathrm{M}
\end{array}
$$

- The $\mathrm{Q}_{\mathrm{c}}$ expression for the system would be:

$$
\mathbf{Q}_{\mathrm{c}}=\frac{\left[\mathrm{NH}_{3}\right]^{2}}{\left[\mathbf{N}_{2}\right]\left[\mathrm{H}_{2}\right]^{3}}
$$

## A Problem to Consider

- First, calculate concentrations from moles of substances.

$$
\mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NH}_{3}(\mathrm{~g})
$$

$\begin{array}{lll}0.0200 & \text { M } & 0.0600\end{array} \quad 0.0100 \mathrm{M}$

- Substituting these concentrations into the reaction quotient gives:

$$
Q_{c}=\frac{(0.0100)^{2}}{(0.0200)(0.0600)^{3}}=23.1
$$

## A Problem to Consider

- First, calculate concentrations from moles of substances.

$$
\begin{aligned}
& \mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NH}_{3}(\mathrm{~g}) \\
& 0.0200 \mathrm{M} \\
& 0.0600 \mathrm{M}
\end{aligned}
$$

- Because $Q_{c}=23.1$ is greater than $K_{c}=0.500$, the reaction will go to the left (toward reactants) as it approaches equilibrium.


## Calculating Equilibrium Concentrations

- Once you have determined the equilibrium constant for a reaction, you can use it to calculate the concentrations of substances in the equilibrium mixture.


## Solving Equilibrium Problems

- 1. Balance the equation.
- 2. Write the equilibrium expression.
- 3. List the initial concentrations.
- 4. Calculate $Q$ and determine the shift to equilibrium.


## Solving Equilibrium Problems <br> (continued)

- 5. Define equilibrium concentrations.
- 6. Substitute equilibrium concentrations into equilibrium expression and solve.
- 7. Check calculated concentrations by calculating $K$.


## Calculating Equilibrium Concentrations

- For example, consider the following equilibrium.

$$
\mathrm{CO}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftarrows \mathrm{CH}_{4}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{~g})
$$

- Suppose a gaseous mixture contained 0.30 mol $\mathrm{CO}, 0.10 \mathrm{~mol} \mathrm{H}_{2}, 0.020 \mathrm{~mol} \mathrm{H}_{2} \mathrm{O}$, and an unknown amount of $\mathrm{CH}_{4}$ per liter.
- What is the concentration of $\mathrm{CH}_{4}$ in this mixture? The equilibrium constant $\mathrm{K}_{\mathrm{c}}$ equals 3.92.


## Calculating Equilibrium Concentrations

- First, calculate concentrations from moles of substances.

$$
\begin{array}{ccc}
\mathrm{CO}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) & \mathrm{CH}_{4}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{~g}) \\
\mathbf{0 . 3 0} \mathbf{M} & \mathbf{0 . 1 0} \mathbf{M} & \text { ?? }
\end{array}
$$

- Substituting the known concentrations and the value of $\mathrm{K}_{\mathrm{c}}$ gives:

$$
3.92=\frac{\left[\mathrm{CH}_{4}\right](0.020 \mathrm{M})}{(0.30 \mathrm{M})(0.10 \mathrm{M})^{3}}
$$

## Calculating Equilibrium Concentrations

- First, calculate concentrations from moles of substances.

| $\mathrm{CO}(\mathrm{g})+3 \mathrm{H}_{2}(\mathrm{~g})$ | $\mathrm{CH}_{4}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |  |
| :---: | :---: | :---: |
| $\mathbf{0 . 3 0} \mathbf{M}$ | $\mathbf{0 . 1 0} \mathbf{M}$ | $? ?$ |$\quad \mathbf{0 . 0 2 0} \mathbf{M}$

- You can now solve for $\left[\mathrm{CH}_{4}\right]$.

$$
\left[\mathrm{CH}_{4}\right]=\frac{(3.92)(0.30 \mathrm{M})(0.10 \mathrm{M})^{3}}{(0.020 \mathrm{M})}=0.059
$$

- The concentration of $\mathrm{CH}_{4}$ in the mixture is $\mathbf{0 . 0 5 9} \mathbf{~ m o l} / \mathrm{L}$.


## Calculating Equilibrium Concentrations

- First, calculate concentrations from moles of substances.

| $\mathrm{CO}(\mathrm{g})+3 \mathrm{H}_{2}(\mathrm{~g})$ | $\mathrm{CH}_{4}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |  |
| :--- | :--- | :--- |
| $\mathbf{0 . 3 0} \mathbf{M}$ | $\mathbf{0 . 1 0} \mathbf{M}$ | $? ?$ |

- The equilibrium-constant expression is:

$$
\mathrm{K}_{\mathrm{c}}=\frac{\left[\mathrm{CH}_{4}\right]\left[\mathrm{H}_{2} \mathrm{O}\right]}{[\mathrm{CO}]\left[\mathrm{H}_{2}\right]^{3}}
$$

## Calculating Equilibrium Concentrations

- Consider the following equilibrium.

$$
\mathrm{CO}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{~g}) \rightleftarrows \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g})
$$

- Suppose you start with 1.000 mol each of carbon monoxide and water in a 50.0 L container. Calculate the molarity of each substance in the equilibrium mixture at $1000{ }^{\circ} \mathrm{C}$.
- $\mathrm{K}_{\mathrm{c}}$ for the reaction is 0.58 at $1000^{\circ} \mathrm{C}$.


## Calculating Equilibrium Concentrations

- First, calculate the initial molarities of CO and $\mathrm{H}_{2} \mathrm{O}$.

$$
\mathrm{CO}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{~g}) \rightleftarrows \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g})
$$

$1.000 \mathrm{~mol} \quad 1.000 \mathrm{~mol}$

$$
50.0 \mathrm{~L} \quad 50.0 \mathrm{~L}
$$

## Calculating Equilibrium Concentrations

- First, calculate the initial molarities of CO and $\mathrm{H}_{2} \mathrm{O}$.

$$
\begin{array}{ccc}
\mathrm{CO}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{~g}) & \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g}) \\
\mathbf{0 . 0 2 0 0} \mathbf{M} & \mathbf{0 . 0 2 0} \mathbf{M} & \mathbf{0} \mathbf{M}
\end{array} \mathbf{0} \mathbf{M} .
$$

- The starting concentrations of the products are 0 .
- We must now set up a table of concentrations (starting, change, and equilibrium expressions in x ).


## Calculating Equilibrium Concentrations

- Let $\mathbf{x}$ be the moles per liter of product formed.

| $\mathrm{CO}(\mathrm{g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |  |  |  | $\longleftrightarrow \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g})$ |
| :---: | :---: | :---: | :---: | :---: |
| Starting | 0.0200 | 0.0200 |  |  |
| Change | -x | -x |  |  |
| Equilibrium | $0.0200-\mathrm{x}$ | $0.0200-\mathrm{x}$ |  |  |$\quad$| 0 | 0 |
| :---: | :---: | :---: |
| +x | +x |
| x | x |

- The equilibrium-constant expression is:

$$
\mathbf{K}_{\mathrm{c}}=\frac{\left[\mathrm{CO}_{2}\right]\left[\mathrm{H}_{2}\right]}{[\mathrm{CO}]\left[\mathrm{H}_{2} \mathrm{O}\right]}
$$

## Calculating Equilibrium Concentrations

- Solving for $\mathbf{x}$.

| $\mathrm{CO}(\mathrm{g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |  |  |  | $\longleftrightarrow \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g})$ |
| :---: | :---: | :---: | :---: | :---: |
| Starting | 0.0200 | 0.0200 |  |  |
| Change | -x | -x |  |  |
| Equilibrium | $0.0200-\mathrm{x}$ | $0.0200-\mathrm{x}$ |  |  |$\quad$| 0 | 0 |
| :---: | :---: | :---: |
| +x | +x |
| x | x |

- Substituting the values for equilibrium concentrations, we get:
$(x)(x)$

$$
0.58=\frac{(X)(X)}{(0.0200-x)(0.0200-x)}
$$

## Calculating Equilibrium Concentrations

- Solving for $\mathbf{x}$.

| $\mathrm{CO}(\mathrm{g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |  |  | $\rightleftarrows \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g})$ |  |
| :---: | :---: | :---: | :---: | :---: |
| Starting | 0.0200 | 0.0200 | 0 | 0 |
| Change | -x | -x | +x | + x |
| Equilibrium | 0.0200-x | 0.0200-x | x | x |
| - Or: |  | $8=$ |  |  |

## Calculating Equilibrium Concentrations

- Solving for $\mathbf{x}$.

| $\mathrm{CO}(\mathrm{g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |  |  |  | $\rightleftarrows \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g})$ |
| :---: | :---: | :---: | :---: | :---: |
| Starting | 0.0200 | 0.0200 |  |  |
| Change | -x | -x |  |  |
| Equilibrium | $0.0200-\mathrm{x}$ | $0.0200-\mathrm{x}$ |  |  |$\quad$| 0 | 0 |
| :---: | :---: | :---: |
| +x | +x |
| x | x |

- Taking the square root of both sides we get:

$$
0.76=\frac{x}{(0.0200-x)}
$$

## Calculating Equilibrium Concentrations

- Solving for $\mathbf{x}$.

| $\mathrm{CO}(\mathrm{g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |  |  |  | $\rightleftarrows \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g})$ |
| :---: | :---: | :---: | :---: | :---: |
| Starting | 0.0200 | 0.0200 |  |  |
| Change | -x | -x |  |  |
| Equilibrium | $0.0200-\mathrm{x}$ | $0.0200-\mathrm{x}$ |  |  |$\quad$| 0 | 0 |
| :---: | :---: | :---: |
| +x | +x |
| x | x |

- Rearranging to solve for x gives:

$$
x=\frac{0.0200 \times 0.76}{1.76}=0.0086
$$

## Calculating Equilibrium Concentrations

- Solving for equilibrium concentrations.

| $\mathrm{CO}(\mathrm{g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |  |  |  | $\mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2}(\mathrm{~g})$ |
| :---: | :---: | :---: | :---: | :---: |
| Starting | 0.0200 | 0.0200 |  |  |
| Change | -x | -x |  |  |
| Equilibrium | $0.0200-\mathrm{x}$ | $0.0200-\mathrm{x}$ |  |  |$\quad$| 0 | 0 |
| :---: | :---: | :---: |
| +x | +x |
| x | x |

- If you substitute for $x$ in the last line of the table you obtain the following equilibrium concentrations.

$$
\begin{array}{ll}
0.0114 \text { M CO } & 0.0086 \mathrm{M} \mathrm{CO}_{2} \\
0.0114 \mathrm{M} \mathrm{H}_{2} \mathrm{O} & 0.0086 \mathrm{M} \mathrm{H}_{2}
\end{array}
$$

## Calculating Equilibrium Concentrations

- In some cases it is necessary to solve a quadratic equation to obtain equilibrium concentrations.
- The next example illustrates how to solve such an equation.


## Calculating Equilibrium Concentrations

- Consider the following equilibrium.

$$
\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{HI}(\mathrm{~g})
$$

- Suppose $1.00 \mathrm{~mol} \mathrm{H}_{2}$ and $2.00 \mathrm{~mol}_{2}$ are placed in a $1.00-\mathrm{L}$ vessel. How many moles per liter of each substance are in the gaseous mixture when it comes to equilibrium at $458^{\circ} \mathrm{C}$ ?
- $\mathrm{K}_{\mathrm{c}}$ at this temperature is 49.7.


## Calculating Equilibrium Concentrations

- The preceding example illustrates the three steps in solving for equilibrium concentrations.

1. Set up a table of concentrations (starting, change, and equilibrium expressions in x ).
2. Substitute the expressions in x for the equilibrium concentrations into the equilibrium-constant equation.
3. Solve the equilibrium-constant equation for the values of the equilibrium concentrations.

## Calculating Equilibrium Concentrations

- The concentrations of substances are as follows.

| $\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g})$ | $\longleftrightarrow 2 \mathrm{HI}(\mathrm{g})$ |  |
| :---: | :---: | :---: |
| Starting | 1.00 | 2.00 |
| Change | -x | -x |
| Equilibrium | $1.00-\mathrm{x}$ | $2.00-\mathrm{x}$ |

- The equilibrium-constant expression is:

$$
\mathbf{K}_{\mathrm{c}}=\frac{[\mathrm{HII}]^{2}}{\left[\mathrm{H}_{2}\right]\left[\mathrm{I}_{2}\right]}
$$

## Calculating Equilibrium Concentrations

- The concentrations of substances are as follows.

- Substituting our equilibrium concentration expressions gives:

$$
\mathbf{K}_{\mathrm{c}}=\frac{(2 \mathrm{x})^{2}}{(1.00-\mathrm{x})(2.00-\mathrm{x})}
$$

## Calculating Equilibrium Concentrations

- Solving for $\mathbf{x}$.

| $\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g})$ | $\longleftrightarrow 2 \mathrm{HII}(\mathrm{g})$ |  |
| :---: | :---: | :---: |
| Starting | 1.00 | 2.00 |
| Change | -x | -x |
| Equilibrium | $1.00-\mathrm{x}$ | $2.00-\mathrm{x}$ |

- The two possible solutions to the quadratic equation are:

$$
x=2.33 \quad \text { and } \quad x=0.93
$$

## Calculating Equilibrium Concentrations

- Solving for $\mathbf{x}$.

| $\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g})$ | $\rightleftarrows 2 \mathrm{HI}(\mathrm{g})$ |  |
| :---: | :---: | :---: |
| Starting | 1.00 | 2.00 |
| Change | -x | -x |
| Equilibrium | $1.00-\mathrm{x}$ | $2.00-\mathrm{x}$ |

- However, $\mathrm{x}=2.33$ gives a negative value to $1.00-\mathrm{x}$ (the equilibrium concentration of $\mathrm{H}_{2}$ ), which is not possible.

Only $x=0.93$ remains.

## Calculating Equilibrium Concentrations

## - Solving for $\mathbf{x}$.

| $\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g}) \longleftrightarrow 2 \mathrm{HI}(\mathrm{g})$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Starting | 1.00 | 2.00 | 0 |
| Change | -x | -x | +2x |
| Equilibrium | 1.00-x | 2.00-x | 2 x |

- The equation rearranges to give:

$$
\begin{gathered}
0.920 x^{2}-3.00 x+2.00=0 \\
x=\left\{-b \pm\left(b^{2}-4 a c\right)^{1 / 2}\right\} / 2 a
\end{gathered}
$$

## Calculating Equilibrium Concentrations

- Solving for equilibrium concentrations.

| $\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g})$ |  |  |  | $2 \mathrm{HI}(\mathrm{g})$ |
| :---: | :---: | :---: | :---: | :---: |
| Starting | 1.00 | 2.00 |  |  |
| Change | -x | -x |  |  |
| Equilibrium | $1.00-\mathrm{x}$ | $2.00-\mathrm{x}$ |  |  |

- If you substitute 0.93 for x in the last line of the table you obtain the following equilibrium concentrations.

$$
0.07 \mathrm{M} \mathrm{H}_{2} \quad 1.07 \mathrm{M} \mathrm{I}_{2} \quad 1.86 \mathrm{M} \mathrm{HI}
$$

| Procedure for Solving |
| :---: |
| Procedure for Solving Equilibrium Problems <br> 1 Write the balanced equation for the reaction. <br> 2 Write the equilibrium expression using the law of mass action. <br> 3 List the initial concentrations. <br> 4 Calculate $Q$, and determine the direction of the shift to equilibrium. <br> 5 Define the change needed to reach equilibrium, and define the equilibrium concentrations by applying the change to the initial concentrations. <br> 6 Substitute the equilibrium concentrations into the equilibrium expression, and solve for the unknown. <br> 7 Check your calculated equilibrium concentrations by making sure they give the correct value of $K$. |

## Le Châtelier's Principle

- . . . if a change is imposed on a system at equilibrium, the position of the equilibrium will shift in a direction that tends to reduce that change.


## Effects of Changes on the System

- 1. Concentration: The system will shift away from the added component.
- 2. Temperature: $K$ will change depending upon the temperature (treat the energy change as a reactant).


## Effects of Changes on the System (continued)

- 3. Pressure:
- a. Addition of inert gas does not affect the equilibrium position.
- b. Decreasing the volume shifts the equilibrium toward the side with fewer moles.

Figure 13.8: (a) The initial equilibrium mixture of $\mathrm{N} 2, \mathrm{H} 2$, and NH3. (b) Addition of N2.(c) The new equilibrium position for the system containing more N2 (due to addition of N2), less H2, and more NH3 than in (a).



## Effects of Pressure Change

- A pressure change caused by changing the volume of the reaction vessel can affect the yield of products in a gaseous reaction only if the reaction involves a change in the total moles of gas present


## Effects of Pressure Change

- If the products in a gaseous reaction contain fewer moles of gas than the reactants, it is logical that they would require less space.
- So, reducing the volume of the reaction vessel would, therefore, favor the products.
- Conversely, if the reactants require less volume (that is, fewer moles of gaseous reactant), then decreasing the volume of the reaction vessel would shift the equilibrium to the left (toward reactants).


## Effects of Pressure Change

- Literally "squeezing" the reaction will cause a shift in the equilibrium toward the fewer moles of gas.
- It's a simple step to see that reducing the pressure in the reaction vessel by increasing its volume would have the opposite effect.
- In the event that the number of moles of gaseous product equals the number of moles of gaseous reactant, vessel volume will have no effect on the position of the equilibrium.


## Effect of Temperature Change

- Temperature has a significant effect on most reactions
- Reaction rates generally increase with an increase in temperature. Consequently, equilibrium is established sooner.
- In addition, the numerical value of the equilibrium constant $K_{c}$ varies with temperature.

Figure 13.9: (a) A mixture of $\mathrm{NH} 3(g), \mathrm{N} 2(g)$, and $\mathrm{H} 2(g)$ at equilibrium.
(b) The volume is suddenly decreased. (c) The new equilibrium position for the system containing more NH 3 and less N 2 and H 2 . The reaction $\mathrm{N} 2(g)+3 \mathrm{H} 2(g) \rightarrow 2 \mathrm{NH} 3(g)$ shifts to the right (toward the side with fewer molecules) when the container volume is decreased.


Figure
15.10:

The effect of changing the temperature on chemical equilibrium.

Photo courtesy of

American Color.


## Effect of Temperature Change

- Let's look at "heat" as if it were a product in exothermic reactions and a reactant in endothermic reactions.
- We see that increasing the temperature is analogous to adding more product (in the case of exothermic reactions) or adding more reactant (in the case of endothermic reactions).
- This ultimately has the same effect as if heat were a physical entity.


## Effect of Temperature Change

- For an endothermic reaction, the opposite is true.
"heat"+reactants $\rightleftarrows$ products $(\Delta \mathrm{H}$ is positive)
- Increasing temperature would be analogous to adding more reactant, causing the equilibrium to shift right.
- This change results in more product at equilibrium, amd a larger numerical value for $\mathbf{K}_{\mathbf{c}}$.


## Effect of Temperature Change

- For example, consider the following generic exothermic reaction.
reactants $\rightleftarrows$ products+"heat" ( $\Delta \mathrm{H}$ is negative)
- Increasing temperature would be analogous to adding more product, causing the equilibrium to shift left.
- Since "heat" does not appear in the equilibriumconstant expression, this change would result in a smaller numerical value for $\mathbf{K}_{\mathbf{c}}$.


## Effect of Temperature Change

- In summary:
- For an endothermic reaction ( $\Delta \mathrm{H}$ positive) the amounts of products are increased at equilibrium by an increase in temperature ( $\mathbb{K}_{\mathrm{c}}$ is larger at higher temperatures).
- For an exothermic reaction ( $\Delta \mathrm{H}$ is negative) the amounts of reactants are increased at equilibrium by an increase in temperature ( $\mathrm{K}_{\mathrm{c}}$ is smaller at higher temperatures).


| TABLE 13.4 | Shifts in the |
| :--- | :--- |
| Equilibrium Position for the |  |
| Reaction $58 \mathrm{~kJ}+\mathrm{N}_{2} \mathrm{O}_{4}(g)$ |  |
| $2 \mathrm{NO}_{2}(g)$ | Shift |
| Change | Right |
| Addition of $\mathrm{N}_{2} \mathrm{O}_{4}(g)$ | Left |
| Addition of $\mathrm{NO}_{2}(g)$ | Left |
| Removal of $\mathrm{N}_{2} \mathrm{O}_{4}(g)$ | Right |
| Removal of $\mathrm{NO}_{2}(g)$ | None |
| Addition of $\mathrm{He}^{(g)}$ | Left |
| Decrease container <br> volume | Right |
| Increase container |  |
| volume |  |
| Increase temperature | Right |
| Decrease temperature | Left |

## Effect of a Catalyst

- A catalyst is a substance that increases the rate of a reaction but is not consumed by it.
- It is important to understand that a catalyst has no effect on the equilibrium composition of a reaction mixture.
- A catalyst merely speeds up the attainment of equilibrium.

