Nuclear Physics - Problem Set 8 – Due MONday, 11/19

Note: You have LOTS of time (nearly 2 weeks) for this problem but don't wait too long to start – better finish this one earlier and get started on #9!
P.S.: Please forgive the funny symbols for vectors – instead of an arrow, they ended up looking like an "r" above the verctors.

Problem 1)

In lecture, we claimed that a wave function of the form

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f(\theta,\varphi)\frac{e^{ikr}}{r}$$

is an "approximate" Eigenfunction of the free Hamiltonian,

$$H_0 = \frac{\vec{p}^2}{2m} = -\hbar^2\frac{\vec{\nabla}^2}{2m}.$$

a) Using the expression for the Laplacian in spherical coordinates,

$$\vec{\nabla}^2 = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$, for the second part,

show that indeed both parts of this wave function fulfill the equation $H\psi = E\psi$ with the same Eigenvalue E in the limit when r -> ∞ (you may assume that $f(\theta,\phi)$ and its derivatives are well-behaved and finite for all angles).

b) Calculate the **radial** flux density (i.e., the r-component of the flux density vector) of the second part of the wave function and show that it indeed has the form $j_r = \frac{\hbar}{2mi}\left[\psi^*\left(\frac{\partial}{\partial r}\psi\right) - \left(\frac{\partial}{\partial r}\psi^*\right)\psi\right] = v|\psi|^2$ with velocity v.

Problem 2)

At a kinetic energy of $E = T_{kin} = 2.5$ MeV in the center of mass, the p-p scattering (ℓ=0, spin singlet) phase shift δ_0 is about 50 degrees.

a) Calculate the cross section $\frac{d\sigma}{d\Omega}$ (it's independent of angle, since we assume that only ℓ=0 contributes). Note that $p = (2\mu T_{kin})^{1/2}$

b) Given that the root mean square radius of a proton is about 0.8 fm, what can you infer about the depth V_0 of the "square well potential" between protons? (Assume the potential is constant – $V(r) = -V_0$ – for proton-proton distance $r < R = 1.6$ fm and zero for larger distance – obviously a very crude approximation. The factor 2 comes about because the protons feel their mutual attraction as soon as they touch, i.e., if their centers are 2 radii apart). *Hint:* You'll have to use numerical methods to find a solution for the equation relating the phase shift to the potential depth and radius. There are several solutions; please take the one with the smallest value for the potential depth (it will be similar to what we found for the depth of the nucleon potential in the Fermi gas model). Also, please remember to use the REDUCED mass μ for the p-p system. All necessary formulae can be found in our books - you may have to hunt from them, though (way in the back). *CAREFUL:* While I gave you the phase shift itself in degrees, you'll have to use the radians setting on your calculator throughout your calculation (and convert the phase shift into radians as well).