Scattering

Nuclear Physics 722/822

Sebastian Kuhn
What Do we Need?

- (Polarized) Beam
  - Electrons or muons $\rightarrow \gamma^*$; pions, protons, antiprotons, light nuclei, heavy ions...
- (Polarized) Targets (or counterrotating beams)
  - Protons, deuterons, $^3$He, heavier nuclei, heavy ions, antiprotons
- Detectors
  - For scattered/produced electrons/muons/... and hadrons/nuclei
- Facilities
Low-Medium Energy Accelerators

- (Tandem) Van de Graaf
- Cyclotron
- Synchrotron

Elektronen-Stretcher-Anlage (ELSA)
High-energy Accelerators

Accelerating cavity: disk loaded cylindrical wave guide
use TM$_{01}$ mode to get a longitudinal electric field
match phase and velocity

Surf the microwaves!

DESY

Jefferson Lab
High Energy Accelerators – 2 Examples

• **Superconducting Linear Accelerators (CEBAF at JLab)**
  - 2K niobium cavities, very low resistive losses
  - Recirculate few times, 100’s of µA
  - High gradient (5-50 MeV/m ⇒ 4-12 GeV)
  - CW extracted beam on external targets
  - Thick targets ⇒ high luminosity

• **Storage rings (HERA at DESY, RHIC at BNL, LHC at CERN)**
  - Large circulating currents (mA)
  - Recirculate millions of times
  - Require only modest (re)acceleration
  - CW internal beam on thin gas targets or counterrotating beams
    (typically lower Luminosity)
1. Charged particle passes through wire chamber and knocks out electrons from the gas.
2. Electrons drift in the E field to the cathode wire, colliding with gas molecules.
3. Close to the wire, the mean free path times the electric field is large enough to ionize the gas molecules. *Avalanche!*
4. Read the signal on the cathode wire (time gives distance).

Similar: Gas Electron Multipliers, μMEGAs,…

Applications: VDC, Multi-layer drift chamber (track $\rightarrow$), TIPTP Projection Chamber
Typical Detector Elements

**Scintillators**: time ($\Rightarrow \beta \Rightarrow$ particle type) and energy measurement
(typical resolution: down to 50-100 ps for plastic)
- Typically a doped plastic or crystal (eg: Ge, NaI, BaF₂)
- Charged particle passes through scintillator (or neutral particle interacts) and excites atomic electrons. These de-excite and emit light.
- Minimum energy loss (when $\beta\gamma \approx 1$) is $dE/dx = 2 \text{ MeV}/(\text{g/cm}^2)$

**Cherenkov counter**: threshold velocity measurement.
- Typically an empty box with smoke (ie: a gas) and mirrors
- Local light speed is $v = 1/n < c$
- Particles travelling faster than $v$ will emit Cherenkov light (an electromagnetic ‘sonic boom’) $\Rightarrow$ threshold CC (yes/no)
- The opening angle of the Cherenkov cone is related to the particle’s velocity $\Rightarrow R_{\text{Imaging CH}} \text{erenkov}$ (measure $\beta \Rightarrow$ particle type)

Also: Transition Radiation Detector, DIRC,
Typical Detector Elements

**Electromagnetic shower counters:**
measure energy (+ time), discriminate electrons and detect neutral particles

- Electrons and photons passing through material shower
- After one radiation length of material on average:
  - Electrons emit a bremsstrahlung photon
  - Photons convert to an electron/positron pair or Compton-scatter
- After ≈10 radiation lengths, one e⁻ or γ is now ~1000 particles
- Simple design: alternating layers of lead \((R_L = 6 \text{ mm})\) and scintillant
  - Higher resolution: Heavy metal glass (Pb glass, PbWO₄) combine both
    - Particles shower in the lead
    - Charged particles deposit energy in the scintillant

Also: Hadronic Calorimeter, μ counter...
Detectors+Magnets = Spectrometers

SLAC End Station A – where the quarks were discovered experimentally
Jefferson Lab Hall A (Hall C similar)

Typically small acceptance but high resolution, very good shielding (→ high luminosity)
Large Acceptance Spectrometers

COMPASS

HERMES
CLAS12

**Base equipment**
- Forward Detector
  - TORUS magnet
  - Forward vertex tracker
  - HT Cherenkov Counter
  - Drift chamber system
  - LT Cherenkov Counter
  - Forward ToF System
  - Preshower calorimeter
  - E.M. calorimeter
- Central Detector
  - SOLENOID magnet
  - Barrel Silicon Tracker
  - Central Time-of-Flight

**Additional equipment**
- Micromegas (CD & FD)
- RICH counter (FD)
- Neutron detector (CD)
- Small angle tagger (FD)
Jefferson Lab in Perspective

Past:
- $Q^2 = <6$ GeV$^2$
- $x > 0.1...0.6$
- $W = 0.9...3$ GeV

Now:
- $Q^2 = 1...13$ GeV$^2$
- $x = 0.06...0.8$
- $W < 4$ GeV
Count rate (L = luminosity):

\[ \dot{N} = P \cdot \dot{n}_{el} = \Delta \sigma \cdot \frac{N_T}{A} \cdot \dot{n}_{el} = \Delta \sigma \cdot \frac{N_T}{A} \cdot \frac{I}{e} = \Delta \sigma \cdot L \]

In general:

\[ \Delta \sigma = \Delta \sigma (E'...E', \theta_e...\theta_e + \Delta \theta_e, \varphi...\varphi + \Delta \varphi) \]

Limit of infinitesimal acceptance:

\[ \Delta \sigma = \frac{d^{3}\sigma}{dE'd\theta_e d\varphi} (E', \theta_e, \varphi) \Delta E' \Delta \theta_e \Delta \varphi = \frac{d\sigma}{dE'd\Omega} \Delta E' \Delta \Omega \]

(Use Jacobian to transform variables)

In case of more particles/observables and finite phase space:

\[ \Delta \sigma = \iiint_{\text{Phase Space}} \frac{d^n\sigma}{dk_1 dk_2 ... dk_n} (k_1...k_n) \text{Acc}(k_1...k_n) dk_1 dk_2 ... dk_n \]

where Acceptance is defined by detector boundaries and bin sizes as well as number of observed particles

(inclusive = only 1, semi-inclusive, exclusive)
Kinematic Variables

\[ \nu = E - E' = |\vec{k}| - |\vec{k}'| \]
\[ \vec{q} = \vec{k} - \vec{k}'; \quad q^\mu = (\nu, \vec{q}) = k^\mu - k'^\mu \]
\[ Q^2 = -(k - k')^2 = \vec{q}^2 - \nu^2 \approx 4EE'\sin^2 \frac{\theta_c}{2} \]
\[ y = \frac{q^\mu P_\mu}{k^\mu P_\mu} = \frac{\nu}{E}; \quad x = \frac{Q^2}{2q^\mu P_\mu} = \frac{Q^2}{2M \nu} \]
\[ W = \sqrt{(P_\mu + q^\mu)^2} = \sqrt{M^2 + (1/x - 1)Q^2} \]

Inclusive

Lepton variables

Semi-Inclusive

Hadron variables

\[ P_{\text{had}}^\mu = \left( z\nu, \sqrt{z^2 \nu^2 - m_h^2 - P_{hT}^2}, \vec{P}_{hT} \right) \]
\[ z = E_h / \nu \]
\[ \vec{P}_{hT} = |\vec{P}_{hT}| (\sin \theta_h \cos \phi_h, \sin \theta_h \sin \phi_h) \]
\[ x_F = \frac{\vec{P}_{h}^{c.m.} \cdot \hat{q}}{|\vec{P}_{h}^{c.m.}|_{\text{max}}} \]

Rapidity

\[ \operatorname{artanh} \frac{|\vec{p}| c}{E} = \frac{1}{2} \ln \frac{E + |\vec{p}| c}{E - |\vec{p}| c} \]

Pseudo-Rapidity

\[ \eta = - \ln \left[ \tan \left( \frac{\theta}{2} \right) \right] \]
Electron Scattering - what can we measure?

Accel. \[ \rightarrow \] \[ E \rightarrow e^- \]
\[
\text{N centers} \]
\[
\text{TARGET} \]
\[
\theta \rightarrow \text{Det.} \]
\[
\text{N} \rightarrow E' \rightarrow \Delta \theta \]

What is the likelihood to find the electron scattered into the detector? \( P \)

\[
P \sim N_T \cdot L = \frac{N_T}{A \cdot L} \cdot L = \frac{N_T}{A} \]

\[ \Rightarrow \text{call } \Delta \sigma = \frac{P}{(N_T)} \text{ (cross section)} \]

\( \Delta \sigma \) \text{ DEPENDS on the kinematics } (E, E', \theta) \text{ and is } \propto \text{proportional to SIZE of kinematic bin spanned by the detector} \]

*Note: \( \frac{N_T}{A} \sim \rho \left[ \frac{g}{cm^3} \right] \cdot L [cm] \cdot \text{Avogadro} \)

\[
\frac{\text{Avogadro}}{\text{Atomic Weight} \cdot \text{Lu}} \]

Electron Scattering - Theorist's View

\( \psi_{in} \rightarrow \psi_f \)

Target (1 center)

What is the transition rate \( W_i \rightarrow f \)?

\[
N_{i \rightarrow f} = N_{i, \text{in}} \cdot P(i \rightarrow f) = \frac{N_{i, \text{in}}}{A} \cdot N_T \cdot \Delta \sigma
\]

\[ = \frac{I_{e, \text{in}}}{A} \cdot N_T \cdot \Delta \sigma = \left( \frac{I_{e, \text{in}}}{A} \right) \cdot N_T \cdot \Delta \sigma \]

\[ \Rightarrow W_i \rightarrow f = \frac{I_{e, \text{in}}}{A} \cdot N_T \cdot \Delta \sigma \]

Fermi's GOLDEN Rule:

\[ W_i \rightarrow f = \frac{2\pi}{h} \left( M_{fi} \right) \Delta \phi \]

\[ M_{fi} = \langle \psi_f | H_{int} | \psi_{in} \rangle \]
Elastic Scattering - Feynman diagram

Form Factors

Low-medium energy: Distribution of charge and magnetism inside the hadron

$$\nabla^2 V(r) = -4\pi \rho(r) \quad \Rightarrow$$

$$q^2 V \propto \int e^{iqr} \rho(r) d^3r = F(q)$$

$$\mathcal{H} \approx -eV \propto \frac{F(q)}{q^2}$$

$$\frac{d\sigma}{d\Omega} \propto |\langle f | \mathcal{H} | i \rangle|^2 \propto \frac{F^2(q)}{q^4}$$

Ex: $$\rho(r) = a e^{-ar} \Rightarrow F(q^2) = (1 + q^2/a^2)^{-2} \text{ (Dipole Form)}$$

High energy: "Stability" of internal structure against hard "blows"

Can the hadron absorb a high momentum virtual photon without breaking apart?

magnetic interaction
$\mu$ due to electron spin
$\chi$ due to target spin
Elastic Scattering off Nuclei

-> Nuclear radii, density distributions
Inelastic Scattering off Nuclei

$e + ^{12}\text{C} \rightarrow e' + ^{12}\text{C}$

$p_e = 495 \text{ MeV/c}$
$\theta = 65.4^\circ$
$lq/lh = 2.68 \text{ fm}^{-1}$

$->$ Excited states of nuclei, transition matrix elements
Quasi-elastic Scattering off Nuclei

$E^f = 3.5 \text{ GeV}$, $\nu = 0.932 \text{ GeV}$, $q = 1.62$

$|p_f| = 2.00 \text{ GeV/c}$

$\frac{p_f^2}{2m} \approx 0.02$

$2\sqrt{q_f} \approx 0.65$

(actually $q_f \approx 100 \text{ km/d}$)

-> scattering of single nucleons bound in the nucleus; momentum distribution, energy of residual system
Quasi-elastic Scattering off Nuclei with coincident detection of proton

-> momentum and energy distribution of bound proton; “Spectroscopic Factor”

Figure 5-12: Extracted $\rho(p_m)$ for the 1s shell of $^{12}\text{C}$. The spectral function has been integrated over $30 < E_m < 80$ MeV. The solid line represents the result of the PWIA calculation, normalized to the measured transparency; the data points are shown with statistical errors only.

Fig. 17.8 Distribution of energies required to remove a proton from $^{16}\text{O}$ by the (p,2p) reaction, showing the single-particle states (Tyrén et al. 1958).
Quasi-elastic Scattering off Nuclei with coincident detection of proton

Spectroscopic factors show that roughly 30-35% of nucleons are not in (close in energy to) a well-defined shell.
Some kinematics - in the lab system

\[ k^\mu = (E, 0, 0, E) = (E, \vec{k}) \]

\[ k'^\mu = (E', E's\sin\theta, 0, E'\cos\theta) = (E', \vec{k}') \]

\[ q^\mu = (E-E', -E's\sin\theta, 0, E-E'\cos\theta) = (\nu, \vec{q}) \]

\[ Q^2 = -q^\mu q_\mu = -\nu^2 + \vec{q}^2 = (E-E'\cos\theta) + E'^2\sin^2\theta - (E-E') \]

\[ = E^2 - 2EE'\cos\theta + E'^2 - E^2 + 2EE' \]

\[ = 2EE'(1-\cos\theta) = 4EE'\sin^2\theta \]

\[ p^\mu = (M, 0, 0, 0) = (M, \vec{0}) \]

\[ p'^\mu = p^\mu + q^\mu = (M+\nu, \vec{q}) \]

\[ p^\mu p'_\mu = W^2 = M^2 + 2M\nu + \nu^2 - \frac{Q^2}{4} \]

\[ = M^2 + 2M\nu - Q^2 \]

Elastic Scattering: \( W^2 = M^2 \frac{Q^2}{2M} \Rightarrow \frac{Q^2}{2M} \Rightarrow \frac{Q^2}{2M^2} = 1 \)

Elastic Cross section - final form

\[ \Delta\sigma = \frac{4\pi^2\alpha^2(hc)^2}{Q^4} \cos^3\frac{\theta}{2} \left[ \frac{G_E^2 + \frac{1}{4} G_M^2}{1 + \frac{Q^2}{4M^2}} + 2\frac{G_E}{G_M} \tan\frac{\theta}{2} \right] \]

\[ \cdot E^2 \Delta\Omega \]

\( \tau^2 = \frac{\nu^2}{Q^2}, G_E(Q^2), G_M(Q^2) \): Form Factors

Dirac Particle: \( G_E = G_M = 1 \) (const)

Anomalous magnetic moment: \( G_M \propto (1+\frac{Q^2}{a^2})G_E \)

Extended Charge distribution:

\[ G_E(Q^2) \approx \text{Fourier transform of} \; p(x) \]

Ex: \( p(x) \approx \frac{a^3}{3\pi} e^{-ax^2} \Rightarrow G_E(Q^2) \approx \left(1 + \frac{Q^2}{a^2}\right)^{-\frac{1}{2}} \)

(C Dipole Form). \( a^2 = 0.71 \text{ GeV}^2 \)
Elastic Form Factors of Hadrons

- Example: Pion
- Nucleons discussed in separate presentation
(e,e') spectrum

**Generic Electron Scattering at fixed momentum transfer**
Inelastic Cross Section - What's different

Phase space factor \( d^3k' = k'^2 d\Sigma_k, dk' \)
(\( S\)-function drops since \( E'\) can have any value)

- conversion

\[
\begin{align*}
  k'^2 d\Sigma d\Omega & = E'^2 dE' d\omega \Omega d\theta d\phi \\
  & = -2E'^2 d\omega \Omega dE' \\
  & = \frac{\pi E'}{E} (-2E d\omega \Omega) dE' \\
  & = \frac{\pi E'}{E} \frac{dE'}{dQ^2} d\nu \quad \text{since } \nu = E - E'
\end{align*}
\]

Interpretation of \( W_i(Q^2, \nu) \)

(transverse) electromagnetic transition probability to final state characterized by \( \nu \), with resolution \( \sim \frac{1}{\sqrt{Q^2}} \)

Transition to discrete final states (resonances):

\[
W_i(Q^2, \nu) \propto \tau G_m^2 (\text{transition}) \cdot S(\nu - \nu_R)
\]

\( \nu_R \) is given by

\[
M_{\text{Res}}^2 \approx W_{\text{Res}}^2 = \frac{M^2 + 2M\nu_k - Q^2}{2}
\]

In reality: resonances have finite width \( \Gamma \sim \frac{1}{\text{life}} \Rightarrow \rightarrow n\)

(except elastic transition) \( S\)-function

Threshold: \( W_{\text{min}}^2 = (M + m_{\pi})^2 = (1.08 \text{ GeV})^2 \)

Replace \( \frac{G_E^2 + \tau G_m^2}{1 + \tau} \) with \( W_2(Q^2, \nu) \)

and \( \tau G_m^2 \) with \( W_1(Q^2, \nu) \)

\[
\Delta \sigma = \frac{4\pi \alpha^2(\Delta Q^2)^2}{Q^4} \frac{W_2^2 + 2\tan^2 \theta \nu W_1}{E} \Delta Q^2 \Delta \nu
\]

\(*\Rightarrow W_2(Q^2, \nu) = \frac{G_E^2 + \tau G_m^2}{1 + \tau} \cdot S(\nu - \nu_R)\)

\( W_1(Q^2, \nu) = \tau G_m^2 \cdot S(\nu - \nu_R) \)
Elastic scattering

\[
\frac{\Delta \sigma}{\Delta \Omega} = \frac{4\alpha^2 (hc)^2 E'^2 \cos^2 \theta/2}{Q^4} \frac{E'}{E} \left( \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau \tan^2 \theta/2 \frac{G_M^2(Q^2)}{2} \right)
\]

where \( \tau = \nu^2/Q^2 \).

Inelastic Scattering

\[
\frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} = \frac{4\pi \alpha^2 (hc)^2 E' \cos^2 (\theta/2)}{Q^4 E} \left( W_2(Q^2, \nu) + 2 \tan^2 (\theta/2) W_1(Q^2, \nu) \right)
\]

\[
\frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} = \frac{4\pi \alpha^2 (hc)^2 E' \cos^2 (\theta/2)}{Q^4 E} \frac{W_1(Q^2, \nu)}{\epsilon(1 + \tau)} \left( 1 + \epsilon R(Q^2, \nu) \right)
\]

with \( \epsilon = (1 + 2(1 + \tau) \tan^2 (\theta/2))^{-1} \)

\[
\frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} = \frac{4\pi \alpha^2 (hc)^2 E' \cos^2 (\theta/2)}{Q^4 E} \left( \frac{1}{\nu} F_2(x) + 2 \tan^2 (\theta/2) \frac{1}{M} F_1(x) \right)
\]

\[
F_1(x) = MW_1(Q^2, \nu) \quad F_2(x) = \nu W_2(Q^2, \nu)
\]


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**Picture from F. Gross,**

«Making the case for Jefferson Lab»

*The first decade of Science at Jefferson Lab*  
*JoP, Conf. Series 299 (2011) 012001*
p(e,e') at 9.71 GeV and 7 degree

\[
\frac{d\sigma}{dQ^2 dW} [\text{nb/sr/GeV}]
\]

\[
W, Q^2
\]

\[
W^2 [\text{GeV}^2] = M_{\text{final}}^2
\]

\[
S_{11}/D_{13}, F_{15} \quad \Delta_{23}
\]
Deep Inelastic Scattering (DIS)

Reminder: Elastic scattering

\[
\frac{\Delta \sigma}{\Delta \Omega} = \frac{4\alpha^2 (\hbar c)^2 E' \cos^2 \theta/2}{Q^4} \frac{E'}{E} \left( \frac{G_E^2(Q^2)}{1 + \tau} + 2\tau \tan^2 \theta/2 \ G_M^2(Q^2) \right)
\]

Elastic scattering from quarks:

\[
\Delta \sigma = \frac{4\pi z_q^2 \alpha^2 (\hbar c)^2 E' \cos^2 (\theta/2)}{Q^4 E} \left( q(x) \Delta x + 2\nu^2/Q^2 \tan^2 (\theta/2) q(x) \Delta x \right) \Delta Q^2.
\]

We can use the relation \( \Delta x = -Q^2/(2M\nu^2) \Delta \nu = -x \Delta \nu/\nu \) to rewrite this as

\[
\frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} = \frac{4\pi \alpha^2 (\hbar c)^2 E' \cos^2 (\theta/2)}{Q^4 E} \left( \frac{x}{\nu} z_q^2 q(x) + \frac{1}{M} \tan^2 (\theta/2) z_q^2 q(x) \right).
\]

Reminder: IN-Elastic scattering

\[
\frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} = \frac{4\pi \alpha^2 (\hbar c)^2 E' \cos^2 (\theta/2)}{Q^4 E} \left( \frac{1}{\nu} F_2(x) + 2 \tan^2 (\theta/2) \frac{1}{M} F_1(x) \right)
\]

\[
\Rightarrow \quad F_1(x) = \frac{1}{2} \left( \frac{4}{9} [u(x) + \bar{u}(x)] + \frac{1}{9} [d(x) + \bar{d}(x) + s(x) + \bar{s}(x)] + ... \right) \quad \text{No } Q^2!
\]

NOTE: \( \Rightarrow F_2 = 2xF_1 \)

Callan-Gross
Quark-Parton Structure of the Proton

\[
q(x) \sim \left\langle P,s \left| \bar{q} \gamma^\mu q \right| P,s \right\rangle
\]

\[
\Delta q(x) = q \uparrow \uparrow (x) - q \uparrow \downarrow (x) + \bar{q} \uparrow \uparrow (x) - \bar{q} \uparrow \downarrow (x) \sim \left\langle P,s \left| \bar{q} \gamma^\mu \gamma^5 q \right| P,s \right\rangle
\]

“axial charge”, similarly \( G(x) \) and \( \Delta G(x) \) for gluons

Spin Sum Rule:

\[
S_P = \frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \Delta G + L_q + L_G
\]
Structure Functions

\[
\frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} = \frac{4 \pi \alpha^2 (\hbar c)^2 E' \cos^2(\theta/2)}{Q^4 E} \left( \frac{1}{\nu} F_2(x) + 2 \tan^2(\theta/2) \frac{1}{M} F_1(x) \right)
\]

\[
\frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} \uparrow\!\downarrow = \frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} \uparrow\!\uparrow = \frac{4 \pi \alpha^2}{M \nu Q^2 E^2} \left[ (E + E' \cos \theta) g_1 - 2 x M g_2 \right]
\]

Unpolarized: \( F_1(x, Q^2) \) and \( F_2(x, Q^2) \)

Polarized: \( g_1(x, Q^2) \) and \( g_2(x, Q^2) \)

Parton model:

\[
F_1(x) = \frac{1}{2} \sum_i e_i^2 q_i(x) \quad \text{and} \quad F_2(x) = 2 x F_1(x) \quad \text{\( i = \) quark flavor} \quad e_i = \text{quark charge}
\]

\[
g_1(x) = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x) \quad \text{and} \quad g_2(x) = 0
\]

the structure functions \( g_1 \) and \( g_2 \) are linear combinations of \( A_1 \) and \( A_2 \)

\[
g_1(x, Q^2) = \frac{\tau}{1 + \tau} (A_1 + \frac{1}{\sqrt{\tau}} A_2) F_1
\]

\[
g_2(x, Q^2) = \frac{\tau}{1 + \tau} (\sqrt{\tau} A_2 - A_1) F_1 \quad \tau = \frac{\nu^2}{Q^2}
\]
Quark-Parton Structure of the Proton

\[ q(x) \sim \left\langle P,s \mid \bar{q}\gamma^\mu q \right| P,s \right\rangle \]

\[ \Delta q(x) = q \uparrow \uparrow (x) - q \uparrow \downarrow (x) + \bar{q} \uparrow \uparrow (x) - \bar{q} \uparrow \downarrow (x) \sim \left\langle P,s \mid \bar{q}\gamma^\mu \gamma^5 q \right| P,s \right\rangle \]

“axial charge”, similarly \( G(x) \) and \( \Delta G(x) \) for gluons

Spin Sum Rule:

\[ S_P = \frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \Delta G + L_q + L_G \]
Structure Functions

\[
\frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} = \frac{4\pi\alpha^2(\hbar c)^2E'\cos^2(\theta/2)}{Q^4E} \left( \frac{1}{\nu} F_2(x) + 2\tan^2(\theta/2) \frac{1}{M} F_1(x) \right)
\]

\[
\frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} \downarrow \uparrow - \frac{\Delta \sigma}{\Delta Q^2 \Delta \nu} \uparrow \downarrow = \frac{4\pi\alpha^2}{M\nu Q^2 E^2} \left[ (E + E'\cos\theta) g_1 - 2xMg_2 \right]
\]

Unpolarized: \( F_1(x,Q^2) \) and \( F_2(x, Q^2) \)

Polarized: \( g_1(x,Q^2) \) and \( g_2(x, Q^2) \)

Parton model:

\[
F_1(x) = \frac{1}{2} \sum_i e_i^2 q_i(x) \quad \text{and} \quad F_2(x) = 2x F_1(x)
\]

\[
g_1(x) = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x) \quad \text{and} \quad g_2(x) = 0
\]

the structure functions \( g_1 \) and \( g_2 \) are linear combinations of \( A_1 \) and \( A_2 \)

\[
g_1(x, Q^2) = \frac{\tau}{1 + \tau} \left( A_1 + \frac{1}{\sqrt{\tau}} A_2 \right) F_1
\]

\[
g_2(x, Q^2) = \frac{\tau}{1 + \tau} \left( \sqrt{\tau} A_2 - A_1 \right) F_1
\]

\[
\tau = \frac{\nu^2}{Q^2}
\]