Physics 722/822
9/25/2018

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Why use electrons and photons?

- Probe structure understood (point particles)
- Electromagnetic interaction understood (QED)
- Interaction is weak ($\alpha = 1/137$)
  - Perturbation theory works!
    - First Born Approx / one photon exchange
  - Probe interacts only once
  - Study the entire nuclear volume

BUT:
- Cross sections are small
- Electrons radiate
(e,e') spectrum

Generic Electron Scattering at fixed momentum transfer
Experimental goals:

- Elastic scattering
  - Structure of the nucleus
    - Form factors, charge distributions, spin dependent FF
- Quasielastic (QE) scattering
  - Shell structure
    - Momentum distributions
    - Occupancies
  - Short Range Correlated nucleon pairs
  - Nuclear transparency and color transparency
- Deep Inelastic Scattering (DIS)
  - The EMC Effect and Nucleon modification in nuclei
  - Quark hadronization in nuclei
Energy vs length

Select spatial resolution and excitation energy independently

- Photon energy $\nu$ determines excitation energy
- Photon momentum $q$ determines spatial resolution: $\lambda \approx \frac{\hbar}{q}$

Three cases:

- **Low q**
  - Photon wavelength $\lambda$ larger than the nucleon size ($R_p$)

- **Medium q: $0.2 < q < 1$ GeV/c**
  - $\lambda \sim R_p$
  - Nucleons resolvable

- **High q: $q > 1$ GeV/c**
  - $\lambda < R_p$
  - Nucleon structure resolvable
Inclusive electron scattering \((e,e')\)

\[
k'_{\mu} = (E', \vec{k}')
\]

Lab frame kinematics

\[
k_{\mu} = (E, \vec{k})
\]

\[
q_{\mu} = k_{\mu} - k'_{\mu}
\]

\[
p_{\mu} = (M, \vec{0})
\]

\[
p'_{\mu} = (E_p, \vec{p}_p)
\]

(not detected)

**Invariants:**

\[
p_{\mu} p'_{\mu} = M^2
\]

\[
q_{\mu} q_{\mu} = |\vec{q}|^2 - \omega^2
\]

\[
Q^2 = -q_{\mu} q_{\mu}
\]

\[
W^2 = (q_{\mu} + p_{\mu})^2 = p'_{\mu} p'^{\mu}
\]

\[
p_{\mu} q_{\mu} = M \omega
\]
Elastic cross section \((p'^2 = m^2)\)

Recoil factor

\[
\frac{d\sigma}{d\Omega} = \sigma_M \left( \frac{E'}{E} \right) \left\{ \left[ F_1^2(Q^2) + \frac{Q^2}{4M^2} \kappa^2 F_2^2(Q^2) \right] + \frac{Q^2}{2M^2} \left[ F_1(Q^2) + \kappa F_2(Q^2) \right]^2 \tan^2 \frac{\theta}{2} \right\}
\]

\[
= \sigma_M \left( \frac{E'}{E} \right) \left[ G_E^2(Q^2) + \frac{\tau G_M^2(Q^2)}{1 + \tau} + 2\tau \tan^2 \frac{\theta}{2} G_M^2(Q^2) \right]
\]

\[
= \sigma_M \left( \frac{E'}{E} \right) \left[ \frac{Q^4}{\bar{q}^4} R_L(Q^2) + \left( \frac{Q^2}{2\bar{q}^2} + \tan^2 \frac{\theta}{2} \right) R_T(Q^2) \right]
\]

Mott cross section

\[
\sigma_M = \frac{\alpha^2 \cos^2 \left( \frac{\theta_e}{2} \right)}{4E^2 \sin^4 \left( \frac{\theta_e}{2} \right)}
\]

\(F_1, F_2\): Dirac and Pauli form factors

\(G_E, G_M\): Sachs form factors (electric and magnetic)

\[
G_E(Q^2) = F_1(Q^2) - \tau \kappa F_2(Q^2) \quad \tau = \frac{Q^2}{4M^2}
\]

\[
G_M(Q^2) = F_1(Q^2) + \kappa F_2(Q^2) \quad \kappa = \text{anomalous magnetic moment}
\]

\(R_L, R_T\): Longitudinal and transverse response fn
Notes on form factors

• $G_E$, $G_M$, $F_1$ and $F_2$ refer to nucleons
  • $F_1^p(0) = 1$, $F_2^p(0) = \kappa_p = 1.79$
  • $F_1^n(0) = 0$, $F_2^n(0) = \kappa_n = -1.91$
  • $G_E^p(0) = 1$, $G_M^p(0) = 1 + \kappa_p = 2.79$
  • $G_E^n(0) = 0$, $G_M^n(0) = \kappa_n = -1.91$

• $R_L$ and $R_T$ refer to nuclei
Electron-nucleus interactions
Electrons as Waves

Scattering process is quantum mechanical

De Broglie wavelength:

\[ \lambda = \frac{h}{p} \]

Electron energy:

\[ E_e \approx pc \]

\( \lambda \) resolving “scale”:

\[ \lambda = \frac{2\pi(197 \text{ MeV} \cdot \text{fm})}{E_e} \]
Analogy between elastic electron scattering and diffraction
Simple analogy for elastic electron scattering....

Classical Fraunhofer Diffraction

Amplitude of wave at screen:

\[
\Phi \propto \int_0^a \int_0^{2\pi} \exp(i br \cos\phi) r d\phi dr
\]
Classical Fraunhofer Diffraction

Intensity: $\Phi^2 \propto \left( \frac{J_1 \left( \frac{2\pi a}{\lambda} \sin \theta \right)}{\sin \theta} \right)^2$

Minima occur at zeroes of Bessel function. 1st zero: $x = 3.8317$

...some algebra...

Hence $2a \approx \frac{1.22\lambda}{\sin \theta}$
Excursion: Babinet’s principle

Screen with apertures

Complementary screen

Patterns appear the same
Example: $^{30}\text{Si}(e,e')$

$1^{\text{st}}$ minimum = 1.3 fm$^{-1}$

$\Rightarrow \theta = 32.8^\circ$

Electron energy = 454.3 MeV

$\Rightarrow \lambda = 2.73$ fm

Calculated radius = 3.07 fm

Measured rms radius = 3.19 fm

(from fit to entire curve)

$(1 \text{ fm}^{-1} = 197 \text{ MeV}/c)$
I. Elastic Electron Scattering from Nuclei (done formally)

Fermi’s Golden Rule

\[ \frac{d\sigma}{dQ} = \frac{2\pi}{\hbar} |M_{fi}|^2 D_f \]

- \( M_{fi} \): scattering amplitude
- \( D_f \): density of the final states (or phase factor)

\[
M_{fi} = \int \Psi_f^* V(x) \Psi_i d^3x \\
= \int e^{-k_f \cdot x} V(x) e^{-k_i \cdot x} d^3x \\
= \int e^{i q \cdot x} V(x) d^3x
\]

Plane wave approximation for incoming and outgoing electrons

Born approximation (interact only once)
I. Elastic Electron Scattering from (spin-0) Nuclei

Form Factor and Charge Distribution

Using Coulomb potential from a charge distribution, $\rho(x)$,

$$V(x) = -\frac{Ze^2}{4\pi\varepsilon_0} \int \frac{\rho(x')}{|x-x'|} d^3x'$$

$$M_{fi} = -\frac{Ze^2}{4\pi\varepsilon_0} \int e^{iq\cdot x} \left[ \int \frac{\rho(x')}{|x-x'|} d^3x' d^3x \right]$$

$$= -\frac{Ze^2}{4\pi\varepsilon_0} \int e^{iq\cdot x} \left[ \int \frac{e^{iq\cdot x'}}{|x-x'|} \rho(x') d^3x' \right] d^3R$$

$$= -\frac{Ze^2}{4\pi\varepsilon_0} \int \frac{e^{iq\cdot x}}{R} d^3R \left[ \int e^{iq\cdot x'} \rho(x') d^3x' \right]$$

$$F(q) = \int e^{iq\cdot x'} \rho(x') d^3x'$$

Charge form factor $F(q)$ is the Fourrier transform of the charge distribution $\rho(x)$.
I. Elastic (e,e') Scattering ⇒ charge distributions

Elastic electron scattering measured for many nuclei over a wide range of $Q^2$ (mainly at Saclay in the 1970s)

Measured charge distributions agree well with mean field theory calculations.
Measure spin/parity of excited states and transition matrix elements
Structure of the nucleus

- nucleons are bound
  - energy ($E$) distribution
  - shell structure
- nucleons are not static
  - momentum ($k$) distribution

determined by the N-N potential

on average:
Net binding energy: $\approx 8$ MeV
distance: $\approx 2$ fm

Strong repulsion
NN correlations

attractive part
repulsive core
short-range
long-range
\[ V(r) \text{ [MeV]} \]
\[ d \text{ [fm]} \]
\[ \omega, \rho \text{ short-range} \]
\[ \pi \text{ long-range} \]
II. Quasielastic scattering

\[ \frac{Q^2}{2A} \]

Giant resonance

Quasielastic

\[ \frac{Q^2}{2m} \]

\[ \frac{Q^2}{2m} + 300 \text{ MeV} \]

NUCLEUS

DEEP INELASTIC “EMC”

PROTON

DEEP INELASTIC “QUARKS”
Fermi gas model:
how simple a model can you make?

Initial nucleon energy: \( KE_i = \frac{p_i^2}{2m_p} \)
Final nucleon energy: \( KE_f = \frac{p_f^2}{2m_p} = \frac{(\vec{q} + \vec{p}_i)^2}{2m_p} \)

Energy transfer: \( \nu = KE_f - KE_i = \frac{\vec{q}^2}{2m_p} + \frac{\vec{q} \cdot \vec{p}_i}{m_p} \)

Expect:

- Peak centroid at \( \nu = q^2/2m_p + \varepsilon \)
- Peak width \( 2qp_{\text{fermi}}/m_p \)
- Total peak cross section \( = Z\sigma_{ep} + N\sigma_{en} \)
Early 1970’s Quasielastic Data

- getting the bulk features

500 MeV, 60 degrees

\( \vec{q} \approx 500 \text{MeV/c} \)

compared to Fermi model: fit parameter \( k_F \) and \( \varepsilon \)

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**Table:**

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>( k_F ) MeV/c</th>
<th>( \varepsilon ) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^6\text{Li})</td>
<td>169</td>
<td>17</td>
</tr>
<tr>
<td>(^{12}\text{C})</td>
<td>221</td>
<td>25</td>
</tr>
<tr>
<td>(^{24}\text{Mg})</td>
<td>235</td>
<td>32</td>
</tr>
<tr>
<td>(^{40}\text{Ca})</td>
<td>251</td>
<td>28</td>
</tr>
<tr>
<td>nat Ni</td>
<td>260</td>
<td>36</td>
</tr>
<tr>
<td>(^{89}\text{Y})</td>
<td>254</td>
<td>39</td>
</tr>
<tr>
<td>nat Sn</td>
<td>260</td>
<td>42</td>
</tr>
<tr>
<td>(^{181}\text{Ta})</td>
<td>265</td>
<td>42</td>
</tr>
<tr>
<td>(^{208}\text{Pb})</td>
<td>265</td>
<td>44</td>
</tr>
</tbody>
</table>
Scaling

• The dependence of a cross section, in certain kinematic regions, on a single variable.
  • If the data scales, it validates the scaling assumption
  • Scale-breaking indicates new physics
• At moderate $Q^2$ and $x>1$ we expect to see evidence for $y$-scaling, indicating that the electrons are scattering from quasifree nucleons
  • $y =$ minimum momentum of struck nucleon
• At high $Q^2$ we expect to see evidence for $x$-scaling, indicating that the electrons are scattering from quarks.
  (next lecture)
  • $x = Q^2/2mv =$ fraction of nucleon momentum carried by struck quark (in infinite momentum frame)
$\gamma$-scaling in inclusive electron scattering from $^3\text{He}$

$^3\text{He}(e,e')$ at various $Q^2$

$$F(y) = \frac{\sigma^{\text{exp}}}{(Z\bar{\sigma}_p + N\bar{\sigma}_n)} \cdot K$$

$$n(k) = -\frac{1}{2\pi y} \frac{dF(y)}{dy}$$

**Assumption:** scattering takes place from a quasi-free proton or neutron in the nucleus.

$y$ is the momentum of the struck nucleon parallel to the momentum transfer:

$$y \approx -q/2 + mv/q \text{ (nonrelativistically)}$$

**IF** the scattering is quasifree, **then** $F(y)$ is the integral over all perpendicular nucleon momenta (nonrelativistically).

**Goal:** extract the momentum distribution $n(k)$ from $F(y)$. 
Assumptions & Potential Scale Breaking Mechanisms

- No Final State Interactions (FSI)
- No internal excitation of (A-1)
- Full strength of Spectral function can be integrated over at finite $q$
- No inelastic processes (choose $y<0$)
- No medium modifications (discussed later)
Y-scaling works!
Get more information:
Detect the knocked out nucleon (e,e'p)

coincidence experiment

measure: momentum, angles

electron energy: $E_e$
proton: $\vec{P}_{p'}$
scattered electron: $\vec{k}_{e'}$ $E_{e'} = |\vec{k}_{e'}|$

reconstructed quantities:
missing energy: $E_m = \nu - T_{p'} - T_{A-1}$
missing momentum: $\vec{P}_m = \vec{q} - \vec{P}_{p'}$

in Plane Wave Impulse Approximation (PWIA):
direct relation between measured quantities and theory:

$|E| = E_m$ $\vec{P}_{\text{init}} = -\vec{P}_m$
And then there were four (response functions, that is)

(When you include electron and proton spin, there are 18. Yikes!)

(And if you scatter from a polarized spin-1 target, there are 41. Double Yikes!!)
Single nucleon pickup reactions [eg: (p,d), (d,^3\text{He}) ...] are also sensitive to $S(p,E)$ but only sensitive to surface nucleons due to strong absorption in the nucleus.

**DWIA:** If the struck nucleon interacts with the rest of the nucleus, then the cross section still factorizes (usually) but we measure a distorted spectral function.
Measuring $O(e,e'p)$ in Hall A

Fissum et al, PRC 70, (2004) 034606
O(e,e’p) and shell structure

$\Theta_{\text{pq}} = 2.5^\circ$, $\langle P_{\text{miss}} \rangle = 50 \text{ MeV/c}$

$\Theta_{\text{pq}} = 8^\circ$, $\langle P_{\text{miss}} \rangle = 145 \text{ MeV/c}$

$\Theta_{\text{pq}} = 16^\circ$, $\langle P_{\text{miss}} \rangle = 280 \text{ MeV/c}$

$\Theta_{\text{pq}} = 20^\circ$, $\langle P_{\text{miss}} \rangle = 340 \text{ MeV/c}$

$1p_{1/2}, 1p_{3/2}$ and $1s_{1/2}$ shells visible

Momentum distribution as expected for $l = 0, 1$

Fissum et al, PRC 70, 034606 (2003)
But we do not see enough protons!
(e,e’p) summary

• Measure shell structure directly
• Measure nucleon momentum distributions
• But:
  • Not enough nucleons seen!
Short Range Correlations (SRCs)

Mean field contributions: $p < p_{\text{Fermi}} \approx 250 \text{ MeV/c}$
Well understood, Spectroscopic Factors $\approx 0.65$

High momentum tails: $p > p_{\text{Fermi}}$
Calculable for few-body nuclei, nuclear matter.
Dominated by two-nucleon short range correlations

Poorly understood part of nuclear structure
NN potential models not applicable at $p > 350 \text{ MeV/c}$

Uncertainty in SR interaction leads to uncertainty at $p > p_{\text{Fermi}}$, even for simplest systems

Nucleons are like people ...
What are Correlations?

Average Two Nucleon Properties in the Nuclear Ground State

Not Two-Body Currents

An Experimentalist’s Definition:

• A high momentum nucleon whose momentum is balanced by one other nucleon
  • NN Pair with
    • Large Relative Momentum
    • Small Total Momentum
• Whatever a theorist says it is
Why are Correlations Interesting?

Responsible for high momentum part of Nuclear WF

Ciofi degli Atti, PRC 53 (1996) 1689
Correlations should be universal

Many-body calculations predict that the high momentum distribution for all nuclei has the same shape:

\[ n_A(k) / n_d(k) = a_2(A/d) \]

O. Benhar, Phys Lett B 177 (1986) 135
Inclusive Electron Scattering at $x_B > 1$

- At fixed $Q^2$, $x_B$ determines a minimum initial momentum for the scattered nucleon (remember $y$-scaling?)
- If the momentum distributions of two nuclei have the same shape, then the ratios of their cross sections should be flat

$momentum\ scaling\ \leftrightarrow\ x_B\ scaling$
Correlations are Universal

Scaling (flat ratios) indicates a common momentum distribution.

1 < x < 1.5: dominated by different mean field n(k)
1.5 < x < 2: dominated by 2N SRC n(k)

α_{2N} ≈ 20%
α_{3N} ≈ 1%

Day et al, PRL 59, 427 (1987)
Frankfurt et al, PRC 48 2451 (1993)
Egiyan et al., PRL 96, 082501 (2006)
Fomin et al., PRL 108, 092502 (2012)
Short Range Correlations (SRC)

- 2N-SRC are pairs of nucleons that:
  - Are close together (overlap) in the nucleus.
  - Have high relative momentum and low center-of-mass (cm) momentum, where high and low is compared to the Fermi momentum of the nucleons (≈250 MeV/c in heavy nuclei)
Exclusive SRC Studies

\[ A(e,e'pN): \text{detect electron + two nucleons} \]

- **Pros:** Measure the both nucleons to characterize the 2N-SRC pairs

- **Cons:**
  - **Interpretation difficulties:**
    - Competing processes,
    - Final State Interactions (FSI)
    - Transparency.
  - **Experimental difficulties:**
    - Large backgrounds,
    - Low rates,
    - Large installation,
    - Dedicated detectors

Who knew nuclear physics could be so complicated
Measurement Concept:

1. Hit a high momentum proton hard ($Q^2 > 1 \text{ GeV}^2$)

2. Reconstruct the initial (missing) momentum of the struck nucleon

3. Look for a recoil nucleon with momentum that balanced that of the struck proton

$$\vec{p}_{\text{miss}} = \vec{q} - \vec{p}_p = -\vec{p}_{\text{initial}}$$
Goal: Study both $pn$ and $pp$ SRC in $^{12}$C over an $(e,e'p)$ missing momentum range of 300-600 MeV/c
JLab Hall-A

Physicists Tend To Fill Empty Space©
JLab Hall-A E01-015

Physicists Tend To Fill Empty Space ©
Physicists Tend To Fill Empty Space
Now detect yet another nucleon
JLab Hall A C(e,e′pN) - selected kinematics

\[ Q^2 = 2 \text{ GeV}^2 \]
\[ x_B = \frac{Q^2}{2mv} = 1.2 \]
\[ P_{\text{miss}} = 300-600 \text{ MeV/c} \]

Detect the proton, look for its partner nucleon

\[ E_e = 4.627 \text{ GeV} \]
\[ E_{e'} = 3.724 \text{ GeV} \]
The $(e,e'pN)/(e,e'p)$ ratio gives the probability for a high momentum proton to be part of a pN-SRC pair.

**All** high $p_{\text{initial}}$ protons have a correlated partner

np pairs dominate

- Importance of tensor force at $0.3<p_{\text{initial}}<0.6$ GeV/c

![Graph showing SRC pair fractions](image)

**pp-SRC**

$96 \pm 23\%$

**pn-SRC**

$9 \pm 2\%$

2N-SRC from inclusive and exclusive measurements

1. The probability for a nucleon to have $p \geq 300$ MeV/c in medium nuclei is 20-25%.

2. More than ~90% of all nucleons with $p \geq 300$ MeV/c belong to 2N-SRC.

3. 2N-SRC dominated by np pairs.

~80% of kinetic energy of nucleon in nuclei is carried by nucleons in 2N-SRC.
Quasielastic summary: \((e,e')\), \((e,e'p)\) and \((e,e'pN)\)

- \((e,e')\) scaling shows the electron is (mostly) scattering from single nucleons
- \((e,e')\) ratios measure the probability of short range correlations (SRC) in nuclei
- \((e,e'p)\) measures \(E\) and \(p\) distributions of single nucleons
- \((e,e'pN)\) measures \(E\) and \(p\) distributions of nucleon pairs

The nucleus:
- 60-70% single particle - \(E + p\) dists measured
- 20\(\pm\)5\% SRC - starting to measure
- 10-20\% LRC