Nuclear physics

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I. HISTORICAL

Nuclear physics started in 1894 with the discovery of the radioactivity of uranium by A. H. Becquerel. Marie and Pierre Curie investigated this phenomenon in detail: to their astonishment they found that raw uranium ore was far more radioactive than the refined uranium from the chemist's store. By chemical methods, they could separate (and name) several new elements from the ore which were intensely radioactive: radium ($Z=88$), polonium ($Z=84$), a gas they called emanation ($Z=86$) (radon), and even a form of lead ($Z=82$).

Ernest Rutherford, at McGill University in Montreal, studied the radiation from these substances. He found a strongly ionizing component which he called α rays, and a weakly ionizing one, β rays, which were more penetrating than the α rays. In a magnetic field, the α rays showed positive charge, and a charge-to-mass ratio corresponding to ^{4}He. The β rays had negative charge and were apparently electrons. Later, a still more penetrating, uncharged component was found, γ rays.

Rutherford and F. Soddy, in 1903, found that after emission of an α ray, an element of atomic number Z was transformed into another element, of atomic number $Z-2$. (They did not yet have the concept of atomic number, but they knew from chemistry the place of an element in the periodic system.) After β-ray emission, Z was transformed into $Z+1$, so the dream of alchemists had become true.

It was known that thorium ($Z=90$, $A=232$) also was radioactive, also decayed into radium, radon, polonium and lead, but obviously had different radioactive behavior from the decay products of uranium ($Z=92$, $A=238$). Thus there existed two or more forms of the same chemical element having different atomic weights and different radioactive properties (lifetimes) but the same chemical properties. Soddy called these isotopes.

Rutherford continued his research at Manchester, and many mature collaborators came to him. H. Geiger and J. M. Nuttall, in 1911, found that the energy of the emitted α particles, measured by their range, was correlated with the lifetime of the parent substance: the lifetime decreased very rapidly (exponentially) with increasing α-particle energy.

By an ingenious arrangement of two boxes inside each other, Rutherford proved that the α particles really were He atoms: they gave the He spectrum in an electric discharge.

Rutherford in 1906 and Geiger in 1908 put thin solid foils in the path of a beam of α particles. On the far side of the foil, the beam was spread out in angle—not surprising because the electric charges in the atoms of the foil would deflect the α particles by small angles and multiple deflections were expected. But to their surprise, a few α particles came back on the front side of the foil, and their number increased with increasing atomic weight of the material in the foil. Definitive experiments with a gold foil were made by Geiger and Marsden in 1909.

Rutherford in 1911 concluded that this backward scattering could not come about by multiple small-angle scatterings. Instead, there must also occasionally be single deflections by a large angle. These could only be produced by a big charge concentrated somewhere in the atom. Thus he conceived the nuclear atom: each atom has a nucleus with a positive charge equal to the sum of the charges of all the electrons in the atom. The nuclear charge Ze increases with the atomic weight.

Rutherford had good experimental arguments for his concept. But when Niels Bohr in 1913 found the theory of the hydrogen spectrum, Rutherford declared, "Now I finally believe my nuclear atom."

The scattering of fast α particles by He indicated also a stronger force than the electrostatic repulsion of the two He nuclei, the first indication of the strong nuclear force. Rutherford and his collaborators decided that this must be the force that holds α particles inside the nucleus and thus was attractive. From many scattering experiments done over a decade they concluded that this attractive force was confined to a radius

$$R=1.2\times10^{-13}A^{1/3}\ \text{cm}, \tag{1}$$

which may be considered to be the nuclear radius. This result is remarkably close to the modern value. The volume of the nucleons, according to Eq. (1), is proportional to the number of particles in it.

When α particles were sent through material of low atomic weight, particles were emitted of range greater than the original α particle. These were interpreted by Rutherford and James Chadwick as protons. They had observed the disintegration of light nuclei, from boron up to potassium.

Quantum mechanics gave the first theoretical explanation of natural radioactivity. In 1928 George Gamow, and simultaneously K. W. Gurney and E. U. Condon, discovered that the potential barrier between a nucleus and an α particle could be penetrated by the α particle coming from the inside, and that the rate of penetration depended exponentially on the height and width of the barrier. This explained the Geiger-Nuttall law that the lifetime of α-radioactive nuclei decreases enormously as the energy of the α particle increases.

On the basis of this theory, Gamow predicted that protons of relatively low energy, less than one million electron volts, should be able to penetrate into light nuclei, such as Li, Be, and B, and disintegrate them. When Gamow visited Cambridge, he encouraged the experimenters at the Cavendish Laboratory to build accelerators of relatively modest voltage, less than one million volts. Such accelerators were built by M. L. E. Oliphant on the one hand, and J. D. Cockcroft and E. T. S. Walton on the other.

By 1930, when I spent a semester at the Cavendish, the Rutherford group understood α particles very well. The penetrating γ rays, uncharged, were interpreted as high-frequency electromagnetic radiation, emitted by a nucleus after an α ray: the α particle had left the nucleus in an excited state, and the transition to the ground state was accomplished by emission of the γ ray.

The problem was with β rays. Chadwick showed in 1914 that they had a continuous spectrum, and this was repeatedly confirmed. Rutherford, Chadwick, and C. D. Ellis, in their book on radioactivity in 1930, were baffled. Bohr was willing to give up conservation of energy in this instance. Pauli violently objected to Bohr's idea, and suggested in 1931 and again in 1933 that together with the electron (β-particle) a neutral particle was emitted, of such high penetrating power that it had never been observed. This particle was named the neutrino by Fermi, "the small neutral one."

II. THE NEUTRON AND THE DEUTERON

In 1930, when I first went to Cambridge, England, nuclear physics was in a peculiar situation: a lot of experimental evidence had been accumulated, but there was essentially no theoretical understanding. The nucleus was supposed to be composed of protons and electrons, and its radius was supposed to be $<10^{-12}$ cm. The corresponding momentum, according to quantum mechanics, was

$$P>P_{\rm min}=\frac{\hbar}{R}=\frac{10^{-27}}{10^{-12}}=10^{-15}\ \ \mathrm{erg}/c, \tag{2}$$

while from the mass m_e of the electron

$$m_e c=3\times10^{-17}\ \ \mathrm{erg}/c. \tag{3}$$

Thus the electrons had to be highly relativistic. How could such an electron be retained in the nucleus, indeed, how could an electron wave function be fitted into the nucleus?

Further troubles arose with spin and statistics: a nucleus was supposed to contain A protons to make the correct atomic weight, and $A-Z$ electrons to give the net charge Z. The total number of particles was $2A-Z$, an odd number if Z was odd. Each proton and electron was known to obey Fermi statistics, hence a nucleus of odd Z should also obey Fermi statistics. But band spectra of nitrogen, N_2, showed that the N nucleus, of $Z=7$, obeyed Bose statistics. Similarly, proton and electron had spin $\frac{1}{2}$, so the nitrogen nucleus should have half-integral spin, but experimentally its spin was 1.

These paradoxes were resolved in 1932 when Chadwick discovered the neutron. Now one could assume that the nucleus consisted of Z protons and $A-Z$ neutrons. Thus a nucleus of mass A would have Bose (Fermi) statistics if A was even (odd) which cleared up the ^{14}N paradox, provided that the neutron obeyed Fermi statistics and had spin $\frac{1}{2}$, as it was later shown to have.

Chadwick already showed experimentally that the mass of the neutron was close to that of the proton, so the minimum momentum of $10^{15}\,\mathrm{erg}/c$ has to be compared with

$$M_n c=1.7\times10^{-24}\times3\times10^{10}=5\times10^{-14}\ \ \mathrm{erg}/c, \tag{4}$$

where M_n is the mass of the nucleon. $P_{\rm min}=10^{-15}$ is small compared to this, so the wave function of neutron and proton fits comfortably into the nucleus.

The discovery of the neutron had been very dramatic. Walther Bothe and H. Becker found that Be, bombarded by α particles, emitted very penetrating rays that they interpreted as γ rays. Curie and Joliot exposed paraffin to these rays, and showed that protons of high energy were ejected from the paraffin. If the rays were actually γ rays, they needed to have extremely high energies, of order 30 MeV. Chadwick had dreamed about neutrons for a decade, and got the idea that here at last was his beloved neutron.

Chadwick systematically exposed various materials to the penetrating radiation, and measured the energy of the recoil atoms. Within the one month of February 1932 he found the answer: indeed the radiation consisted of particles of the mass of a proton, they were neutral, hence neutrons. A beautiful example of systematic experimentation.

Chadwick wondered for over a year: was the neutron an elementary particle, like the proton, or was it an excessively strongly bound combination of proton and electron? In the latter case, he argued, its mass should be less than that of the hydrogen atom, because of the binding energy. The answer was only obtained when Chadwick and Goldhaber disintegrated the deuteron by γ rays (see below): the mass of the neutron was 0.8 MeV greater than that of the H atom. So, Chadwick decided, the neutron must be an elementary particle of its own.

If the neutron was an elementary particle of spin $\frac{1}{2}$, obeying Fermi statistics, the problem of spin and statistics of ^{14}N was solved. And one no longer needed to squeeze electrons into the too-small space of a nucleus. Accordingly, Werner Heisenberg and Iwanenko independently in 1933 proposed that a nucleus consists of neutrons and protons. These are two possible states of a more general particle, the nucleon. To emphasize this, Heisenberg introduced the concept of the isotopic spin τ_z the proton having $\tau_z=+\frac{1}{2}$ and the neutron $\tau_z=-\frac{1}{2}$. This concept has proved most useful.

Before the discovery of the neutron, in 1931 Harold Urey discovered heavy hydrogen, of atomic weight 2. Its nucleus, the deuteron, obviously consists of one proton and one neutron, and is the simplest composite nucleus. In 1933, Chadwick and Goldhaber succeeded in disintegrating the deuteron by γ rays of energy 2.62 MeV, and measuring the energy of the proton resulting from the disintegration. In this way, the binding energy of the deuteron was determined to be 2.22 MeV.

This binding energy is very small compared with that of ^{4}He, 28.5 MeV, which was interpreted as meaning that the attraction between two nucleons has very short range and great depth. The wave function of the deuteron outside the potential well is then determined simply by the binding energy ε. It is

$$\psi = \exp(-\alpha r)/r, \tag{5}$$

$$\alpha = (M\varepsilon)^{1/2}/\hbar, \tag{6}$$

with M the mass of a nucleon.

The scattering of neutrons by protons at moderate energy can be similarly determined, but one has to take into account that the spins of the two nucleons may be either parallel (total $S=1$) or antiparallel ($S=0$). The spin of the deuteron is 1. The $S=0$ state is not bound. The scattering, up to at least 10 MeV, can be described by two parameters for each value of S, the scattering length and the effective range r_0. The phase shift for $L=0$ is given by

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} k^2 r_0, \tag{7}$$

where k is the wave number in the center-of-mass system, δ the phase shift, a the scattering length, and r_0 the effective range. Experiments on neutron-proton scattering result in

$$a_t = 5.39 \text{ fm}, \quad r_{ot} = 1.72 \text{ fm},$$
$$a_s = -23.7 \text{ fm}, \quad r_{os} = 2.73 \text{ fm}, \tag{8}$$

where t and s designate the triplet and singlet $L=0$ states, ^3S and ^1S. The experiments at low energy, up to about 10 MeV, cannot give any information on the shape of the potential. The contribution of $L>0$ is very small for $E<10$ MeV, because of the short range of nuclear forces.

Very accurate experiments were done in the 1930s on the scattering of protons by protons, especially by Tuve and collaborators at the Carnegie Institution of Washington, D.C., and by R. G. Herb *et al.* at the University of Wisconsin. The theoretical interpretation was mostly done by Breit and collaborators. The system of two protons, at orbital momentum $L=0$, can exist only in the state of total spin $S=0$. The phase shift is the shift relative to a pure Coulomb field. The scattering length resulting from the analysis is close to that of the ^1S state of the proton-neutron system. This is the most direct evidence for charge independence of nuclear forces. There is, however, a slight difference: the proton-neutron force is slightly more attractive than the proton-proton force.

Before World War II, the maximum particle energy available was less than about 20 MeV. Therefore only the S-state interaction between two nucleons could be investigated.

III. THE LIQUID DROP MODEL

A. Energy

The most conspicuous feature of nuclei is that their binding energy is nearly proportional to A, the number of nucleons in the nucleus. Thus the binding per particle is nearly constant, as it is for condensed matter. This is in contrast to electrons in an atom: the binding of a $1S$ electron increases as Z^2.

The volume of a nucleus, according to Eq. (1), is also proportional to A. This and the binding energy are the basis of the liquid drop model of the nucleus, used especially by Niels Bohr: the nucleus is conceived as filling a compact volume, spherical or other shape, and its energy is the sum of an attractive term proportional to the volume, a repulsive term proportional to the surface, and another term due to the mutual electric repulsion of the positively charged protons. In the volume energy, there is also a positive term proportional to $(N-Z)^2 = (A-2Z)^2$ because the attraction between proton and neutron is stronger than between two like particles. Finally, there is a pairing energy: two like particles tend to go into the same quantum state, thus decreasing the energy of the nucleus. A combination of these terms leads to the Weizsäcker semi-empirical formula

$$E = -a_1 A + a_2 A^{2/3} + a_3 Z^2 A^{-1/3}$$
$$+ a_4 (A-2Z)^2 A^{-1} + \lambda a_5 A^{-3/4}. \tag{9}$$

Over the years, the parameters $a_1, \ldots, a_5$ have been determined. Green (1954) gives these values (in MeV):

$$a_1 = 15.75, \quad a_2 = 17.8,$$
$$a_3 = 0.710, \quad a_4 = 23.7,$$
$$a_5 = 34. \tag{10}$$

The factor λ is +1 if Z and $N=A-Z$ are both odd, $\lambda = -1$ if they are both even, and $\lambda = 0$ if A is odd. Many more accurate expressions have been given.

For small mass number A, the symmetry term $(N-Z)^2$ puts the most stable nucleus at $N=Z$. For larger A, the Coulomb term shifts the energy minimum to $Z < A/2$.

Among very light nuclei, the energy is lowest for those which may be considered multiples of the α particle, such as ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{40}Ca. For $A=56$, ^{56}Ni ($Z=28$) still has strong binding but ^{56}Fe ($Z=26$) is more strongly bound. Beyond $A=56$, the preference for multiples of the α particle ceases.

For nearly all nuclei, there is preference for even Z and even N. This is because a pair of neutrons (or protons) can go into the same orbital and can then have maximum attraction.

Many nuclei are spherical; this giving the lowest surface area for a given volume. But when there are many nucleons in the same shell (see Sec. VII), ellipsoids, or even more complicated shapes (Nielsen model), are often preferred.

B. Density distribution

Electron scattering is a powerful way to measure the charge distribution in a nucleus. Roughly, the angular distribution of elastic scattering gives the Fourier transform of the radial charge distribution. But since $Ze^2/\hbar c$ is quite large, explicit calculation with relativistic electron wave functions is required. Experimentally, Hofstadter at Stanford started the basic work.

In heavy nuclei, the charge is fairly uniformly distributed over the nuclear radius. At the surface, the density falls off approximately like a Fermi distribution,

$$\rho/\rho_0 \approx [1+\exp(r-R)/a]^{-1}, \tag{11}$$

with $a \approx 0.5$ fm; the surface thickness, from 90% to 10% of the central density, is about 2.4 fm.

In more detailed studies, by the Saclay and Mainz groups, indications of individual proton shells can be discerned. Often, there is evidence for nonspherical shapes. The neutron distribution is more difficult to determine experimentally; sometimes the scattering of π mesons is useful. Inelastic electron scattering often shows a maximum at the energy where scattering of the electron by a single free proton would lie.

C. α radioactivity

Equation (9) represents the energy of a nucleus relative to that of free nucleons, $-E$ is the binding energy. The mass excess of Z protons and $(A-Z)$ neutrons is

$$\Delta M = 7.3Z + 8.1(A-Z)\ \text{MeV}, \tag{12}$$

which complies with the requirement that the mass of ^{12}C is 12 amu. The mass excess of the nucleus is

$$E+\Delta M = E + 7.3Z + 8.1(A-Z)\ \text{MeV}. \tag{13}$$

The mass excess of an α particle is 2.4 MeV, or 0.6 MeV per nucleon. So the excess of the mass of nucleus (Z,A) over that of $Z/2$ α particles plus $A-2Z$ neutrons is

$$\begin{aligned} E' &= E + \Delta M - (Z/2)0.6\ \text{MeV} \\ &= E + 7.0Z + 8.1(A-Z). \end{aligned} \tag{14}$$

The (smoothed) energy available for the emission of an α particle is then

$$E''(Z,A) = E'(Z,A) - E'(Z-2,A-4). \tag{15}$$

This quantity is negative for small A, positive from about the middle of the periodic table on. When it becomes greater than about 5 MeV, emission of α particles becomes observable. This happens when $A \geqslant 208$. It helps that $Z=82$, $A=208$ is a doubly magic nucleus.

D. Fission

In the mid 1930s, Fermi's group in Rome bombarded samples of most elements with neutrons, both slow and fast. In nearly all elements, radioactivity was produced. Uranium yielded several distinct activities. Lise Meitner, physicist, and Otto Hahn, chemist, continued this research in Berlin and found some sequences of radioactivities following each other. When Austria was annexed to Germany in Spring 1938, Meitner, an Austrian Jew, lost her job and had to leave Germany; she found refuge in Stockholm.

Otto Hahn and F. Strassmann continued the research and identified chemically one of the radioactive products from uranium $(Z=92)$. To their surprise they found the radioactive substance was barium, $(Z=56)$. Hahn, in a letter to Meitner, asked for help. Meitner discussed it with her nephew, Otto Frisch, who was visiting her. After some discussion, they concluded that Hahn's findings were quite natural, from the standpoint of the liquid drop model: the drop of uranium split in two. They called the process "fission."

Once this general idea was clear, comparison of the atomic weight of uranium with the sum of the weights of the fission products showed that a very large amount of energy would be set free in fission. Frisch immediately proved this, and his experiment was confirmed by many laboratories. Further, the fraction of neutrons in the nucleus, $N/A=(A-Z)/A$, was much larger in uranium than in the fission products hence neutrons would be set free in fission. This was proved experimentally by Joliot and Curie. Later experiments showed that the average number of neutrons per fission was $\nu=2.5$. This opened the prospect of a chain reaction.

A general theory of fission was formulated by Niels Bohr and John Wheeler in 1939. They predicted that only the rare isotope of uranium, U-235, would be fissionable by slow neutrons. The reason was that U-235 had an odd number of neutrons. After adding the neutron from outside, both fission products could have an even number of neutrons, and hence extra binding energy due to the formation of a neutron pair. Conversely, in U-238 one starts from an even number of neutrons, so one of the fission products must have an odd number. Nier then showed experimentally that indeed U-235 can be fissioned by slow neutrons while U-238 requires neutrons of about 1 MeV.

E. The chain reaction

Fission was discovered shortly before the outbreak of World War II. There was immediate interest in the chain reaction in many countries.

To produce a chain reaction, on average at least one of the 2.5 neutrons from a U-235 fission must again be captured by a U-235 and cause fission. The first chain reaction was established by Fermi and collaborators on 2 December 1942 at the University of Chicago. They used a "pile" of graphite bricks with a lattice of uranium metal inside.

The graphite atoms served to slow the fission neutrons, originally emitted at about 1 MeV energy, down to thermal energies, less than 1 eV. At those low energies, capture by the rare isotope U-235 competes favorably with U-238. The carbon nucleus absorbs very few neutrons, but the graphite has to be very pure C. Heavy water works even better.

The chain reaction can either be controlled or explosive. The Chicago pile was controlled by rods of boron absorber whose position could be controlled by the operator. For production of power, the graphite is cooled by flowing water whose heat is then used to make steam. In 1997, about 400 nuclear power plants were in operation (see Till, 1999).

In some experimental ''reactors,'' the production of heat is incidental. The reactor serves to produce neutrons which in turn can be used to produce isotopes for use as tracers or in medicine. Or the neutrons themselves may be used for experiments such as determining the structure of solids.

Explosive chain reactions are used in nuclear weapons. In this case, the U-235 must be separated from the abundant U-238. The weapon must be assembled only immediately before its use. Plutonium-239 may be used instead of U-235 (see Drell, 1999).

IV. THE TWO-NUCLEON INTERACTION

A. Experimental

A reasonable goal of nuclear physics is the determination of the interaction of two nucleons as a function of their separation. Because of the uncertainty principle, this requires the study of nuclear collisions at high energy. Before the second World War, the energy of accelerators was limited. After the war, cyclotrons could be built with energies upward of 100 MeV. This became possible by modulating the frequency, specifically, decreasing it on a prescribed schedule as any given batch of particles, e.g., protons, is accelerated. The frequency of the accelerating electric field must be

$$\omega \sim B/m_{\mathrm{eff}},$$

in order to keep that field in synchronism with the orbital motion of the particles. Here B is the local magnetic field which should decrease (slowly) with the distance r from the center of the cyclotron in order to keep the protons focused; $m_{\mathrm{eff}}=E/c^2$ is the relativistic mass of the protons which increases as the protons accelerate and r increases. Thus the frequency of the electric field between the dees of the cyclotron must decrease as the protons accelerate.

Such frequency modulation (FM) had been developed in the radar projects during World War II. At the end of that war, E. McMillan in the U.S. and Veksler in the Soviet Union independently suggested the use of FM in the cyclotron. It was introduced first at Berkeley and was immediately successful. These FM cyclotrons were built at many universities, including Chicago, Pittsburgh, Rochester, and Birmingham (England).

The differential cross section for the scattering of protons by protons at energies of 100 to 300 MeV was soon measured. But since the proton has spin, this is not enough: the scattering of polarized protons must be measured for two different directions of polarization, and as a function of scattering angle. Finally, the change of polarization in scattering must be measured. A complete set of required measurements is given (Walecka, 1995). The initial polarization, it turns out, is best achieved by scattering the protons from a target with nuclei of zero spin, such as carbon.

TABLE I. P and D phase shifts at 300 MeV, in degrees.

^1P	−28	$^1D_2$	+25
$^3P_0$	−10	$^3D_1$	−24
$^3P_1$	−28	$^3D_2$	+25
$^3P_2$	+17	$^3D_3$	+4

Proton-proton scattering is relatively straightforward, but in the analysis the effect of the Coulomb repulsion must, of course, be taken into account. It is relatively small except near the forward direction. The nuclear force is apt to be attractive, so there is usually an interference minimum near the forward direction.

The scattering of neutrons by protons is more difficult to measure, because there is no source of neutrons of definite energy. Fortunately, when fast protons are scattered by deuterons, the deuteron often splits up, and a neutron is projected in the forward direction with almost the full energy of the initial proton.

B. Phase shift analysis

The measurements can be represented by phase shifts of the partial waves of various angular momenta. In proton-proton scattering, even orbital momenta occur only together with zero total spin (singlet states), odd orbital momenta with total spin one (triplet states). Phase shift analysis appeared quite early, e.g., by Stapp, Ypsilantis, and Metropolis in 1957. But as long as only experiments at one energy were used, there were several sets of phase shifts that fitted the data equally well. It was necessary to use experiments at many energies, derive the phase shifts and demand that they depend smoothly on energy.

A very careful phase shift analysis was carried out by a group in Nijmegen, Netherlands, analyzing first the *pp* and the *np* (neutron-proton) scattering up to 350 MeV (Bergervoet *et al.*, 1990). They use *np* data from well over 100 experiments from different laboratories and energies. Positive phase shifts means attraction.

As is well known, S waves are strongly attractive at low energies, e.g., at 50 MeV, the ^3S phase shift is 60°, ^1S is 40°. ^3S is more attractive than ^1S, just as, at $E=0$, there is a bound ^3S state but not of ^1S. At high energy, above about 300 MeV, the S phase shifts become repulsive, indicating a repulsive core in the potential.

The P and D phase shifts at 300 MeV are shown in Table I (Bergervoet *et al.*, 1990). The singlet states are attractive or repulsive, according to whether L is even or odd. This is in accord with the idea prevalent in early nuclear theory (1930s) that there should be exchange forces, and it helps nuclear forces to saturate. The triplet states of $J=L$ have nearly the same phase shifts as the corresponding singlet states. The triplet states show a tendency toward a spin-orbit force, the higher J being more attractive than the lower J.

C. Potential

In the 1970s, potentials were constructed by the Bonn and the Paris groups. Very accurate potentials, using the Nijmegen data base were constructed by the Nijmegen and Argonne groups.

We summarize some of the latter results, which include the contributions of vacuum polarization, the magnetic moment interaction, and finite size of the neutron and proton. The longer range nuclear interaction is one-pion exchange (OPE). The shorter-range potential is a sum of central, L^2, tensor, spin-orbit and quadratic spin-orbit terms. A short range core of $r_0 = 0.5$ fm is included in each. The potential fits the experimental data very well: excluding the energy interval 290–350 MeV, and counting both pp and np data, their $\chi^2 = 3519$ for 3359 data.

No attempt is made to compare the potential to any meson theory. A small charge dependent term is found. The central potential is repulsive for $r < 0.8$ fm; its minimum is -55 MeV. The maximum tensor potential is about 50 MeV, the spin-orbit potential at 0.7 fm is about 130 MeV.

D. Inclusion of pion production

Nucleon-nucleon scattering ceases to be elastic once pions can be produced. Then all phase shifts become complex. The average of the masses of π^+, π^0, and π^- is 138 MeV. Suppose a pion is made in the collision of two nucleons, one at rest (mass M) and one having energy $E > M$ in the laboratory. Then the square of the invariant mass is initially

$$(E+M)^2 - P^2 = 2M^2 + 2EM. \tag{16}$$

Suppose in the final state the two nucleons are at rest relative to each other, and in their rest system a pion is produced with energy ε, momentum π, and mass μ. Then the invariant mass is

$$(2M+\varepsilon)^2 - \pi^2 = 4M^2 + 4M\varepsilon + \mu^2. \tag{17}$$

Setting the two invariant masses equal,

$$E - M = 2\varepsilon + \mu^2/2M, \tag{18}$$

a remarkably simple formula for the initial kinetic energy in the laboratory. The absolute minimum for meson production is 286 MeV. The analysts have very reasonably chosen $E - M = 350$ MeV for the maximum energy at which nucleon-nucleon collision may be regarded as essentially elastic.

V. THREE-BODY INTERACTION

The observed binding energy of the triton, ^{3}H, is 8.48 MeV. Calculation with the best two-body potential gives 7.8 MeV. The difference is attributed to an interaction between all three nucleons. Meson theory yields such an interaction based on the transfer of a meson from nucleon i to j, and a second meson from j to k. The main term in this interaction is

$$V_{ijk} = A\,Y(mr_{ij})Y(mr_{jk})\sigma_i\cdot\sigma_j\sigma_j\cdot\sigma_k\tau_i\cdot\tau_j\tau_j\cdot\tau_k\,, \tag{19}$$

where Y is the Yukawa function,

$$Y(mr) = \frac{\exp(-mcr/\hbar)}{mcr/\hbar}. \tag{20}$$

The cyclic interchanges have to be added to V_{123}. There is also a tensor force which has to be suitably cut off at small distances. It is useful to also add a repulsive central force at small r.

The mass m is the average of the three π mesons, $m = \frac{1}{3}m_{\pi^0} + \frac{2}{3}m_{\pi^\pm}$. The coefficient A is adjusted to give the correct ^{3}H binding energy and the correct density of nuclear matter. When this is done, the binding energy of ^{4}He automatically comes out correctly, a very gratifying result. So no four-body forces are needed.

The theoretical group at Argonne then proceed to calculate nuclei of atomic weight 6 to 8. They used a Green's function Monte Carlo method to obtain a suitable wave function and obtained the binding energy of the ground state to within about 2 MeV. For very unusual nuclei like ^{7}He or ^{8}Li, the error may be 3–4 MeV. Excited states have similar accuracy, and are arranged in the correct order.

VI. NUCLEAR MATTER

"Nuclear matter" is a model for large nuclei. It assumes an assembly of very many nucleons, protons, and neutrons, but disregards the Coulomb force. The aim is to calculate the density and binding energy per nucleon. In first approximation, each nucleon moves independently, and because we have assumed a very large size, its wave function is an exponential, $\exp(i\mathbf{k}\cdot\mathbf{r})$. Nucleons interact, however, with their usual two-body forces; therefore, the wave functions are modified wherever two nucleons are close together. Due to its interactions, each nucleon has a potential energy, so a nucleon of wave vector $\mathbf{k}$ has an energy $E(k) \neq (\hbar^2/2m)k^2$.

Consider two particles of momenta $\mathbf{k}_1$ and $\mathbf{k}_2$; their unperturbed energy is

$$W = E(k_1) + E(k_2), \tag{21}$$

and their unperturbed wave function is

$$\phi = \exp[i\mathbf{P}\cdot(\mathbf{r}_1+\mathbf{r}_2)] \times \exp[i\mathbf{k}_0\cdot(\mathbf{r}_1-\mathbf{r}_2)], \tag{22}$$

where $\mathbf{P} = (\mathbf{k}_1+\mathbf{k}_2)/2$ and $\mathbf{k}_0 = 1/2(\mathbf{k}_1-\mathbf{k}_2)$. We disregard the center-of-mass motion and consider

$$\phi = e^{i\mathbf{k}_0\cdot\mathbf{r}}, \tag{23}$$

as the unperturbed wave function. Under the influence of the potential v this is modified to

$$\psi = \phi - (Q/e)v\psi. \tag{24}$$

Here $v\psi$ is considered to be expanded in plane wave states k_1', k_2', and

$$e=E(k_1')+E(k_2')-W, \tag{25}$$

$$Q=1 \text{ if states } k_1' \text{ and } k_2' \text{ are both unoccupied,}$$
$$Q=0 \text{ otherwise.} \tag{26}$$

Equation (26) states the Pauli principle and ensures that $e>0$ always. It is assumed that the occupied states fill a Fermi sphere of radius k_F.

We set

$$v\psi=G\phi, \tag{27}$$

and thus define the reaction matrix G, which satisfies the equation

$$\left.\begin{aligned}&\langle \mathbf{k}|G|\mathbf{k}_0;\mathbf{P},W\rangle=\langle \mathbf{k}|v|\mathbf{k}_0\rangle-(2\pi)^{-3}\int d^3k' \\ &\langle \mathbf{k}|v|\mathbf{k}'\rangle\frac{Q(\mathbf{P},k')}{E(\mathbf{P}+\mathbf{k}')+E(\mathbf{P}-\mathbf{k}')-W}\langle \mathbf{k}'|G|\mathbf{k};\mathbf{P},W\rangle\end{aligned}\right\}. \tag{28}$$

This is an integral equation for the matrix $\langle k|G|k_0\rangle$. P and W are merely parameters in this equation.

The diagonal elements $\langle k|G|k_0,P\rangle$ can be transcribed into the k_1, k_2 of the interacting nucleons. The one-particle energies are then

$$W(k_1)=\sum_{k_2}\langle k_1k_2|G|k_1k_2\rangle+(\hbar^2/2M)k_1^2. \tag{29}$$

With modern computers, the matrix Eq. (28) can be solved for any given potential v. In the 1960s, approximations were used. First it was noted that for states outside the Fermi sphere, G was small; then $E(\mathbf{P}\pm\mathbf{k}')$ in the denominator of Eq. (28) was replaced by the kinetic energy. Second, for the occupied states, the potential energy was approximated by a quadratic function,

$$W(k)=(\hbar^2/2M^*)k^2, \tag{30}$$

M^* being an effective mass.

It was then possible to obtain the energy of nuclear matter as a function of its density. But the result was not satisfactory. The minimum energy was found at too high a density, about 0.21 fm^{-3} instead of the observed 0.16 fm^{-3}. The binding energy was only 11 MeV instead of the observed 16 MeV.

Modern theory has an additional freedom, the three-body interaction. Its strength can be adjusted to give the correct density. But the binding energy, according to the Argonne-Urbana group, is still only 12 MeV. They believe they can improve this by using a more sophisticated wave function.

In spite of its quantitative deficiencies nuclear matter theory gives a good general approach to the interaction of nucleons in a nucleus. This has been used especially by Brown and Kuo (1966) in their theory of interaction of nucleons in a shell.

VII. SHELL MODEL

A. Closed shells

The strong binding of the α particle is easily understood; a pair of neutrons and protons of opposite spin, with deep and attractive potential wells, are the qualitative explanation. The next proton or neutron must be in a relative p state, so it cannot come close, and, in addition, by the exchange character of the forces (see Sec. IV.C), the interaction with the α particle is mainly repulsive: thus there is no bound nucleus of $A=5$, neither ^{5}He nor ^{5}Li. The α particle is a closed unit, and the most stable light nuclei are those which may be considered to be multiples of the α particles, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, etc.

But even among these α-particle nuclei, ^{16}O is special: the binding energy of α to ^{12}C, to form ^{16}O, is considerably larger than the binding of α to ^{16}O. Likewise, ^{40}Ca is special: it is the last nucleus "consisting" of α particles only which is stable against β decay.

The binding energies can be understood by considering nuclei built up of individual nucleons. The nucleons may be considered moving in a square well potential with rounded edges, or more conveniently, an oscillator potential of frequency ω. The lowest state for a particle in that potential is a $1s$ state of energy ε_0. There are two places in the $1s$ shell, spin up and down; when they are filled with both neutrons and protons, we have the α particle.

The next higher one-particle state is $1p$, with energy $\varepsilon_0+\hbar\omega$. The successive eigenstates are

$$(1s),\quad (1p),\quad (1d2s),\quad (1f2p),\quad (1g2d3s)$$

with energies

$$(\varepsilon_0),\quad (\varepsilon_0+\hbar\omega),\quad (\varepsilon_0+2\hbar\omega),\quad (\varepsilon_0+3\hbar\omega).$$

The principal quantum number is chosen to be equal to the number of radial nodes plus one. The number of independent eigenfunctions in each shell are

$$(2),\quad (6),\quad (12),\quad (20),\quad (30),$$

so the total number up to any given shell are

$$(2),\quad (8),\quad (20),\quad (40),\quad (70),\quad \ldots.$$

The first three of these numbers predict closed shells at ^{4}He, ^{10}O, and ^{40}Ca, all correct. But $Z=40$ or $N=40$ are not particularly strongly bound nuclei.

The solution to this problem was found independently by Maria Goeppert-Mayer and H. Jensen: nucleons are subject to a strong spin-orbit force which gives added attraction to states with $j=\ell+1/2$, repulsion to $j=\ell-1/2$. This becomes stronger with increasing j. The strongly bound nucleons beyond the $1d2s$ shell, are

$$(1f_{7/2}),\quad (2p1f_{5/2}1g_{9/2}),\quad (2d3s1g_{7/2}1h_{11/2}),$$
$$(2f3p1h_{9/2}1i_{13/2}).$$

The number of independent eigenfunctions in these shells are, respectively,

$$(8),\quad (22),\quad (32),\quad (44).$$

So the number of eigenstates up to $1f_{7/2}$ is 28, up to $1g_{9/2}$ is 50, up to $1h_{11/2}$ is 82, and up to $1i_{13/2}$ is 126. Indeed, nuclei around $Z=28$ or $N=28$ are particularly strongly bound. For example, the last α particle in ^{56}Ni ($Z=N=28$) is bound with 8.0 MeV, while the next α particle, in ^{60}Zn ($Z=N=30$) has a binding energy of only 2.7 MeV. Similarly, ^{90}Zr ($N=50$) is very strongly bound and Sn, with $Z=50$, has the largest number of stable isotopes. ^{208}Pb ($Z=82, N=126$) has closed shells for protons as well as neutrons, and nuclei beyond Pb are unstable with respect to α decay. The disintegration $^{212}\mathrm{Po}\rightarrow{}^{208}\mathrm{Pb}+\alpha$ yields α particles of 8.95 MeV while $^{208}\mathrm{Pb}\rightarrow{}^{204}\mathrm{Hg}+\alpha$ would release only 0.52 MeV, and an α particle of such low energy could not penetrate the potential barrier in 10^{10} years. So there is good evidence for closed nucleon shells.

Nuclei with one nucleon beyond a closed shell, or one nucleon missing, generally have spins as predicted by the shell model.

B. Open shells

The energy levels of nuclei with partly filled shells are usually quite complicated. Consider a nucleus with the 44-shell about half filled: there will be of the order of $2^{44}\approx 10^{13}$ different configurations possible. It is obviously a monumental task to find the energy eigenvalues.

Some help is the idea of combining a pair of orbitals of the same j and m values of opposite sign. Such pairs have generally low energy, and the pair acts as a boson. Iachello and others have built up states of the nucleus from such bosons.

VIII. COLLECTIVE MOTIONS

Nuclei with incomplete shells are usually not spherical. Therefore their orientation in space is a significant observable. We may consider the rotation of the nucleus as a whole. The moment of inertia θ is usually quite large; therefore, the rotational energy levels which are proportional to $1/\theta$ are closely spaced. The lowest excitations of a nucleus are rotations.

Aage Bohr and Ben Mottleson have worked extensively on rotational states and their combination with intrinsic excitation of individual nucleons. There are also vibrations of the nucleus, e.g., the famous vibration of all neutrons against all protons, the giant dipole state at an excitation energy of 10–20 MeV, depending on the mass number A.

Many nuclei, in their ground state, are prolate spheroids. Their rotations then are about an axis perpendicular to their symmetry axis, and an important characteristic is their quadrupole moment. Many other nuclei have more complicated shapes such as a pear; they have an octopole moment, and their rotational states are complicated.

IX. WEAK INTERACTIONS

Fermi, in 1934, formulated the the first theory of the weak interaction on the basis of Pauli's neutrino hypothesis. An operator of the form

$$\overline{\phi_e}\phi_\nu\overline{\psi_p}\psi_n \tag{31}$$

creates an electron ϕ_e and an antineutrino $\overline{\phi_\nu}$, and converts a neutron ψ_n into a proton ψ_p. The electron and the neutrino are not in the nucleus, but are created in the β process. All operators are taken at the same point in space-time.

Fermi assumed a vector interaction in his first β-decay paper.

The Fermi theory proved to be essentially correct, but Gamov and Teller later introduced other covariant combinations allowed by Dirac theory. Gamov and Teller said there could be a product of two 4-vectors, or tensors, or axial vectors, or pseudoscalars. Experiment showed later on that the actual interaction is

$$\text{Vector minus Axial vector,} \tag{32}$$

and this could also be justified theoretically.

The β-process, Eq. (31), can only happen if there is a vacancy in the proton state ψ_p. If there is in the nucleus a neutron of the same orbital momentum, we have an allowed transition, as in $^{13}\mathrm{N}\rightarrow{}^{13}\mathrm{C}$. If neutron and proton differ by units in angular momentum, so must the leptons. The wave number of the leptons is small, then the product $(kR)^L$ is very small if L is large: such β transitions are highly forbidden. An example is ^{40}K which has angular momentum $L=4$ while the daughter ^{40}Ca has $L=0$. The radioactive ^{40}K has a half-life of 1.3×10^9 years.

This theory was satisfactory to explain observed β decay, but it was theoretically unsatisfactory to have a process involving four field operators at the same space-time point. Such a theory cannot be renormalized. So it was postulated that a new charged particle W was involved which interacted both with leptons and with baryons, by interactions such as

$$\overline{\phi_e}W\overline{\phi_\nu}, \quad \overline{\psi_p}W\psi_n.$$

This W particle was discovered at CERN and has a mass of 80 GeV. These interactions, involving three rather than four operators, are renormalizable. The high mass of W ensures that in β-decay all the operators ψ_n, ψ_p, ϕ_ν, ϕ_e have to be taken essentially at the same point, within about 10^{-16} cm, and the Fermi theory results.

A neutral counterpart to W, the Z particle, was also found at CERN; it can decay into a pair of electrons, a pair of neutrinos, or a pair of baryons. Its mass has been determined with great accuracy,

$$m(Z)=91 \text{ GeV}. \tag{33}$$

The difference in masses of Z and W is of great theoretical importance. The mass of Z has a certain width from which the number of species of neutrinos can be determined, namely three: ν_e, ν_μ, and ν_τ.

X. NUCLEOSYNTHESIS

It is an old idea that matter consisted "originally" of protons and electrons, and that complex nuclei were gradually formed from these (see Salpeter, 1999). (Modern theories of the big bang put "more elementary" particles, like quarks, even earlier, but this is of no concern here.) At a certain epoch, some neutrons would be formed by

$$H+e^- \rightarrow N+\nu. \tag{34}$$

These neutrons would immediately be captured by protons,

$$\mathrm{N+H \rightarrow D}+\gamma, \tag{35}$$

and the deuterons would further capture protons, giving ^{3}He and ^{4}He. This sequence of reactions, remarkably, leads to a rather definite fraction of matter in ^{4}He nuclei, namely

$$^4\mathrm{He} \approx 23\%, \tag{36}$$

nearly all the rest remaining H. Traces of D, ^{3}He, and ^{7}Li remain.

Again remarkably, there exist very old stars (in globular clusters) in which the fraction of ^{4}He can be measured, and it turns out to be just 23%. This fraction depends primarily on the number of neutrino species which, as mentioned at the end of Sec. IX is three.

In stars like the sun and smaller, nuclear reactions take place in which H is converted into He at a temperature of the order of 10–20 million degrees, and the released energy is sent out as radiation. If, at later stages in the evolution, some of the material of such a star is lost into the galaxy, the fraction of ^{4}He in the galaxy increases, but very slowly.

In a star of three times the mass of the sun or more, other nuclear processes occur. Early in its life (on the main sequence), the star produces energy by converting H into He in its core. But after a long time, say a billion years, it has used up the H in its core. Then the core contracts and gets to much higher temperatures, of the order of 100 million degrees or more. Then α particles can combine,

$$3\ ^4\mathrm{He} \rightarrow {}^{12}\mathrm{C}+\gamma. \tag{37}$$

Two ^{4}He cannot merge, since ^{8}Be is slightly heavier than two ^{4}He, but at high temperature and density, ^{8}Be can exist for a short time, long enough to capture another ^{4}He. Equation (37) was discovered in 1952 by E. E. Salpeter; it is the crucial step.

Once ^{12}C has formed, further ^{4}He can be captured and heavier nuclei built up. This happens especially in the inner part of stars of 10 or more times the mass of the sun. The buildup leads to ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, and on to ^{56}Ni. The latter is the last nucleus in which the α particle is strongly bound (see Sec. VII). But it is unstable against β decay; by two emissions of positrons it transforms into ^{56}Fe. This makes ^{56}Fe one of the most abundant isotopes beyond ^{16}O. After forming all these elements, the interior of the star becomes unstable and collapses by gravitation. The energy set free by gravitation then expels all the outer parts of the star (all except the innermost $1.5M_\odot$) in a supernova explosion and thus makes the elements formed by nucleosynthesis available to the galaxy at large.

Many supernovae explosions have taken place in the galaxy, and so galactic matter contains a fair fraction Z of elements beyond C, called "metals" by astrophysicists, viz., $Z \simeq 2\%$. This is true in the solar system, formed about 4.5 billion years ago. New stars should have a somewhat higher Z, old stars are known to have smaller Z.

Stars of $M \geqslant 3M_\odot$ are formed from galactic matter that already contains appreciable amounts of heavy nuclei up to ^{56}Fe. Inside the stars, the carbon cycle of nuclear reactions takes place, in which ^{14}N is the most abundant nucleus. If the temperature then rises to about 100 million degrees, neutrons will be produced by the reactions

$$\begin{aligned} &^{14}\mathrm{N}+{}^4\mathrm{He} \rightarrow {}^{17}\mathrm{F}+n, \\ &^{17}\mathrm{O}+{}^4\mathrm{He} \rightarrow {}^{20}\mathrm{Ne}+n. \end{aligned} \tag{38}$$

The neutrons will be preferentially captured by the heavy nuclei already present and will gradually build up heavier nuclei by the *s*-process described in the famous article by E.M. and G. R. Burbidge, Fowler, and Hoyle in *Reviews of Modern Physics* (1957).

Some nuclei, especially the natural radioactive ones, U and Th, cannot be built up in this way, but require the *r*-process, in which many neutrons are added to a nucleus in seconds so there is no time for β decay. The conditions for the *r*-process have been well studied; they include a temperature of more than 10^9 K. This condition is well fulfilled in the interior of a supernova a few seconds after the main explosion, but there are additional conditions so that it is still uncertain whether this is the location of the *r*-process.

XI. SPECIAL RELATIVITY

For the scattering of nucleons above about 300 MeV, and for the equation of state of nuclear matter of high density, special relativity should be taken into account. A useful approximation is mean field theory which has been especially developed by J. D. Walecka.

Imagine a large nucleus. At each point, we can define the conserved baryon current $i\bar{\psi}\gamma_\mu\psi$ where ψ is the baryon field, consisting of protons and neutrons. We also have a scalar baryon density $\bar{\psi}\psi$. They couple, respectively, to a vector field V_μ and a scalar field ϕ with coupling constants g_w and g_s. The vector field is identified with the ω meson, giving a repulsion, and the scalar field with the σ meson, giving an attraction. Coupling constants can be adjusted so as to give a minimum energy of -16 MeV per nucleon and equilibrium density of 0.16 fm^{-3}.

The theory can be generalized to neutron matter and thus to the matter of neutron stars. It can give the charge distribution of doubly magic nuclei, like ^{208}Pb, ^{40}Ca, and ^{16}O, and these agree very well with the distributions observed in electron scattering.

The most spectacular application is to the scattering of 500 MeV protons by ^{40}Ca, using the Dirac relativistic impulse approximation for the proton. Not only are cross section minima at the correct scattering angles, but polarization of the scattered protons is almost complete, in agreement with experiment, and the differential cross section at the second, third, and fourth maximum also agree with experiment.

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