Lecture Notes: NN Scattering
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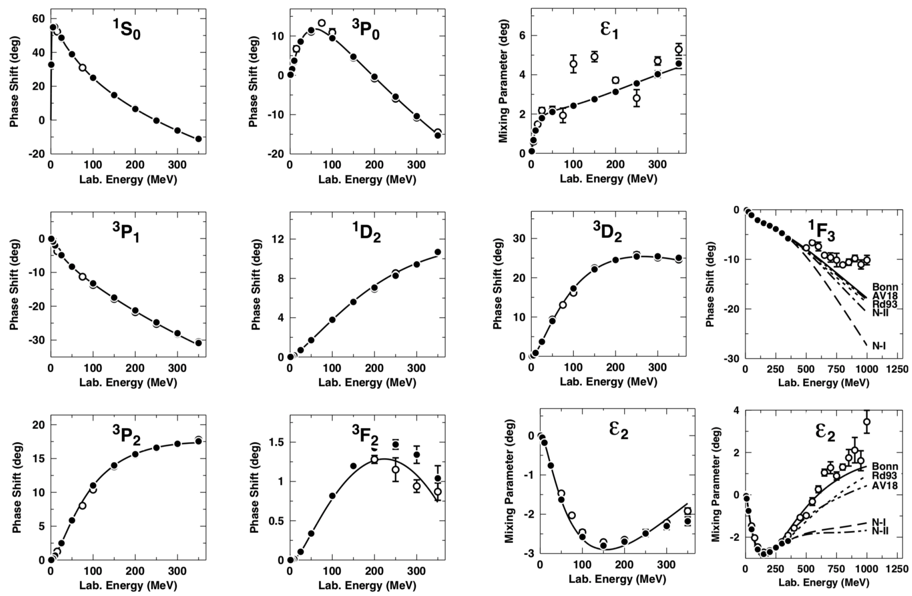
1 Nucleon-Nucleon Scattering

In the previous lecture, we were talking about nucleon-nucleon (NN) scattering events and describing them through phase shifts. These phase shifts are dependent on the spin and angular momentum of the constituent nucleons. The notation for the phase shift is ${}^{(2S+1)}L_J$, where S is the spin, L is orbital angular momentum, and J is total angular momentum and the standard equation $J = L + S$ holds. As far as determining what the phase shift of incoming and outgoing nucleons is, there is nothing that says we have to couple equal orbital angular momenta in the initial and final states. This tells us there has to be something in the Hamiltonian (H) that couples different L. Thus if we express the Hamiltonian as a matrix, we would have non-zero elements in areas surrounded by red circles. These would represent a transition from ${}^3S_1$ to ${}^3D_1$ or vice-versa (The green square will be explained later).

$$H = \begin{array}{c c} & \begin{array}{ccc} {}^3S_1 & {}^3D_1 & \end{array} \\ \begin{array}{c} {}^3S_1 \\ {}^3D_1 \\ \\ \end{array} & \begin{pmatrix} \boxed{*} & * & \cdots \\ * & * & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \end{array}$$

These off diagonal elements are referred to as a tensor force as they are a force that can change angular momentum by more than one unit.

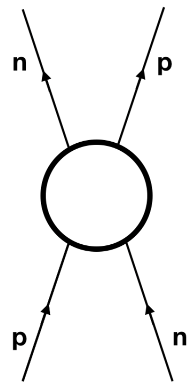
The below plots show measurements for the phase shifts at various energies. From these plots we can see that the only phase shift that isn't 0 at low energies are the S-wave phase shifts. Thus we would expect the element in the green square to be the main contributor in the Hamiltonian at these energies. Also note that the plots labeled with an epsilon represent changes from one phase shift to another.



2 Nucleon-Nucleon Interactions

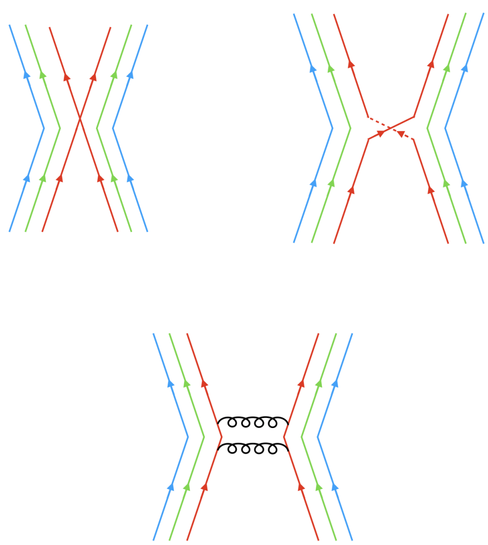
NN interactions need to conserve isospin in the z-direction and the square of the total isospin ($[H, I_3] = [H, I^2] = 0$). Even though the Hamiltonian can't change isospin it can still have an interaction potential that depends on it. Thus the isospin structure can have terms without any isospin dependence and terms that depend on $\hat{\tau}_1 \cdot \hat{\tau}_2$ where $\hat{\tau}_1$ and $\hat{\tau}_2$ are the isospin operators of the first and second nucleon respectively.

There are a few different models we can use to describe NN interactions. Given the diagram below, where the circle represents all possible interactions, each model tries to describe what happens in that circle.



2.1 QCD inspired models

In QCD we try to examine NN interactions on the quark level. The simplest forms we can redraw the interaction diagram is in the following ways.



The first image depicts two nucleons coming in and trading red quarks before parting ways. The second image shows one of the red quarks breaking into a quark anti-quark pair which then interacts with the other incoming red quark and then continuing on. The last image shows two nucleons coming in and exchanging gluons before parting. The last image is also the leading term for glancing elastic NN-collisions at very high energies (Pomeron exchange).

Since we are trying to describe the interaction between two nucleons on the quark level we need to consider interactions due to the strong force. The general quantum mechanical approach to analyzing these interactions is perturbation theory. However, the strong force is too strong at the distances we are considering for perturbation theory to work. Lattice QCD (LQCD) overcomes this problem by breaking space up into very close points (forming a lattice). By making the distance between these points small enough, the strong force from one point to the next becomes weak enough that perturbation theory can work. The problem with this approach is that it requires lots of numerical calculation which can currently only be done on massive super computing clusters. While this field has made large strides in the last few years, there is still lots of work to be done.

2.2 Effective Field Theories

Since LQCD requires lots of computing power, the other method we can use to study NN interactions is making an effective field theory. For an effective field theory to work it just needs to retain the main points of QCD.

2.2.1 Chiral Perturbation Theory

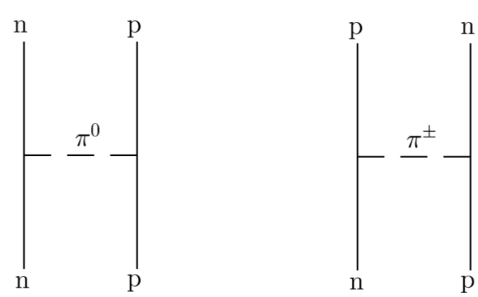
If quarks had 0 mass, then there would be no distinction between left and right handed quarks and thus they would have chiral symmetry. Since quarks do have mass this causes a spontaneous breaking of chiral symmetry which in turn causes the Goldstone bosons carrying out the interaction to not be completely massless (even though we would like them to be). Thus we can instead expand the interaction in small parameters (like mass) to make predictions. This expansion is Chiral Perturbation Theory

2.2.2 Chiral Effective Field Theory

For this theory QCD is described as an infinite sea of pions with vortices. Basically, it has all the main points of QCD, but the smallest components are nucleons and pions (no quarks). This can be made even simpler by removing all of the pions which is a Pion-less Effective Theory.

2.3 Interactions via Meson Exchange

From looking at the previous pictures we can see that we are basically drawing a meson (quark anti-quark pair) exchange between the two nucleons. For one pion exchange this gives us two possible pictures: one where the charge of the incoming particles doesn't change and one where it does.

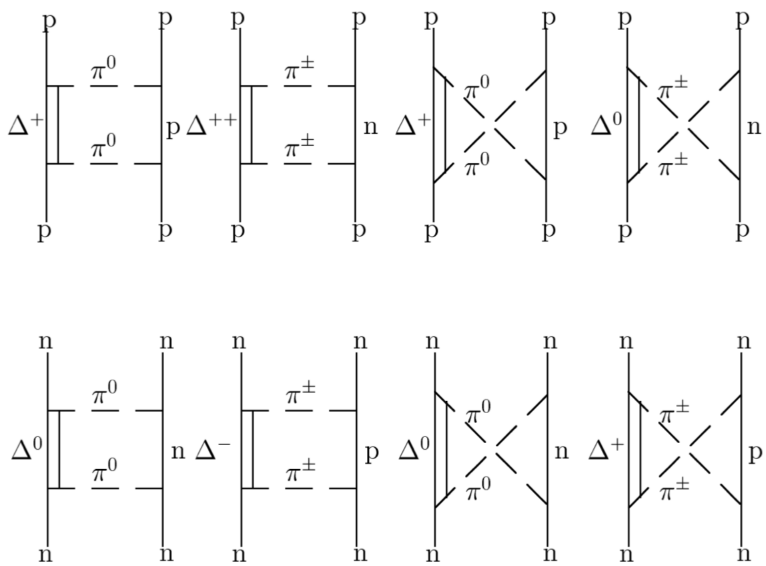


For a free massive particle described by EM waves we can use the Klein-Gordon equation ($(\Box + m^2)\psi = 0$) to describe it. For the situation we are describing (static approximation = no time dependence) we can rewrite the Klein-Gordon equation as follows.

$$(\vec{\nabla}^2 - m)\psi = 4\pi\delta^{(3)}(\vec{r})\psi$$

Where ψ describes a pseudo-scalar pion field with a nuclear source.

There is also the possibility of two pion exchange which has the following diagrams.



The effective NN potential for one and two pion exchange is shown below.

$$v_{12}^{L} = \left[\sum_{l=1}^{6} v_{L}^{l}(r) O_{12}^{l}\right] + v_{L}^{\sigma T}(r) O_{12}^{\sigma T} + v_{L}^{tT}(r) O_{12}^{tT}$$

where

$$O_{12}^{l=1,\ldots,6} = [\mathbf{1}, \hat{\sigma}_1 \cdot \hat{\sigma}_2, S_{12}] \otimes [\mathbf{1}, \hat{\tau}_1 \cdot \hat{\tau}_2]$$

The below plot shows the general form of this potential as a function of the distance between the nucleons. The blue section shows the portion of the potential dominated by the strong repulsive core that can be mocked up through rho and omega exchange (but is more likely from colormagnetic repulsion) and which explains why nucleons can't be too close together. The red section shows where we expect to see two pion exchange (often described through the exchange of an effective "sigma" meson) and the green part shows one pion exchange. The latter is mostly responsible for the tensor force described at the beginning.

NN Interaction Potential