Nuclear Physics - Problem Set 8 – Solution

Problem 1)

a) The first part of the given wave function of course fulfills the equation
\[ H_0 e^{ikr} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial r^2} e^{ikr} = E e^{ikr}, \]

as is well known from elementary Quantum Mechanics. For the second part, we use the expression for the Laplacian in spherical coordinates as given in the problem:

\[ H_0 f(\theta, \phi) \frac{e^{ikr}}{r} = -\frac{\hbar^2}{2m} f(\theta, \phi) \frac{e^{ikr}}{r} = \]

\[ = -\frac{\hbar^2}{2m} \left[ f(\theta, \phi) \frac{1}{r^2} \frac{\partial}{\partial r} \left( \frac{1}{r^2} \frac{\partial}{\partial r} e^{ikr} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} f(\theta, \phi) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} f(\theta, \phi) \right] \frac{e^{ikr}}{r} \]

The first part of the last expression is indeed of the form \( H\psi = E\psi \) with the same eigenvalue \( E \), while the second part falls off with an additional factor \( 1/r^2 \) and therefore becomes negligible in the limit \( r \to \infty \), as long as the angular derivatives inside the parentheses are well-behaved (finite).

b) This is a simple plug-in problem:
Problem 2)

a) Since this is a non-relativistic situation, I find (with $E = 2.5$ MeV)

$$k_{\text{out}} = (2 \mu c^2 E)^{1/2} / \hbar c = (m_{\text{proton}} c^2 2.5 \, \text{MeV})^{1/2} / \hbar c = 48.4 \, \text{MeV} / \hbar c = 0.245 \, \text{fm}^{-1}$$

(remember, $\mu$ is $m_{\text{proton}}/2$ here). The cross section is

$$\frac{d\sigma}{d\Omega} = \left| f(\theta) \right|^2 = \frac{1}{k^2} \sin^2(\delta_\theta) = 9.78 \, \text{fm}^2 = 97.8 \, \text{mb}$$

b) I use equation 17.7 on page 290 of Povh et al. with $k_{\text{out}} = (2 \mu c^2 E)^{1/2} / \hbar c = 0.245 \, \text{fm}^{-1}$ and $a = 1.6 \, \text{fm}$. Subtracting the last term on the rhs from both sides and taking the tan on both sides, I get a transcendental equation that I can solve for $|V|$. The numerical solution I find is

$$|V_0| = 33.355 \, \text{MeV}.$$ (Just plug it in the equation to convince yourself that this is correct.)

This result is a very rough approximation, although it is at least the right order of magnitude (compare to the plot on page 291 - you’d have to average out the exponential, negative potential at $r>0.8 \, \text{fm}$ and the repulsive core at smaller $r$).